Targeted Financial Conditions Indices and Growth-at-Risk

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- Recent applications to 'at-risk' modeling: tighter financial conditions associated with downside risks to activity, etc.
- Issue: FCIs typically designed to capture common variation in financial series (e.g. PCA), not tailored for specific applications (exceptions discusssed below)

This Paper

Targeted Financial Conditions Indices (TFCIs). FCIs that are:

- Extracted from a large panel of financial time series
- Tailored to explain or forecast any part of the distribution of a variable of interest
- How: novel methodology based on rotation of PCA scores

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Application to US GDP-at-risk:

- Revisit Adrian et al. (2019); Adams et al. (2021) by extracting TFCIs from dataset underlying Chicago Fed's NFCI
- TFCI for US GDP downside risk 'nicer' in real time than NFCI/PCA; nuances between left tail and median TFCIs
- Better density forecasting performance than alternatives

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- **Maybe related**: density forecast combination using weighted scoring rules (Opschoor et al., 2017)

Targeted Factor Extraction I

In words:

- **Goal**: extract one or more common factors from a panel of financial series subject to the restriction that factor(s) should maximize forecasting power for a given quantile and horizon of a (macro) target variable
- **How it works**: start from PCA scores, then rotate them based on a suitable loss function
- **Application**: new TFCIs from NFCI dataset, tailored to forecast US GDP growth distribution

Targeted Factor Extraction II

Let $\mathbb{Z}$ be a $T \times n$ panel of series that have mean zero and (for simplicity) unit variance, and $\mathbb{F}$ be any factor decomposition of $\mathbb{Z}$, e.g. the full set of (standardised) PCA scores. Then

$$z_t = \Lambda f_t \tag{1}$$

Λ: $n \times n$ matrix of factor loadings

Targeted Factor Extraction III

Let $G(\theta)$ be a $n \times n$ orthonormal matrix parametrised by the vector of angles θ. Then

$$z_t = \Lambda f_t = \Lambda G(\theta)\, G'(\theta)\, f_t \equiv \tilde{\Lambda}(\theta)\, \tilde{f}_t(\theta) \tag{2}$$

gives me another admissible factor decomposition

Targeted Factor Extraction IV

$G(\theta)$ is the product of suitably chosen Givens matrices:

$$G(\theta) = \prod_{i=1}^{\min(s,n-1)} \prod_{j=i+1}^{r} G_{i,j}(\theta_{i,j}) \tag{3}$$

where the only non-zero elements of $G_{i,j}(\theta_{i,j})$ are $g_{kk} = 1,\ k \neq i,j$, $g_{kk} = \cos\theta_{i,j},\ k = i,j$ and $g_{ji} = -g_{ij} = -\sin\theta_{i,j}$.
$r \leq n$: dimension of the column (sub-) space of Λ that is rotated by $G(\theta)$;
$s < r$: number of factors included in the regression models.
For us: $s = 1$, r picked dynamically from a grid in OOS exercise

Targeted Factor Extraction V

Example: $n = 4$, $r = 3$, $s = 1$

$$G(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & 0 & 0 & -\sin\theta_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_3 & 0 & 0 & \cos\theta_3 \end{bmatrix}$$

Targeted Factor Extraction VI

Conditional quantile function of variable y_{t+h} for quantile τ:

$$
\begin{aligned}
Q\left(y_{t+h}|w_t,\ \tilde{f}_t\left(\theta_\tau\right), \tau\right) &= \alpha'_\tau w_t + \gamma'_\tau\left(\theta_\tau\right) s_\tau \tilde{f}_t\left(\theta_\tau\right) \\
&= \begin{bmatrix} \alpha'_\tau & \gamma'_\tau\left(\theta_\tau\right) \end{bmatrix} \begin{bmatrix} w_t \\ s_\tau \tilde{f}_t\left(\theta_\tau\right) \end{bmatrix} \\
&\equiv \beta'_\tau\left(\theta_\tau\right) x_t\left(\theta_\tau\right) \qquad (4)
\end{aligned}
$$

w_t: any variables not included in z_t, e.g. lagged values of y_t;
s_τ: $s \times n$ selection matrix (here $s = 1$, so just picks first column)

Targeted Factor Extraction VII

$\hat{\beta}_{\tau}(\theta_{\tau})$ solves the quantile regression problem

$$\hat{\beta}_{\tau}(\theta_{\tau}) = \arg\min_{\beta_{\tau}(\theta_{\tau})} \frac{1}{T} \sum_{t=1}^{T} \rho_{\tau}\left(y_t - \beta_{\tau}'(\theta_{\tau})\, x_t(\theta_{\tau})\right) \tag{5}$$

$\rho_{\tau}(u) = u(\tau - \mathbb{I}(u < 0))$: check function

Targeted Factor Extraction VIII

Our object of interest is θ_τ^*, the set of angles, and therefore rotated factors $\tilde{f}_t\left(\theta_\tau^*\right)$, that, given a choice of r and s, maximises the fit of the model:

$$\theta_\tau^* = \arg\min_{\theta_\tau} \frac{1}{T} \sum_{t=1}^{T} \rho_\tau \left(y_t - \hat{\beta}_\tau' \left(\theta_\tau\right) x_t \left(\theta_\tau\right) \right) \tag{6}$$

Solved numerically

Targeted Factor Extraction IX

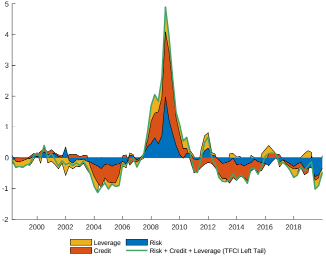
To recap, for each horizon and quantile of interest, we have a fitted model of the following form:

$$\hat{Q}\left(\Delta gdp_{t+h,t}|x_t\left(\theta^*_\tau\right),\tau\right) = \beta'_\tau\left(\theta^*_\tau\right)\begin{bmatrix}1\\ \Delta gdp_{t,t-h}\\ \tilde{f}_t\left(\theta^*_\tau\right)\end{bmatrix} \qquad (7)$$

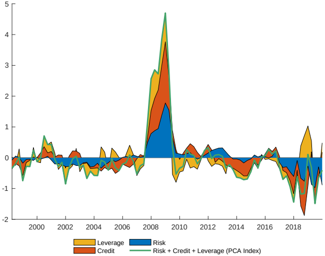
$\Delta gdp_{t+h}, t$: cumulative GDP growth between t and $t+h$;

TFCIs and Growth-at-Risk I

Figure: Left tail TFCI and PCA index - 1 Year Ahead



(a) TFCI left tail index

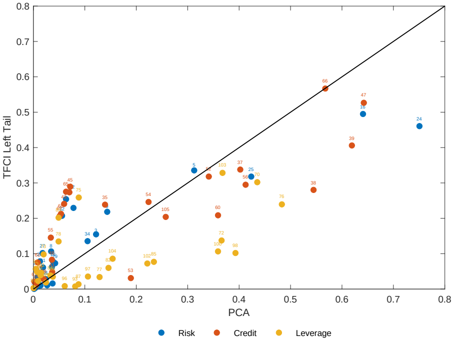


(b) PCA index

Note: The figures plot the real-time (ex ante) time series of the a) Left Tail TFCI (5th Percentile) and b) PCA Index, when forecasting 1 year ahead. The indices comprise three subgroups: leverage (yellow), credit (red) and risk (blue). Both indices have been standardized.

TFCIs and Growth-at-Risk II

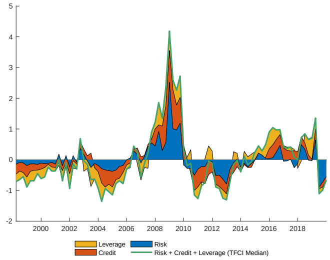
Figure: Average Real-Time Squared Loadings, 1 Year Ahead



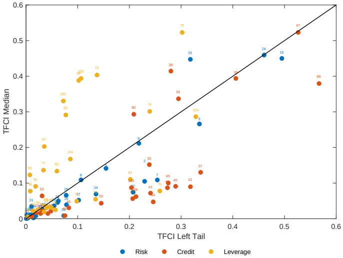
Note: Sample averages of real-time (ex ante) squared loadings for the PCA vs. Left Tail TFCI indices, when forecasting 1 year ahead. Each dot corresponds to one component series.

TFCIs and Growth-at-Risk III

Figure: Median TFCI - 1 Year Ahead



(a) Real-time median TFCI and contributions



(b) Average real-time TFCI squared loadings, median vs. left tail

Note: Panel (a) shows the real-time Median TFCI time series and the contributions of each subgroup. Panel (b) compares the sample averages of real-time squared loadings of the Left Tail TFCI and the Median TFCI.

Forecast Evaluation

Recursive estimation on pseudo-real-time data:

- 1971Q1:2019Q4 data
  - include only financial variables that were available for at least 50% of the sample up until the forecast date
- 1999Q1:2019Q4 forecast evaluation sample
- Benchmarked against PCA, Giglio et al. (2016) (GKP), NFCI (real-time TBU)

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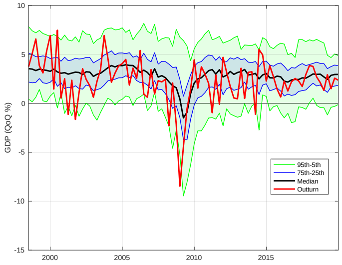
Evaluate local fit (mean tick loss), weighted probability scores (Gneiting and Ranjan, 2011), calibration (PITs):

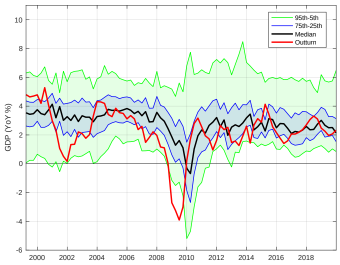
- TFCIs invariably as good or better fit across the distribution
- Statistically significant improvements compared to all alternatives
- Better calibration

Forecast Evaluation Results I

Figure: TFCI Forecasts vs. Outturns



(a) 1 quarter ahead



(b) 1 year ahead

Note: Selected predictive quantiles based on TFCIs over time against outturns, a) 1 quarter and b) 1 year ahead. The QoQ growth rate is seasonally-adjusted and annualized.

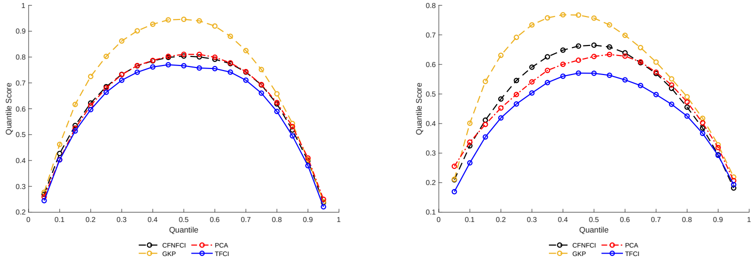
Forecast Evaluation Results II

$$QS_{\tau,h} = \rho_\tau \left(y_{t_v+h} - \hat{P}^{-1}_{v,h}(\tau) \right) \quad (8)$$

The **quantile score** penalises outturns that are more extreme than the predictive quantile $\hat{P}^{-1}_{v,h}(\tau)$

Forecast Evaluation Results III

Figure: Average Quantile Scores



(a) 1 Quarter Ahead (b) 1 Year Ahead

Note: Average quantile scores for all models, a) 1 quarter and b) 1 year ahead. Lower values represent better performance.

Forecast Evaluation Results IV

$$GR_{\tau,h} = \int_0^1 QS_{\tau,h} w(\tau)\, d\tau \tag{9}$$

where w are non-negative weight functions on the real line. Differences are testable within standard Diebold and Mariano (1995); Amisano and Giacomini (2007) framework.

Forecast Evaluation Results V

Table 1 - Average GR Scores and GR Scores Ratios

	1 Quarter Ahead				1 Year Ahead			
	TFCI	GKP	PCA	NFCI	TFCI	GKP	PCA	NFCI
Uniform (w_0)	0.58	**1.19**	**1.05**	**1.05**	0.42	**1.32**	**1.12**	**1.14**
Center (w_1)	0.11	**1.20**	**1.05**	**1.04**	0.08	**1.33**	**1.11**	**1.15**
Tails (w_2)	0.13	**1.16**	**1.05**	**1.06**	0.10	**1.29**	**1.16**	1.12
Right Tail (w_3)	0.18	**1.16**	**1.06**	**1.05**	0.13	**1.23**	**1.11**	**1.10**
Left Tail (w_4)	0.18	**1.21**	1.04	1.05	0.13	**1.40**	**1.15**	**1.17**

Note: The table shows average Gneiting and Ranjan (2011) scores for our model (TFCI) and for different weighting functions: $w_0 = 1$; $w_1(\tau) = \tau(1-\tau)$; $w_2(\tau) = (2\tau - 1)^2$; $w_3(\tau) = \tau^2$; $w_4(\tau) = (1-\tau)^2$. Scores for the remaining models are reported as ratios to the respective TFCI score. A ratio > 1 indicates that a model performs worse than the TFCI, and numbers in bold denote statistically significant differences at the 10% confidence level or better using the same testing strategy as Diebold and Mariano (1995), Amisano and Giacomini (2007).

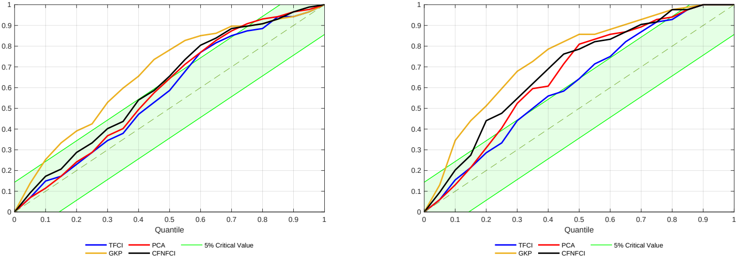
Forecast Evaluation Results VI

$$PIT_{v,t+h} = \int_{-\infty}^{y_{t+h}} \hat{p}_{v,h}(x \mid y_t)dx \equiv \hat{P}_{v,h}(y_{t+h} \mid y_t) \qquad (10)$$

where $\hat{p}_{v,h}(\cdot)$ is the PDF estimated in vintage v for forecast horizon h, and $\hat{P}_{v,h}(\cdot)$ the corresponding CDF. Testing as in Rossi and Sekhposyan (2014)

Forecast Evaluation Results VII

Figure: Probability Integral Transforms (PITs)



(a) 1 Quarter Ahead

(b) 1 Year Ahead

Note: The charts show the probability integral transforms (PITs) for each model and for both predictive horizons. The green band represents the 10% critical region, as in Rossi and Sekhposyan (2014). An ideally-calibrated model lies on the diagonal throughout the quantiles, so the closer to it, the better.

Conclusion

- Novel approach to extract factors from large data sets that maximize covariation with the quantiles of a target distribution of interest
- We build TFCIs for the quantiles of future US GDP growth from ChiFed's NFCI dataset
- Leverage indicators co-move more with median of predictive distribution, credit and risk more informative about downside risks
- Better density forecasting performance than alternatives

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