Imperfect Risk-Sharing and the Business Cycle

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Imperfect risk-sharing and business cycles

- Does households' heterogeneity matter for business cycle analysis?
- New class of models (HANK): answer is "yes"
  - Amplifies/dampens effects of aggregate shocks
  - Transmission mechanisms of fiscal and monetary policy
- Challenging to assess these channels quantitatively
  - Answers depend on set of financial assets and risk-sharing mechanisms
  - Hard to combine realistic asset markets with standard business cycle models
- We develop a framework robust to these considerations
  1. Measure degree of imperfect risk-sharing from households' choices
  2. Provide framework to assess its macroeconomic implications

What we do

Our method has two steps

1. Accounting procedure for micro data
   - *Prototype* model: households' decision problem under complete markets
   - "Wedges" distort risk-sharing and optimal labor supply
   - Measure individual wedges that account for micro data (CEX, PSID)

2. Combine micro wedges with a *class* of HANK models
   - Equivalent representation: RA economy with preference "shocks"
     - State-dependent discount rate
     - State-dependent disutility of labor
   - Preference "shocks" are simple statistics of micro wedges

Counterfactuals: what would have happened with perfect risk-sharing?

What we find

Imperfect risk-sharing $\rightarrow$ drop in aggregate demand in Great Recession

- Mostly due to increase in discount rate in equivalent RA representation
- Higher discount rate increases propensity to save of RA and reduce aggregate demand. At the ZLB, effects sizable

What in the micro data is suggesting higher propensity to save?

- Substantial variation in consumption shares during Great Recession
- Consumption share of income rich/asset poor hh's decreased the most
    - Increase in saving rates for this group in 2008
    - Indication of heightened saving motives

Literature

1. Accounting procedures in macro
   - Chari, Kehoe and McGrattan (2008), Hsieh and Klenow (2009), Boerma and Karabarbounis (2017), Ohanian et al. (2018)

2. Aggregation results for models with incomplete markets
   - Nakajima (2005), Krueger and Lustig (2010), Werning (2016)

3. New Keynesian models with incomplete markets
   - Guerrieri and Lorenzoni (2017), Auclert (2016), Kaplan, Moll and Violante (2017), McKay, Nakamura and Steinsson (2016), Auclert et al. (2018)...
   - Precautionary savings and aggregate demand: Heathcote and Perri (2018), Ravn and Sterk (2017), Bayer et al. (2019)

Outline

1. Prototype model and accounting procedure
2. Measuring the wedges
3. A class of New Keynesian model with heterogeneous agents
4. An application to the US economy

Prototype model

- z_t and v_t are aggregate and idiosyncratic states. Let $z^t = (z_0, z_1, ..., z_t)$, $v^t = (v_0, v_1, ..., v_t)$, $s^t = (z^t, v^t)$, w/ $\Pr(s^t|s^{t-1}) = \Pr(v^t|z^t, v^{t-1})\Pr(z^t|z^{t-1})$

- Decision problem of an household
  - Takes as given wages $W(s^t)$ and the price of financial assets $Q(s^t, s_{t+1})$
  - Chooses consumption, labor and financial positions

- Individual specific "wedges"
  - Idiosyncratic wage (*efficiency wedge*), $W(s^t) = \theta(v^t)W(z^t)$
  - Tax on labor (*labor wedge*), $\tau_l(s^t)$
  - Tax on financial assets (*risk sharing wedge*), $\tau_a(s^t, s_{t+1})$

Households' problem

$$\max_{\{c(s^t),l(s^t),a(s^t,s_{t+1})\}} \sum_{t=0}^{\infty} \sum_{s^t} \Pr\left(s^t\right) \beta^t \left[\frac{c(s^t)^{1-\sigma}}{1-\sigma} - \chi \frac{l(s^t)^{1+\nu}}{1+\nu} \right]$$

subject to

$$c\left(s^t\right) + \sum_{s_{t+1}} Q(s^t, s_{t+1}) a(s^t, s_{t+1}) [1 + \tau_a(s^t, s_{t+1})] \leq$$

$$\leq \theta(v^t) W(z^t) l(s^t) [1 - \tau_l(s^t)] + a(s^t) + T(s^t)$$

Optimality

$$\begin{aligned} l\left(s^t\right)^{\nu} &= \frac{\theta(v^t) W(z^t) [1 - \tau_l\left(s^t, s_{t+1}\right)]}{\chi c(s^t)^{\sigma}} \\ \Pr(s^{t+1}|s^t) \beta \left(\frac{c(s^t, s_{t+1})}{c(s^t)} \right)^{-\sigma} &= Q\left(s^t, s_{t+1}\right) \left[1 + \tau_a\left(s^t, s_{t+1}\right)\right] \end{aligned}$$

The procedure in one slide

- We have panel data on $\{c_{it}, w_{it}, l_{it}\}$
- We assume agents face the following prices for Arrow securities

$$Q(s^t, s_{t+1}) = \Pr(s_{t+1}|s^t)\beta \left(\frac{C(z^t, z_{t+1})}{C(z^t)} \right)^{-\sigma}$$

- We recover wedges from the data using the optimality conditions

$$\begin{aligned} \theta_{it} &= \frac{w_{it}}{W_t} \\ \tau_{a,it+1} &= \left[\frac{C_{t+1}/C_t}{c_{it+1}/c_{it}} \right]^{\sigma} - 1 \\ \tau_{l,it} &= 1 - \chi l_{i,t}^{\nu} \frac{c_{it}^{\sigma}}{w_{it}} \end{aligned}$$

The "no-tax" allocation

Suppose that $\tau_a(s^t, s_{t+1}) = 0$ and $\tau_l(s^t) = 0$ for all (s^t, s_{t+1}). Then

1. Individual consumption constant fraction of aggregate consumption

$$c_{it} = \varphi_i C_t$$

   for some weight φ_i

2. Individual hours given by

$$l_{it} = \frac{(\theta_{it}/\varphi_i)^{1/\nu}}{\mathbb{E}_i\left[(\theta_{it}/\varphi_i)^{1/\nu}\right]} L_t$$

Deviations from this allocation require non-zero wedges:

- Risk sharing wedge allows for time-varying consumption shares φ_{it}

$$\varphi_{it} = \prod_{j=0}^{t} (1 + \tau_{a,ij})^{-\frac{1}{\sigma}} \varphi_{i0}.$$

- Labor wedge allows for deviations from frictionless labor supply

"Detailed" economies impose restrictions on wedges

Detailed economy = structural model w/ given market structure, frictions, ...

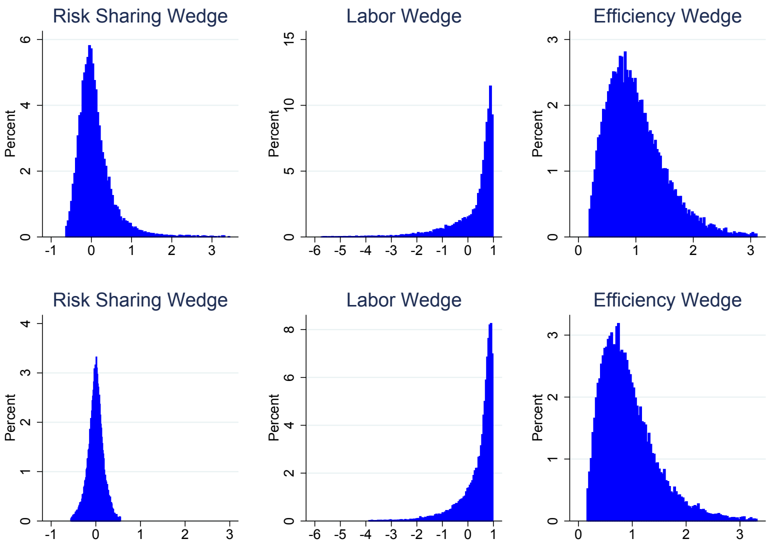
- Predictions of detailed economy for $\{c_{it}, l_{it}, W_{it}\}$ can be replicated in prototype model with appropriate sequence of wedges
- Some examples
  - Huggett (1993) and the risk sharing wedge Details
  - Preference heterogeneity (σ and β) and the risk sharing wedge
  - Sticky wages and idiosyncratic labor wedges

Outline

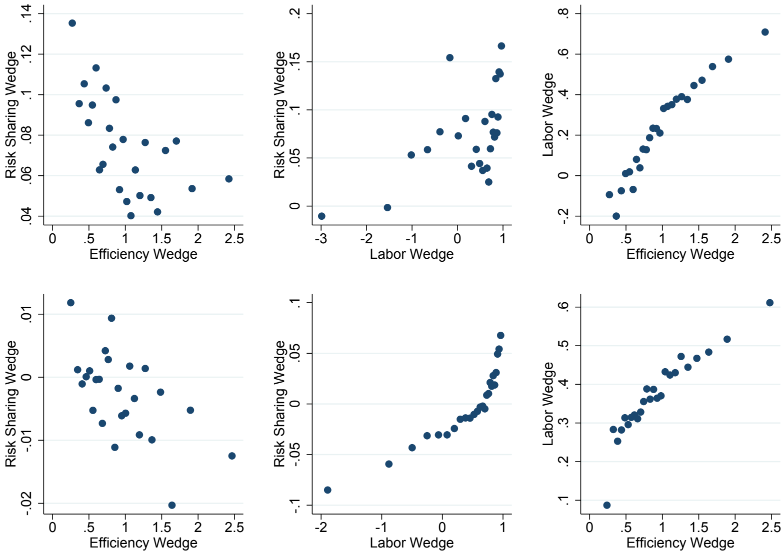
Measuring idiosyncratic wedges

- Need panel on consumption expenditures, wages and hours worked. We use the CEX (1996-2012) and the PSID (1999-2015)
- Data definitions
  - Consumption: Dollar spending in non-durables and services
  - Earnings: Labor + business income
  - Hours: Total hours worked per year
- Mapping between model and data
  - Measure at household level and adjust for number of members
  - Control for characteristics that are typically not included in macro models: education, age, sex, race, state, and family size
- Set $\sigma = 1$, $\nu = 1$ and χ such that labor wedge is on average 0.3 in 2006

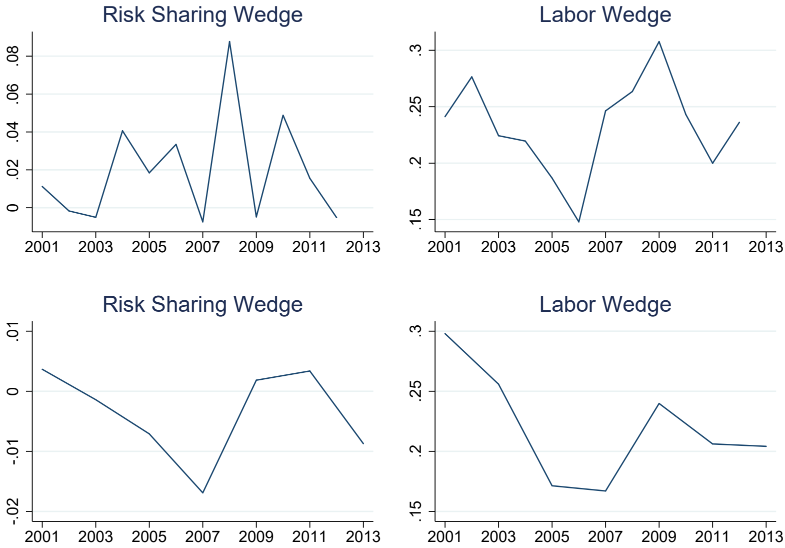
Marginal distribution of the wedges



Cross-sectional patterns



Time-series patterns



Outline

Taking stock

- We have measured micro wedges in the data. We now put them into use
- We consider a *class* of New Keynesian models with heterogeneous agents
  - Macro block: Standard 3 equations NK model (Woodford, 2002)
  - Micro block: unrestricted, allow for a wide range of asset structures
- Key result: Micro block summarized by few statistics of micro wedges
  - Law of motion for aggregates as in RA economy with "taste shocks"
  - Taste shocks simple functions of micro wedges
- Use framework to perform counterfactuals
  - What would happen if risk sharing wedges were set to zero?

Preferences, technology, and monetary policy

- Households' preferences, $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \chi \frac{l^{1+\nu}}{1+\nu}$

- Competitive final good firms use intermediates to produce final good

$$Y(z^t) = \left(\int_0^1 y_i(z^t)^{1/\mu} di \right)^{\mu}$$

- Intermediate good firms are monopolistic competitive, face quadratic adjustment costs á la Rotemberg, production function

$$y_i(z^t) = \exp\{A(z_t)\} n_i(z^t)$$

- Monetary policy described by a Taylor rule

$$i\left(z^t\right) = \max\left\{ \bar{i}^{1-\rho_i} i\left(z^{t-1}\right)^{\rho_i} \left(\frac{\Pi\left(z^t\right)}{\bar{\Pi}} \right)^{\gamma_\pi} \left(\frac{Y\left(z^t\right)}{Y\left(z^{t-1}\right)} \right)^{\gamma_y} \exp\{\epsilon_m(z_t)\}, 1 \right\}$$

The problem of the households

Have access to $\mathcal{J}$ assets and risk-free nominal bond

$$\max_{c,l,b,\{a_j\}} \sum_t \sum_{s^t} \beta^t \Pr\left(s^t|s_0\right) \left[\frac{c\left(s^t\right)^{1-\sigma}}{1-\sigma} - \chi \frac{l(s^t)^{1+\nu}}{1+\nu}\right]$$

subject to

$$\begin{aligned} P\left(z^t\right) c\left(s^t\right) + \sum_{j\in J} a_j\left(s^t\right) + \frac{b\left(s^t\right)}{i\left(z^t\right)} \quad &\leq \quad (1-\tau_l(s^t))W\left(z^t\right)\theta(v_t) l\left(s^t\right) + T(s^t) \\ &+ \quad b\left(s^{t-1}\right) + \sum_{j\in J} R_j\left(s^t\right) a_j\left(s^{t-1}\right) \\ H(b(s^t), \{a_j(s^t)\}_{j\in\mathcal{J}}, s^t) \quad &\geq \quad 0 \qquad H_b(.) \geq 0 \end{aligned}$$

Remark: Nests large class of incomplete market models. Key restriction is that agents with highest marginal valuation for b are on their Euler equation

Heterogeneity and the Euler equation

Euler equation holds for household(s) with highest marginal valuation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{s_{t+1}} \Pr\left(s^{t+1}|s^t\right) \left\{ \frac{\beta}{\Pi\left(z^{t+1}\right)} \left(\frac{c\left(s^t, s_{t+1}\right)}{c\left(s^t\right)} \right)^{-\sigma} \right\}$$

Heterogeneity and the Euler equation

Divide and multiply by $[C(z^{t+1})/C(z^t)]^{-\sigma}$

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{s_{t+1}} \Pr\left(s^{t+1}|s^t\right) \left\{ \frac{\beta}{\Pi\left(z^{t+1}\right)} \underbrace{\left(\frac{C(z^{t+1})/C(z^t)}{c(s^{t+1})/c(s^t)}\right)^{\sigma}}_{[1+\tau_a(s^t,s_{t+1})]} \left(\frac{C\left(z^{t+1}\right)}{C\left(z^t\right)}\right)^{-\sigma} \right\}$$

Heterogeneity and the Euler equation

Aggregate C, Π and i satisfy the Euler equation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{z_{t+1}} \Pr\left(z^{t+1}|z^t\right) \left\{ \frac{\beta\left(v^t, z^{t+1}\right)}{\Pi\left(z^{t+1}\right)} \left(\frac{C\left(z^{t+1}\right)}{C\left(z^t\right)} \right)^{-\sigma} \right\}$$

where

$$\beta(v^t, z^{t+1}) = \beta \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t)[1 + \tau_a(s^t, s_{t+1})]$$

Heterogeneity manifests itself as a "shock" to discount factor (Krueger and Lustig, 2009; Werning, 2016)

- Agents on Euler equation discount more aggregate states characterize by higher average taxes on Arrow securities

Heterogeneity and the Euler equation: complete markets

Aggregate C, Π and i satisfy the Euler equation

$$\frac{1}{i(z^t)} = \max_{v^t} \sum_{z_{t+1}} \Pr\left(z^{t+1}|z^t\right) \left\{ \frac{\beta\left(v^t, z^{t+1}\right)}{\Pi\left(z^{t+1}\right)} \left(\frac{C\left(z^{t+1}\right)}{C\left(z^t\right)} \right)^{-\sigma} \right\}$$

where

$$\beta(v^t, z^{t+1}) = \beta \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t)[1 + \tau_a(s^t, s_{t+1})]$$

With complete markets, $\tau_a(s^t, s_{t+1}) = 0 \; \forall s_{t+1}$ and

$$\beta^c(v^t, z^{t+1}) = \beta$$

- Euler equation as in RA economy

Heterogeneity and labor supply

Optimal labor supply

$$\chi l(s^t)^\nu = (1 - \tau_l(s^t))w(z^t)\theta(v_t)c(s^t)^{-\sigma}$$

Heterogeneity and labor supply

Divide both sides by $\theta(v_t)/C(z^t)^{-\frac{\sigma}{\psi}}$ and aggregate across households

$$\chi^{\frac{1}{\nu}} \underbrace{\left[\sum_{v^t} \Pr(v^t|z^t)\theta(v_t)l(s^t)\right]}_{L_e(z^t)} C(z^t)^{\frac{\sigma}{\nu}} = w(z^t)^{\frac{1}{\nu}} \left\{\sum_{v^t} \Pr(v^t|z^t)(1-\tau_l(s^t))^{\frac{1}{\nu}}\theta(v_t)^{\frac{1+\nu}{\nu}} \left[\frac{c(s^t)}{C(z^t)}\right]^{-\frac{\sigma}{\nu}}\right\}$$

Heterogeneity and labor supply

So, in the aggregate we must have

$$\omega(z^t)\chi L_e(z^t)^\nu = \frac{w(z^t)}{C(z^t)^\sigma}$$

where

$$\omega(z^t) = \left\{ \sum_{v^t} \Pr(v^t|z^t)\varphi(z^t, v^t)^{-\frac{\sigma}{\nu}}\theta(v_t)^{\frac{1+\nu}{\nu}}(1-\tau_l(s^t))^{\frac{1}{\nu}} \right\}^{-\nu}$$

Same FOC of RA agent economy with state-dependent disutility of labor

Heterogeneity and the Phillips curve

Aggregate $\tilde{\Pi} \equiv \Pi(1+\Pi)$, *C and Y must satisfy the Phillips curve*

$$\tilde{\Pi}\left(z^t\right) = \frac{Y\left(z^t\right)}{\kappa\left(\mu-1\right)}\left[\mu\chi\frac{Y(z^t)^{\nu}C\left(z^t\right)^{\sigma}\omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}}-1\right]+\sum_{\mathbf{s}'}Q(z^{t+1}|z^t)\tilde{\Pi}\left(z^{t+1}\right)$$

where

$$\omega(z^t) = \left\{\sum_{v^t}\Pr(v^t|z^t)\varphi(z^t,v^t)^{-\frac{\sigma}{\nu}}\theta(v_t)^{\frac{1+\nu}{\nu}}(1-\tau_l(s^t))^{\frac{1}{\nu}}\right\}^{-\nu}$$

Heterogeneity manifests itself as a shock to the disutility of labor

Suppose high $\theta(v^t)$ also have high consumption shares

- If consumption share of rich decreases $\Rightarrow$ High θ agents work more, low θ agents work less
- Equivalent to positive labor supply shock $\rightarrow$ decrease in marginal cost

Heterogeneity and the Phillips curve: complete markets

Aggregate $\tilde{\Pi} \equiv \Pi(1+\Pi)$, *C and Y must satisfy the Phillips curve*

$$\tilde{\Pi}\left(z^t\right) = \frac{Y\left(z^t\right)}{\kappa\left(\mu-1\right)}\left[\mu\chi\frac{Y(z^t)^{\nu}C\left(z^t\right)^{\sigma}\omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}}-1\right]+\sum_{\mathbf{s}'}Q(z^{t+1}|z^t)\tilde{\Pi}\left(z^{t+1}\right)$$

where

$$\omega(z^t) = \left\{\sum_{v^t}\Pr(v^t|z^t)\varphi(z^t,v^t)^{-\frac{\sigma}{\nu}}\theta(v_t)^{\frac{1+\nu}{\nu}}(1-\tau_l(s^t))^{\frac{1}{\nu}}\right\}^{-\nu}$$

With complete markets, $\tau_a(s^t,s_{t+1}) = 0\ \forall s_{t+1}$ and

$$\omega^c(z^t) = \left\{\sum_{v^t}\Pr(v^t|z^t)\varphi(v_0)^{-\frac{\sigma}{\nu}}\theta(v_t)^{\frac{1+\nu}{\nu}}(1-\tau_l(s^t))^{\frac{1}{\nu}}\right\}^{-\nu}$$

An equivalent representative-agent economy

Suppose that C, Y, Π, i are part of an equilibrium. Then they satisfy

$$\begin{aligned}
\Pi(z^t)\left[1+\Pi(z^t)\right] &= \frac{Y\left(z^t\right)}{\kappa\left(\mu-1\right)}\left[\mu\chi\frac{Y(z^t)^{\nu}C\left(z^t\right)^{\sigma}\omega(z^t)}{\exp\{A(z_t)\}^{1+\nu}}-1\right]+ \\
&+ \sum_{\mathbf{s}'}Q(z^{t+1}|z^t)\Pi(z^{t+1})\left[1+\Pi(z^{t+1})\right] \\
\frac{1}{i\left(z^t\right)} &= \max_{v^t}\sum_{z^{t+1}}\Pr\left(z^{t+1}|z^t\right)\left\{\frac{\beta\left(v^t,z^{t+1}\right)}{\Pi\left(z^{t+1}\right)}\left(\frac{C\left(z^{t+1}\right)}{C\left(z^t\right)}\right)^{-\sigma}\right\} \\
i\left(z^t\right) &= \max\left\{\bar{i}^{1-\rho_i}i\left(z^{t-1}\right)^{\rho_i}\left(\frac{\Pi\left(z^t\right)}{\bar{\Pi}}\right)^{\gamma_\pi}\left(\frac{Y\left(z^t\right)}{Y\left(z^{t-1}\right)}\right)^{\gamma_y}\exp\{\epsilon_m(z_t)\},1\right\} \\
Y\left(z^t\right) &= C\left(z^t\right)+\frac{\kappa}{2}\left[\Pi(z^t)-1\right]^2
\end{aligned}$$

Key observation: Knowledge of $\{\beta(v^t,z^{t+1}),\omega(z^t)\}$ is all we need from the "micro block" to characterize law of motion for aggregate variables

As $\{\beta(v^t,z^{t+1}),\omega(z^t)\}$ varies, the aggregate allocation varies with them

Some examples

"β" shocks important to explain Great Recession in RA economies

- $\beta \uparrow \rightarrow$ RA wants to save more
- Aggregate demand and interest rates fall. Large effects if ZLB binds

HA economies endogenously induce time-variation in β. What mechanisms?

1. Time-varying idiosyncratic risk (Heathcote and Perri, 2018, …) ▸ Example
   - Increase in idiosyncratic income risk + incomplete markets $\rightarrow$ more precautionary savings $\rightarrow$ *as if* $\beta \uparrow$
2. Tightening of borrowing constraints (Eggertson and Krugman, 2012, …)
   - Borrowers cannot borrow $\rightarrow$ Savers cannot save $\rightarrow$ *as if* $\beta \uparrow$

Outline

1. Prototype model and accounting procedure
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Counterfactuals: conceptual experiment

- Suppose we know

$$x = \{A(z_t), \epsilon_m(z_t), \beta(v^t, z^{t+1}), \omega(z^t)\}$$

- Use equivalent RA economy and x to solve for

$$y = \{Y(z^t), \Pi(z^t), i(z^t)\}$$

- Use equivalent RA economy and $x^c = \{A(z_t), \epsilon_m(z_t), \beta^c(v^t, z^{t+1}), \omega^c(z^t)\}$ to solve for counterfactual

$$y^c = \{Y^c(z^t), \Pi^c(z^t), i^c(z^t)\}$$

Contribution of imperfect risk-sharing to macroeconomic aggregates

$$y - y^c$$

Counterfactuals in practice

- Use micro wedges to construct time path for $\{\beta_{it}, \omega_t\}$
- Assume Markov process for $\{A_t, \epsilon_{mt}, \beta_{it}, \omega_t\}$
- Estimate structural parameters of the equivalent RA economy using $\{Y_t, \Pi_t, i_t, \beta_{it}, \omega_t\}$ as observables
- Apply particle filter to estimate state vector and $y = \{Y_t, \Pi_t, i_t\}$
- Solve equivalent RA economy under complete markets and compute counterfactual $y^c = \{Y_t^c, \Pi_t^c, i_t^c\}$ by feeding $\{A_t, \varepsilon_{mt}, \omega_t^{cm}\}$

Contribution of imperfect risk-sharing to macroeconomic aggregates

$$y - y^c$$

Constructing $\beta(v^t, z^{t+1})$

$$
\begin{aligned}
\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t)[1 + \tau_a(s^t, s_{t+1})] \\
&= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^{\sigma}
\end{aligned}
$$

Want:

- Measure change in consumption shares for an individual with history v^t in every possible state v_{t+1}

Problem:

- For each individual, v^t, we observe only one realization of v_{t+1}

Constructing $\beta(v^t, z^{t+1})$

$$\begin{aligned}\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t)[1 + \tau_a(s^t, s_{t+1})] \\ &= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^\sigma\end{aligned}$$

What we do:

- Group individuals with same history v^t
- Compute realized cross-sectional mean of change in consumption shares between z^t and z^{t+1} for individuals in the group
- By law of large numbers, it equals $\beta(v^t, z^{t+1})$

Constructing $\beta(v^t, z^{t+1})$

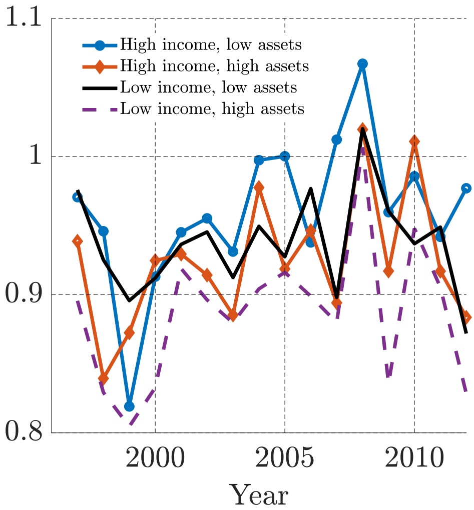
$$\begin{aligned}
\beta(v^t, z^{t+1}) &= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t)[1 + \tau_a(s^t, s_{t+1})] \\
&= \sum_{v_{t+1}} \Pr(v_{t+1}|z^{t+1}, v^t) \left[\frac{C(z^{t+1})/C(z^t)}{c(z^{t+1}, v^t, v_{t+1})/c(z^t, v^t)} \right]^{\sigma}
\end{aligned}$$

In particular:

- Group individuals by income and assets
  - Logic: In Huggett economy income and assets sufficient statistic for v^t
- Within each group i, compute

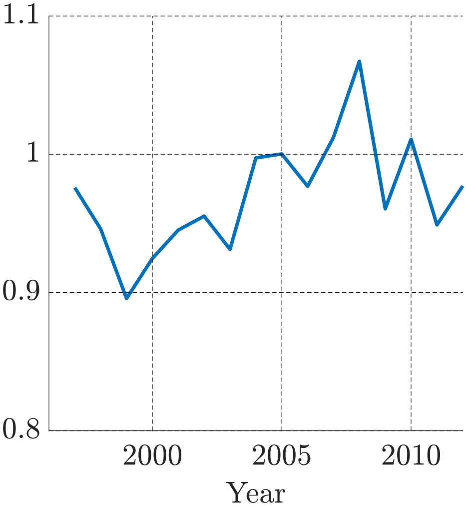
$$\bar{\beta}_{it+1} = \frac{1}{N_i} \sum_{j=1}^{N_i} (1 + \tau_{a,jt+1})$$

Path for $\bar{\beta}_{it}$ for each group



Income rich/asset poor typically have high expected risk-sharing wedges

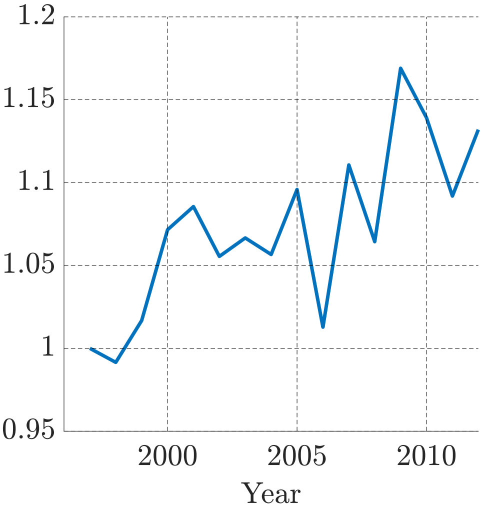
Path for $\max_i \bar{\beta}_{it}$



Imperfect risk-sharing → As if RA is more patient in Great Recession

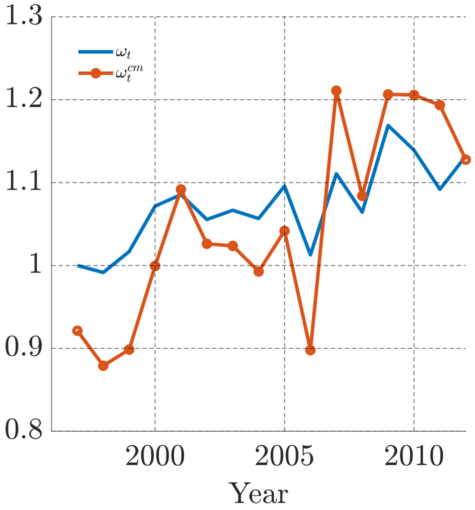
Explain

Path for $\omega(z^t)$



Disutility of labor increases in Great Recession

Path for $\omega^c(z^t)$



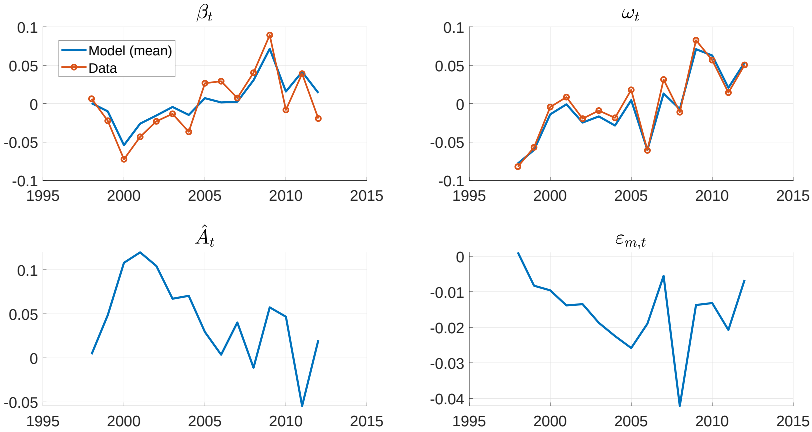
Imperfect risk sharing $\rightarrow$ *As if* RA wants to work more in Great Recession

Estimation and filtering

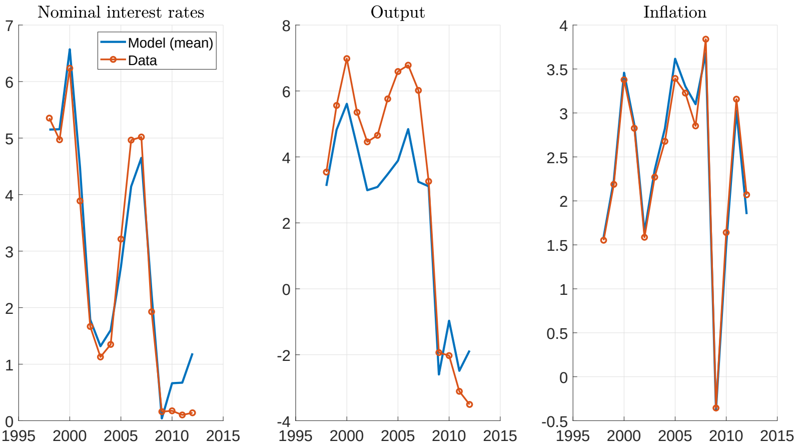
- A_t follows AR(1), $\varepsilon_{m,t}$ iid, $\{\max_i \beta_{it}, \omega_t\}$ follow VAR(1) process
- We set $\sigma = 1$, $\nu = 1$, $\mu = 1.2$, $\Pi^* = 1.02$
- Remaining parameters: $[\kappa, \rho_i, \gamma_\pi, \gamma_{\Delta y}]$ and those of stochastic process $\{A_t, \varepsilon_{m,t}, \max_i \bar{\beta}_{it}, \omega_t\}$
- Use equivalent RA economy to evaluate likelihood function and estimate parameters using $\mathbf{Y}_t = \{\hat{Y}_t, \pi_t, i_t, \beta_{jt}, \omega_t\}$ as observables
- After estimation, back-out structural shocks using particle filter

Parameters

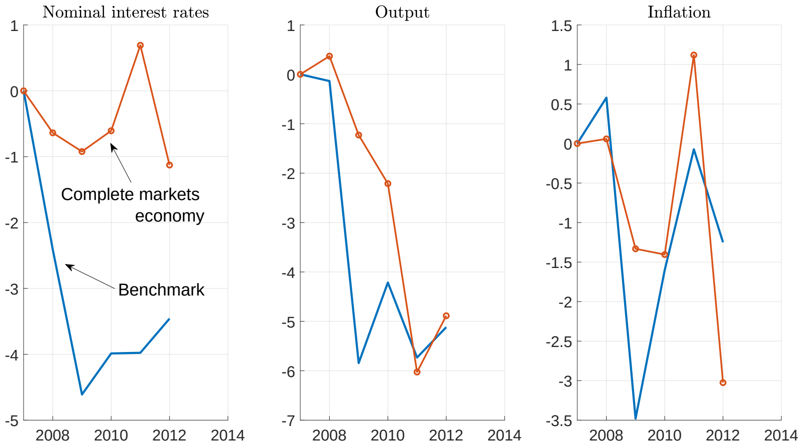
Filtered shocks



Equilibrium outcomes: model and data

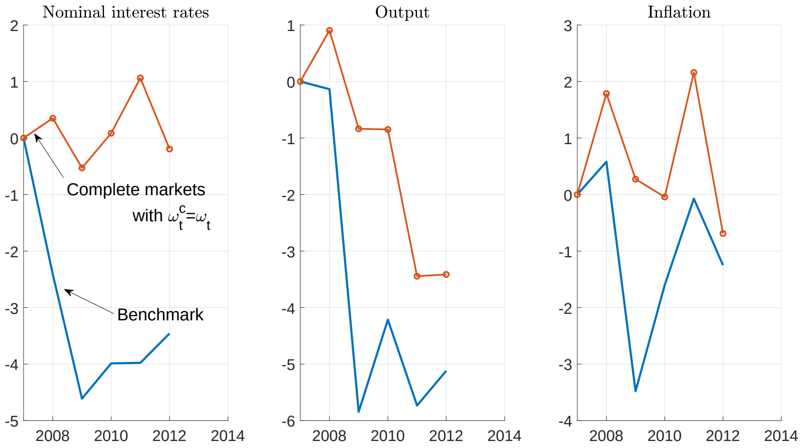


Contribution of imperfect risk-sharing



Milder recession without heterogeneity

Contribution of imperfect risk-sharing



Negative output effects due to increase in propensity to save

Discussion

- In simple NK model, heterogeneity affects aggregates through $[\beta_{it}, \omega_t]$
  - True also in more sophisticated versions (capital, price indexation, etc.)
- Advantages of our procedure
  - Agnostic about nature of idiosyncratic risk and market incompleteness
  - By construction we account for macro and micro data
  - Can perform calculation in benchmark business cycle models
- Disadvantages of our procedure
  - Wedges are not fundamental “shocks”, we cannot say what moves them
  - Cannot study optimal policy
  - No feedback between micro wedges

Conclusion

- Novel framework to evaluate macro models with heterogeneous agents
- Measure micro wedges using CEX and PSID
- Used micro wedges to evaluate business cycle implications of NK models with heterogeneous agents
    - Imperfect risk-sharing during crisis can induce sizable output losses
    - Effects due to increase in propensity to save of income rich/asset poor
- We are working on
    - Disentangling driving forces: precautionary savings vs. debt limits
    - Sensitivity of counterfactuals (adding capital)
    - Other counterfactuals (effects of monetary policy shocks)

Huggett (1993) with tight borrowing limits

- Model details
  - No capital accumulation
  - Aggregate and idiosyncratic risk. Households trade one-period bond
  - Elastic labor supply
  - Competitive labor, goods and financial markets

- Implications for wedges
  - Efficiency wedge due to idiosyncratic income risk
  - Risk sharing wedge due to incomplete markets
  - No labor wedge

Preferences, technology and shocks

- Households have preferences

$$U(c,l) = \log(c) - \chi \frac{l^{1+\nu}}{1+\nu}$$

subject to

$$\begin{aligned} c\left(s^t\right) + b\left(s^t\right) &\leq W(s^t) l\left(s^t\right) + b\left(s^{t-1}\right) R\left(z^{t-1}\right) \\ b\left(s^t\right) &\geq 0 \end{aligned}$$

(Financial autarky in equilibrium: no borrowing $\rightarrow$ no savings)

- Technology for producing final good

$$Y(z^t) = A(z^t) \sum_{v^t} p(v^t|z^t) e(v^t) l(s^t) \qquad \mathbb{E}_{z^t}[e(v^t)] = 1$$

Equilibrium

Optimality and budget constraint

$$\begin{aligned} W(s^t) &= A(z^t)e(v^t) \\ \chi l(s^t)^\nu &= \frac{A(z^t)e(v^t)}{c(s^t)} \\ c(s^t) &= A(z^t)e(v^t)l(s^t) \end{aligned}$$

So, the allocation is given by

$$\begin{aligned} l(s^t) &= \chi^{-\frac{1}{1+\nu}} \\ c(s^t) &= e(v^t)A(z^t)\chi^{-\frac{1}{1+\nu}} \end{aligned}$$

Wedges

- Efficiency wedge:

$$\theta(v^t) = W(s^t)/W(z^t) = \frac{A(z^t)e(v^t)}{A(z^t)} = e(v^t)$$

- Risk-sharing wedge:

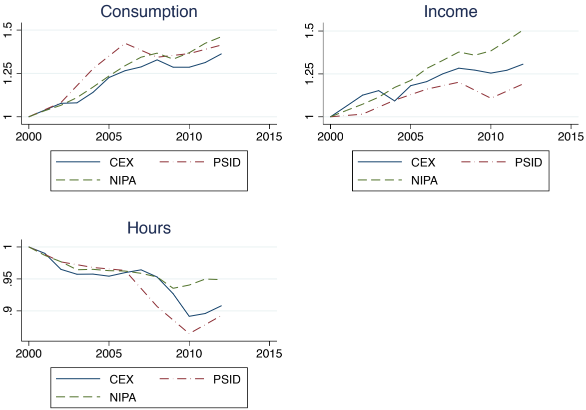
$$\tau_a(s^t, s_{t+1}) = \left[\frac{C(z^{t+1})/C(z^t)}{c(s^{t+1})/c(s^t)}\right] - 1 = \frac{e(v^t)}{e(v^{t+1})} - 1$$

- Labor wedge:

$$\tau_l(s^t) = 0$$

Return

Comparison with NIPA Aggregates



A simple example

- Assume $\sigma = 1$
- Law of motion for idiosyncratic efficiency

$$\Delta \log[\theta(v_t)] = -\frac{\sigma(z_t)}{2} + \sigma(z_t)\varepsilon_{v,t}$$

- Asset market structure
  - Households can only trade a risk-free bond
  - Face a tight borrowing limit: $b(s^t) \geq 0$

In equilibrium financial autarky: every agent is hand-to-mouth

- Labor supply is the same for all households ($\sigma = 1$)
- Individual consumption: $c(s^t) = \theta(v_t)C(z^t)$

Idiosyncratic risk and aggregate demand

The risk sharing wedge in this model is

$$1 + \tau_a(s^t, s_{t+1}) = \frac{\theta(v_t)}{\theta(v_{t+1})} = \exp\{-\Delta \log[\theta(v_{t+1})]\}$$

We can compute the "micro block"

$$\begin{aligned}
\beta(v^t, z^{t+1}) &= \beta \sum_{v^{t+1}} \Pr(v^{t+1}|v^t, z^{t+1}) \exp\{-\Delta \log[\theta(v_{t+1})]\} \\
&= \beta \exp\{\sigma(z^{t+1})\} \\
\omega(z^t) &= 1
\end{aligned}$$

Key mechanism: high expected $\sigma(z_{t+1})$ increases precautionary motives. Higher desired savings manifests itself in the aggregate as increase in β

In benchmark NK models, these shocks lead to a fall in aggregate demand

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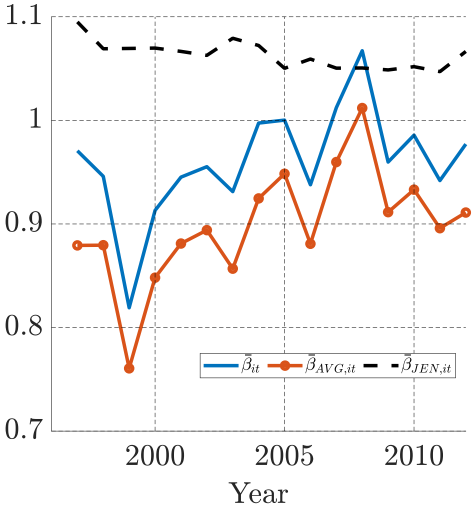
What drives variation in $\max_i \bar{\beta}_{it+1}$?

Focus on income rich/asset poor group

$$\bar{\beta}_{it} = \underbrace{\beta \left[\frac{C_t / C_{t-1}}{\frac{1}{N_i} \sum_{j=1}^{N_i} c_{jt}/c_{jt-1}} \right]}_{\bar{\beta}_{AVG,it}} \underbrace{\sum_{j=1}^{N_i} \left[\frac{\sum_{j=1}^{N_i} c_{jt}/c_{jt-1}}{c_{jt}/c_{jt-1}} \right]}_{\bar{\beta}_{JEN,it}}$$

- $\bar{\beta}_{it}$ can increase if, on average, consumption share of that group between $t-1$ and t falls relative to average
- $\bar{\beta}_{it}$ can increase if Jensen component increases (e.g. higher cross-sectional dispersion in consumption growth)

What drives variation in $\max_i \bar{\beta}_{it+1}$?



Increase in $\max_i \bar{\beta}_{it+1}$ during Great Recession due to a decline, on average, in the consumption share of this group

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Bayesian estimation

We set $\sigma = 1$, $\nu = 1$, $\mu = 1.2$ (Gust et al.), $\Pi^* = 1.02$, and β so that annual nominal rate is 4.5% in deterministic steady state

		Prior		Posterior	
Parameter	Distribution	Mean	Standard deviation	Mean	90% Interval
$4 \times \kappa$	Gamma	85.00	15.00	90.79	[67.41, 115.75]
ρ_i	Beta	0.50	0.28	0.57	[0.29, 0.85]
γ_π	Normal	0.00	1.00	1.51	[0.85, 2.14]
$\gamma_{\Delta y}$	Normal	0.00	1.00	0.58	[0.21, 0.92]
ρ_a	Beta	0.50	0.28	0.52	[0.25, 0.79]
$\Phi_{\beta,\beta}$	Beta	0.50	0.28	0.76	[0.61, 0.92]
$\Phi_{\omega,\omega}$	Beta	0.50	0.28	0.73	[0.33, 0.99]
$100 \times \sigma_a$	InvGamma	1.00	1.00	7.23	[3.36, 10.39]
$100 \times \sigma_m$	InvGamma	1.00	1.00	1.91	[0.98, 2.81]
$100 \times \sigma_\beta$	InvGamma	1.00	1.00	1.88	[1.13, 2.60]
$100 \times \sigma_\omega$	InvGamma	1.00	1.00	3.80	[2.54, 5.05]

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