Inequality and Optimal Monetary Policy in the Open Economy

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November 2023

- **Open-Eco HANK** literature (2021–) focuses on propagation of aggregate & policy shocks
  [Auclert et al. '21, Bayer et al. '23, De Ferra et al. '21, Druedahl et al. '22; Guo et al. '22; Oskolkov '23; Zhou '22...]

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  - individual exposure to idiosyncratic shocks
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- Distinct from a motive to redistribute between households

Main tradeoff and result

Aggregate shocks $\Rightarrow$ output, national income $\Rightarrow$ consumption risk & inequality

TRADE-OFF

Stabilizing consumption inequality

vs

$$\underbrace{\underbrace{\text{Closing output gap} + \text{stabilizing inflation}}_{\text{closed-eco RANK}} + \text{manipulating ToT}}_{\text{open-eco RANK}}$$

RESULT

More output and exchange-rate stabilization than in RANK benchmark

Literature

1. **Positive** monetary policy analysis in open-economy HANK

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2. Optimal monetary policy analysis in **closed-economy** HANK

   [Bhandari et al. '21, Acharya et al. '23, Le Grand et al. '23, McKay & Wolf '23, Davila & Schaab '23]

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3. Optimal monetary policy in open-economy **RANK** or **TANK**

   - 2-country or SOE models **with** int'al risk sharing

     [Clarida et al. '01, '03, Devereux & Engel '03, Benigno & Benigno '03, '05, Galì & Monacelli '05, Corsetti & Pesenti '05, Faia & Monacelli '08, De Paoli '09a, Corsetti et al. '10, Engel '11, Iyer '16, Chen et al. '23]

   - 2-country or SOE models **without** int'al risk sharing

     [Benigno '09, De Paoli '09b; Corsetti et al. '23]

Model

Households

- SOE à la Galì Monacelli (2005) + incomplete markets
- Perpetual youth demographics with turnover rate $1-\vartheta$
- 2 groups of HHs:
  - **Unconstrained** (share $1-\theta$): trade **non-state contingent** 1-period real actuarial bond
  - **Constrained** (share θ): cannot access asset markets (⇒ HtM)
- All HHs subject to idiosyncratic (labour-productivity) risk
- **CARA-Normal** structure as in Acharya et al. '23, Acharya & Dogra '20

Unconstrained households

Newborn i at date s max

$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \Big(\quad u\left(c_t^s(i)\right) - v\left(n_t\right) \quad \Big)$$

s.t.

$$c_t^s(i) + \frac{\vartheta}{R_t} a_{t+1}^s(i) = \mathbf{y}_t^s(i) + (1-\tau^a) a_t^s(i) \qquad a_t^t(i) = a_t$$

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$$\mathbf{y}_t^s(i) = (1-\tau^w) w_t n_t e_t^s(i) + \mathcal{D}_t + \mathcal{T}_t$$

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$$e_t = 1 + \sigma_t \xi_t, \quad \xi_t = \lambda \xi_{t-1} + \upsilon_t$$

Unconstrained households

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$$\mathbb{E}_s \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left(-\frac{1}{\gamma} e^{-\gamma c_t^s(i)} - v\,(n_t) \right)$$

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$$\mathbf{y}_t^s(i) = \underbrace{\frac{P_{H,t}}{P_t} y_t}_{\text{national income}} + \sigma_{y,t} \xi_t^s(i)$$

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Income risk:

$$\sigma_{y,t} = \sigma_y \exp\left\{ -\varphi \left(\frac{y_t}{y} - 1 \right) \right\}$$

Constrained households

- Consume current income:

$$c_t^s(i) = \mathbf{y}_t^s(i) = \frac{P_{H,t}}{P_t} y_t + \sigma_{y,t} \xi_t^s(i)$$

- Consumption changes **one-for-one** with relative price of home goods

Households: Demand system and labour supply

- Demand system a la Gali-Monacelli with home bias $1 - \alpha$ and elasticities details
  - η btw. H vs. F goods
  - ν across countries
  - ε across varieties
- Utilitarian unions set wages and demand uniform labor supplies from the HHs details
- Wages are **flexible** though – prices are sticky

Supply side

- Rotemberg pricing + PCP + optimal payroll subsidy ⇒ **NKPC:**

$$\ln \Pi_{H,t} = \frac{\varepsilon}{\Psi}\left[1 - \left(\frac{1}{1-\tau}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right) p_H(Q_t)\frac{z_t}{w_t}\right] + \beta\left(\frac{z_t w_{t+1} y_{t+1}}{z_{t+1} w_t y_t}\right)\ln \Pi_{H,t+1}$$

where

$$p_{Ht} = \frac{P_{Ht}}{P_t} = \underbrace{\left(\frac{1-\alpha Q_t^{1-\eta}}{1-\alpha}\right)^{\frac{1}{1-\eta}}}_{\text{dynamic ToT manipulation}} \quad \text{and} \quad 1-\tau = \underbrace{\left(\frac{\varepsilon-1}{\varepsilon}\right)\left[\frac{\chi-1+\alpha}{\chi-1}\right]}_{\text{static ToT manipulation}}$$

and $\chi = \eta(1-\alpha)+\nu$ is the trade elasticity

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- **Output:**

$$y_t = \frac{z_t n_t}{1 + \frac{\Psi}{2}\left(\ln \Pi_{Ht}\right)^2}$$

Market clearing and capital flows

- **Cons. demand:**

$$c_t = (1-\theta)c_{u,t} + \theta c_{h,t}, \quad c_{k,t} = (1-\vartheta)\sum_{s=-\infty}^{t} \vartheta^{t-s} \int c_t^s(i,k)di \quad k \in \{u,h\}$$

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- **Fisher parity:**

$$\ln R_t = \ln R_t^* + \ln \frac{Q_{t+1}}{Q_t} - \wp a_{t+1}$$

Household decisions

Consumption functions

- **Constrained** HHs:

$$c_t^s(i;h) = p_H(Q_t)y_t + \sigma_{y,t}\xi_t^s(i;h)$$

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$$c_t^s(i;u) = c_{u,t} + \mu_t\Big[\underbrace{(1-\tau^a)(a_t^s(i) - a_t)}_{\text{(de-meaned) asset wealth}} + \underbrace{\ell_{k,t}^s(i;u)}_{\text{(de-meaned) human wealth}} \Big]$$

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$$\ell_{k,t}^s(i) = \sum_{\tau=0}^{\infty}\left(\frac{\vartheta^\tau}{\prod_{l=0}^{\tau-1} R_{t+l}}\right)[\mathbf{y}_t^s(i) - p_H(Q_t)y_t] = \sigma_{\ell,t}\xi_t^s(i)$$

where

$$\sigma_{\ell,t} = \sigma_{y,t} + \lambda\frac{\vartheta}{R_t}\sigma_{\ell,t+1}$$

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- For **constrained** HHs, consumption (risk) = income (risk)

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- **Monetary policy** affects $\sigma_{c_u,t}$ through both μ_t and $\sigma_{y,t}$!

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- Useful benchmark: **acyclical** consumption risk: $\lambda = 1, \varphi = 0 \Rightarrow \sigma_{c_{u,t}} = \sigma_{c_{h,t}} = \sigma_y$

Aggregate(d) Euler equation

- Cons. growth of **unconstrained** HHs:

$$\Delta c_{u,t+1} = \underbrace{\frac{1}{\gamma}\ln\beta(1-\tau^{a})R_t}_{\text{intertemporal substitution}} + \underbrace{\frac{\gamma}{2}\sigma^2_{c_u,t+1}}_{\text{precautionary savings}}$$

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$$\Delta c_t = (1-\theta)\underbrace{\left\{\frac{1}{\gamma} \ln \beta(1-\tau^a)R_t + \frac{\gamma}{2}\sigma^2_{c_u,t+1}\right\}}_{\text{consumption growth of unconstrained}} + \theta \underbrace{\left\{p_H(Q_{t+1})y_{t+1} - p_H(Q_t)y_t\right\}}_{\text{consumption growth of constrained}}$$

Optimal policy

Social Welfare Function

Utilitarian planner maximises

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \underbrace{\left[(1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int \left(-\frac{1}{\gamma} e^{-\gamma c_t^s(i)} \right) di - v(n_t) \right]}_{\substack{\text{flow utility to planner} \\ \text{at time } t}}$$

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RANK: $\Sigma_t = 1$

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RANK: $\Sigma_t = 1$

HANK: $\Sigma_t > 1$

Welfare cost of inequality Σ_t

- Overall index combines **within** and **between** group inequalities

$$\Sigma_t = (1-\theta)\, e^{-\gamma\theta\mathbb{B}_{c,t}} \Sigma_{u,t} + \theta e^{\gamma(1-\theta)\mathbb{B}_{c,t}} \Sigma_{h,t}$$

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$$\mathbb{B}_{c,t} = c_{u,t} - c_{h,t}$$

- If $\mathbb{B}_{c,t} > 0$, put relatively less weight on inequality within group u

Between-group inequality

- Suppose $\widehat{R}^*_t > 0$ but domestic monetary policy does not respond: $\widehat{R}_t = 0$

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- Depending on domestic mon. policy response, $c_{u,t}$ and $c_{h,t}$ can diverge

Policy Instruments

- **Fiscal policy**: $\{\tau, \tau^w, \tau^a\}$ optimally set ex ante and unresponsive to aggregate shocks

- **Monetary policy**: $\{i_t\}$ adjusted optimally in response to aggregate shocks

Policy Instruments

- **Fiscal policy**: $\{\tau, \tau^w, \tau^a\}$ optimally set ex ante and unresponsive to aggregate shocks
  - τ balances monopolistic distortions
  - τ^w balances labour-wedge distortions
  - τ^a kills steady-state capital outflow
  - Steady state is **constrained-efficient**
- **Monetary policy**: $\{i_t\}$ adjusted optimally in response to aggregate shocks

Domestic productivity shocks

Domestic productivity shock

- **RANK** benchmark: Galì & Monacelli '05

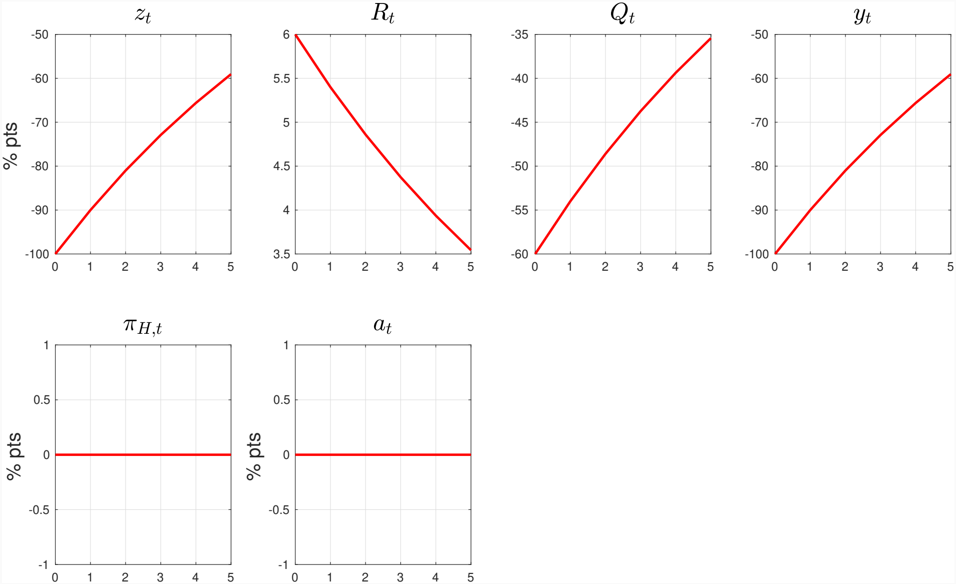
- With $\gamma = \eta = \nu = 1$, **domestic PPI stability** is optimal $\Rightarrow$ "inward-looking" policy

- Optimal allocation features

$$c_t = p_H(Q_t) y_t \qquad a_t = 0 \qquad \Pi_{H,t} = 1 \qquad \forall t \geq 0$$

- Implementable by monetary policy **with or without** international risk sharing (in latter case, HHs **choose** not to borrow/lend from abroad)

Negative z_t shock (RANK)



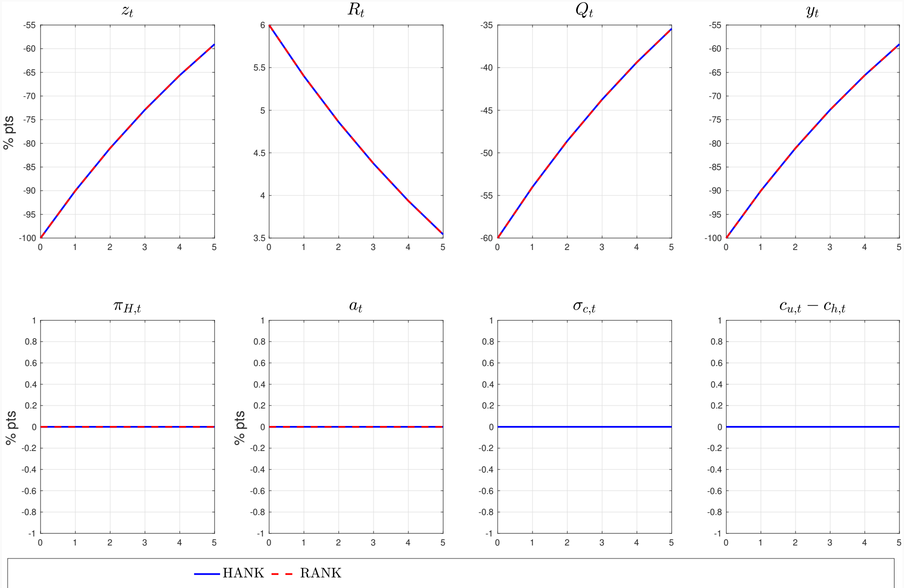
SOE-HANK DIVINE COINCIDENCE

Proposition: Under "Cole-Obstfeld" elasticities ($\gamma = \eta = \nu = 1$), random walk individual risk ($\lambda = 1$) and acyclical income risk ($\varphi = 0$), the optimal allocations in HANK and RANK are identical and independent of the fraction of constrained HHs (θ).

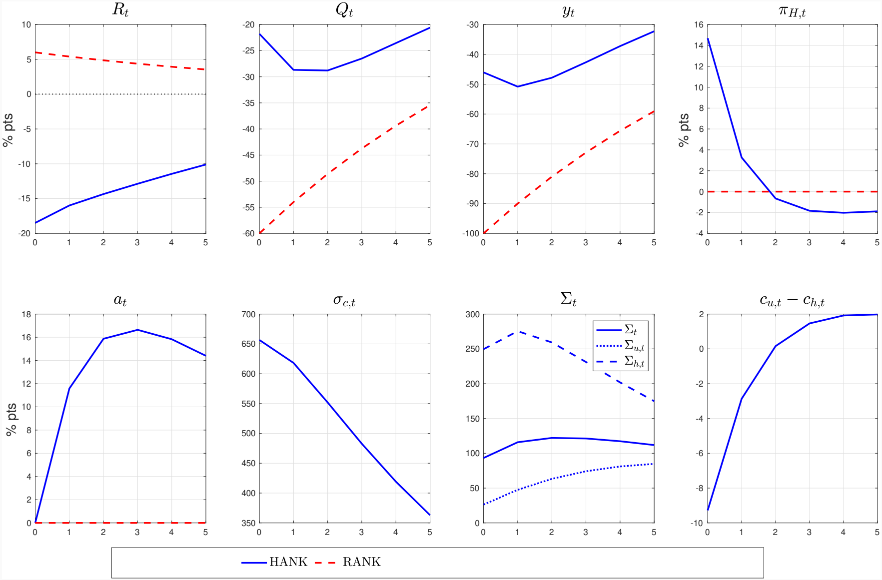
Sketch of proof:

- Cons. growth of **constrained** HHs is $\Delta c_{h,t+1} = p_H(Q_{t+1})y_{t+1} - p_H(Q_t)y_t$
- $\sigma^2_{c_{u,t}} = \sigma^2_y \Rightarrow$ **unconstrained** HHs do not borrow/lend in the aggregate
  $\Rightarrow$ their cons. growth is also $\Delta c_{u,t+1} = p_H(Q_{t+1})y_{t+1} - p_H(Q_t)y_t$
- The two groups are **equally exposed** to the aggregate shock

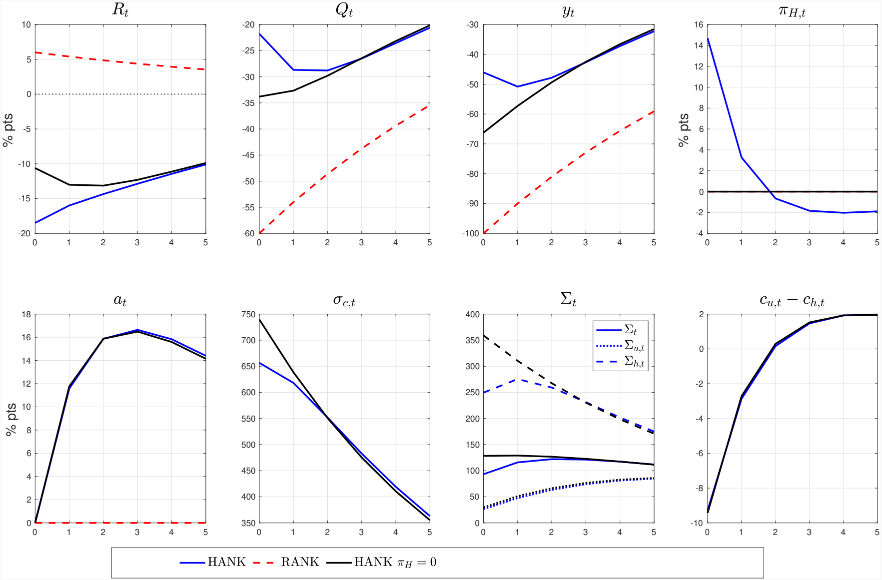
Negative z_t shock (HANK $\varphi = 0, \lambda = 1$)



HANK w. countercyclical income risk ($\varphi > 0$)

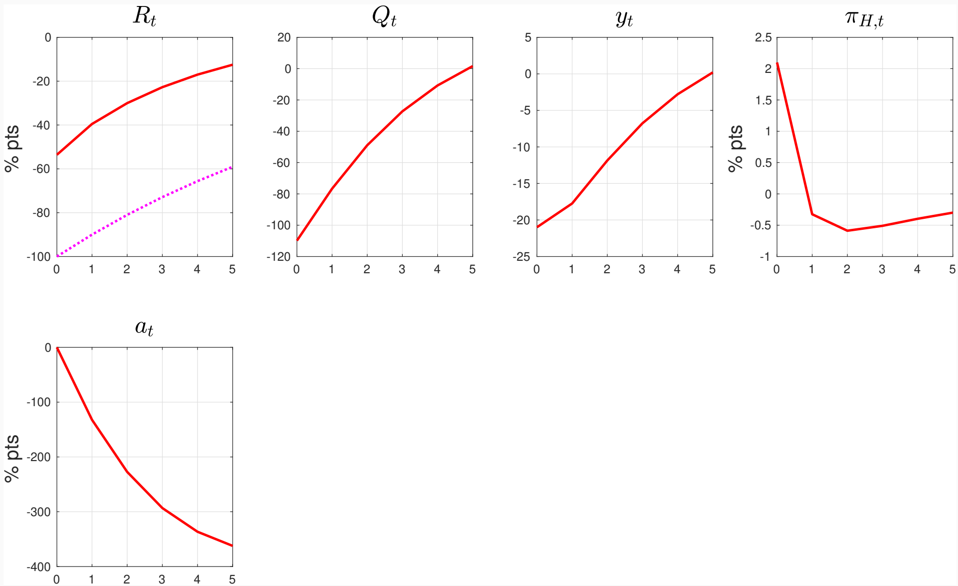


HANK + COUNTERCYCLICAL RISK + PRICE STABILITY

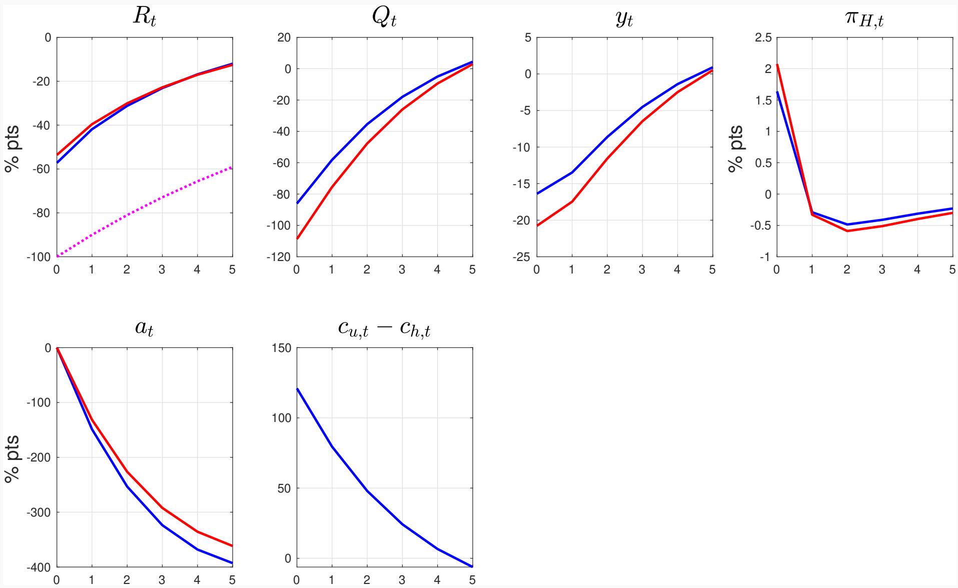


Capital flow shock

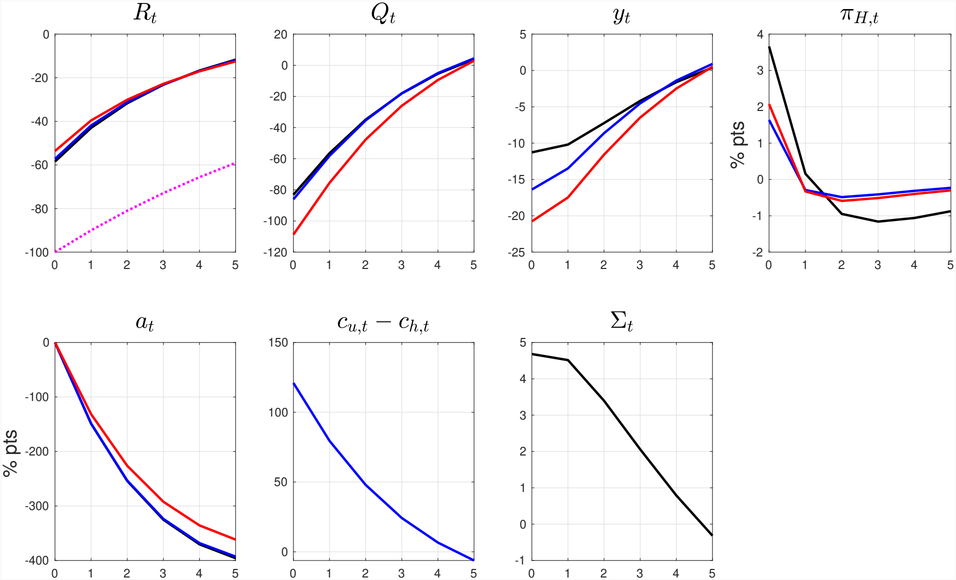
Negative R^* shock (RANK)



Negative R^* shock (TANK)



Negative R^* shock (HANK w. countercyclical risk)



Conclusion

- Optimal policy implements **less volatile** exchange rate and output in HANK
  - **[unequal exposures]** $\Rightarrow$ reduces differences in real incomes btw u and h HHs
  - **[countercyclical risk]** $\Rightarrow$ reduces fluctuations of within-group inequality
- adding lower ERPT, non-unit elasticities **doesn't change prescriptions qualitatively**

Demand system

- Final cons. goods produced by competitive retailers aggregating varieties from all countries
- Their production functions are

$$c = \left[\alpha^{\frac{1}{\eta}} c_F^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} c_H^{\frac{\eta}{\eta-1}}\right]^{\frac{\eta}{\eta-1}} \quad c_H = \left[\int_0^1 c_H(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad c_F = \left[\int_0^1 c_k^{\frac{\nu-1}{\nu}} dk\right]^{\frac{\nu}{\nu-1}}$$

- Let $p_{H,t}, p_{F,t}$ be the prices of the home and foreign baskets in terms of home consumption
- Profit minimisation + zero-profit condition gives the demands

$$c_{H,t} = (1-\alpha)p_{H,t}^{-\eta} c_t \quad c_{F,t} = (1-\alpha)p_{F,t}^{-\eta} c_t$$

where

$$(1-\alpha)p_{H,t}^{1-\eta} + \alpha p_{F,t}^{1-\eta} = 1 \quad \text{and} \quad p_{F,t} = Q_t$$

- Conversely, the demand for home goods by the RoW is

$$c_{Ht}^{*} = \alpha \left(\frac{p_{H,t}}{Q_t}\right)^{-\nu} c^{*}$$

back

Labour supply

- Setup similar to Auclert et al. (2023): Each HH supplies a continuum of labour types to a continuum of unions, each of which demands the same number of hours from all members
- Each union is benevolent and utilitarian, and sets wages accordingly
- With flexible wages, the optimality condition boils down to

$$\underbrace{\left(1-\tau^{w}\right) w_{t}}_{\text{post-tax wage}}=\underbrace{\mathcal{M}_{w}}_{\text{markup}} \times \underbrace{\frac{v'\left(n_{t}\right)}{u'\left(c_{t}\right) \Sigma_{t}}}_{\text{"avg. MRS"}}$$

where

$$\Sigma_{t}=(1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int e^{-\gamma\left[c_{t}^{s}(i)-c_{t}\right]} di$$

captures the dispersion in marginal utility between the members of every union

back