Understanding Growth-at-Risk: A Markov-Switching Approach

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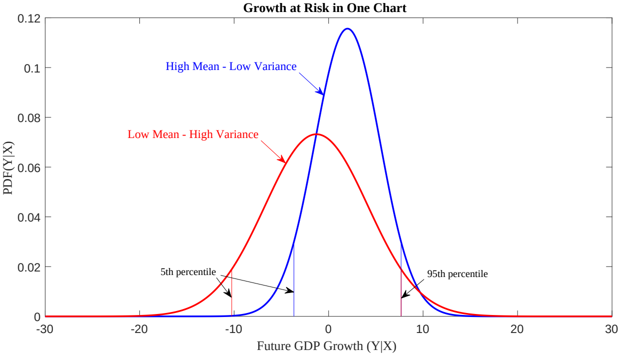
Motivation

- **Risk management is an important consideration for policy decisions**
  - Adrian *et al.* (2019): can reduce probability of a future financial crisis
- **Risk management requires quantification of risks to the outlook**
- **Lively debate about measurement and sources of risks**:
  - Can we reliably detect time-variation in downside risk?
  - What are the drivers of downside risk?
  - How does one interpret downside risk?

An Overview of ”Growth-at-Risk”

- Model entire distribution of future real GDP growth conditional on economic activity and financial conditions.
- Why? Measure uncertainty and risks around forecast.
- Key result: (Conditional) mean and volatility are negatively correlated.
  - High mean - Low volatility: Normal state
  - Low mean - High volatility: Large downside risks $\longrightarrow$ Growth-at-Risk!



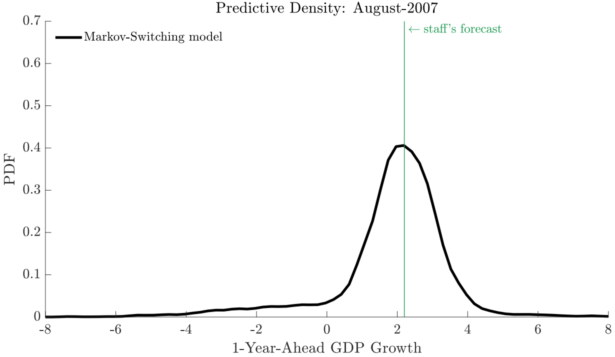
Our Paper

- Standard approach to measure risk: Quantile regressions (QR).
- Our conjecture: Markov-switching (MS) models should work well.
- This paper: MS model of the entire distribution of future real GDP growth conditional on macroeconomic and financial indicators.
  - Transition probabilities depend on macroeconomic and financial conditions
  - Parsimonious model to capture features of “growth-at-risk”
- Advantages of MS model:
  - Explicit about GAR mechanism
  - Reduced-form representation $\rightarrow$ link to non-linear DSGE
  - Well-established parametric approach
  - Structural framework $\rightarrow$ policy experiments

The Paper in One Figure...

"Financial market conditions have deteriorated, and tighter credit conditions and increased uncertainty have the potential to restrain economic growth going forward. In these circumstances, although recent data suggest that the economy has continued to expand at a moderate pace, the Federal Open Market Committee judges that the downside risks to growth have increased appreciably."

August 17, 2007 FOMC statement



- Optimistic forecast but concern about downside risk → MS model left tail

The Paper in One Figure...

"Financial market conditions have deteriorated, and tighter credit conditions and increased uncertainty have the potential to restrain economic growth going forward. In these circumstances, although recent data suggest that the economy has continued to expand at a moderate pace, the Federal Open Market Committee judges that the downside risks to growth have increased appreciably."

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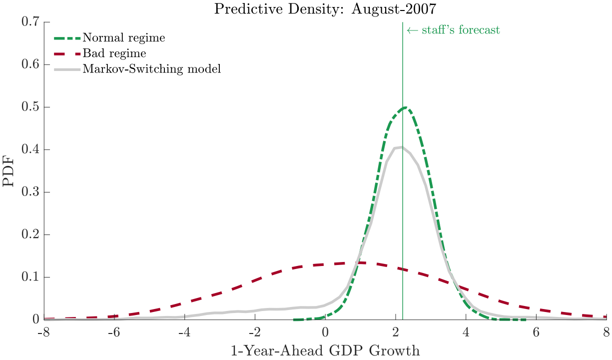


- MS model: endogenously weights on normal and bad regimes

A MS Model of GAR - Direct Approach

$$\underbrace{\bar{\Delta} y_{t+1,t+12}}_{\text{1-Year-Ahead Avg. Growth}} = \alpha_y(s_t) + \sum_{j=0}^{p} \beta_y^j(s_t) f_{t-j} + \sum_{j=0}^{p} \gamma_y^j(s_t) m_{t-j} + \sigma_y(s_t)\varepsilon_t^y,$$

$$\underbrace{f_t}_{\text{Financial Factor}} = \alpha_f(s_t) + \sum_{j=1}^{p} \beta_f^j(s_t) f_{t-j} + \eta_f(s_t) m_t + \sum_{j=1}^{p} \gamma_f^j(s_t) m_{t-j} + \sigma_f(s_t)\varepsilon_t^f,$$

$$\underbrace{m_t}_{\text{Macro Factor}} = \alpha_m(s_t) + \sum_{j=1}^{p} \beta_m^j(s_t) f_{t-j} + \sum_{j=1}^{p} \gamma_m^j(s_t) m_{t-j} + \sigma_m(s_t)\varepsilon_t^m.$$

- **Two regimes:** $s_t = 1$: Normal regime, $s_t = 2$: Bad regime
- **Three ingredients:**
  1. Regime-specific mean and volatility
  2. Regime-specific sensitivity to fundamentals
     - Akin to non-linear dynamics of DSGE models (Gertler *et al.*, 2019; Fernandez-Villaverde *et al.*, 2019; Aruoba *et al.*, 2020)
  3. Financial and macroeconomic conditions influence regime probabilities

A MS Model of GAR - Iterated approach

$$\Delta y_t = \alpha_y(s_t) + \beta_y(s_t) f_t + \gamma_y(s_t) m_t + \sum_{j=1}^{p} \beta_y^j(s_t) f_{t-j} + \sum_{j=1}^{p} \gamma_y^j(s_t) m_{t-j} + \sigma_y(s_t)\varepsilon_t^y,$$

$$f_t = \alpha_f(s_t) + \sum_{j=1}^{p} \beta_f^j(s_t) f_{t-j} + \eta_f(s_t) m_t + \sum_{j=1}^{p} \gamma_f^j(s_t) m_{t-j} + \sigma_f(s_t)\varepsilon_t^f,$$

$$m_t = \alpha_m(s_t) + \sum_{j=1}^{p} \beta_m^j(s_t) f_{t-j} + \sum_{j=1}^{p} \gamma_m^j(s_t) m_{t-j} + \sigma_m(s_t)\varepsilon_t^m.$$

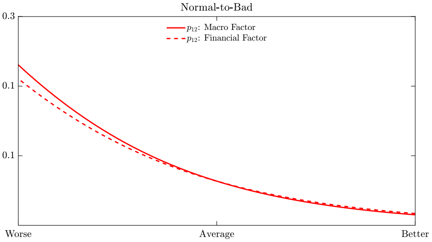
- **If the DGP is a VAR, iterated and direct model are equivalent.**
- **Less parsimonious model, but with several advantages.**
  - Direct connection to existing VAR models
  - Allows to track evolution of risks along the horizon
  - Straightforward to construct IRFs and conditional forecasts

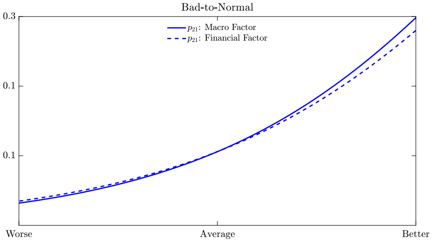
Intuition   Evolution of Risks

Endogenous Transition Probabilities

- s_t follows Markov process with **endogenous transition probabilities**
- Logistic function for $\mathbb{P}(s_{t+1} = 2|s_t = 1)$ and $\mathbb{P}(s_{t+1} = 1|s_t = 2)$:

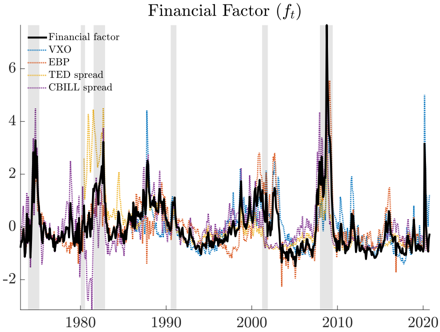
$$\begin{aligned} \mathbb{P}(s_{t+1} = 2|s_t = 1) &= \frac{1}{1 + exp(a_{12} - b_{12} f_t - c_{12} m_t)}, \\ \mathbb{P}(s_{t+1} = 1|s_t = 2) &= \frac{1}{1 + exp(a_{21} - b_{21} f_t - c_{21} m_t)}. \end{aligned}$$

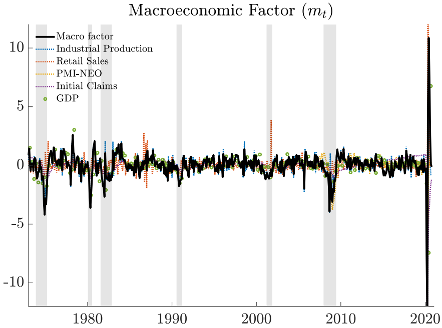




Macro and Financial Conditions

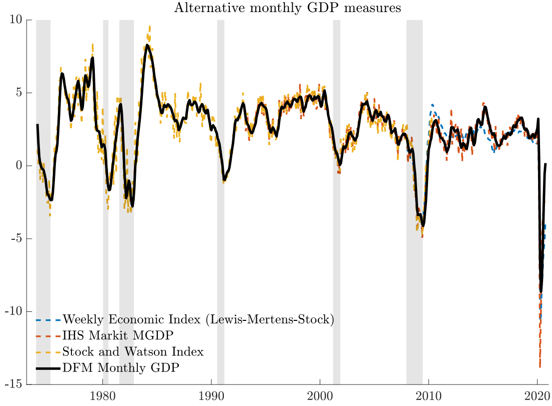
- Mixed-frequency DFM (real-time) estimates of f_t and m_t (Aruoba *et al.*, 2009). Sample January-1973 to May-2020.





Monthly GDP estimate

- DFM also provides real-time estimate of monthly GDP
  $\rightarrow$ timely assessment of buildup of risks
- Monthly GDP tracks well other existing measures:
  - Stock and Watson (1989), IHS-Markit, Lewis *et al.* (2020)



Markov-Switching Model Results

$$\bar{\Delta} y_{t+1,t+12} \quad = \quad \alpha_y(s_t) + \sum_{j=0}^{p} \beta_y^j(s_t) f_{t-j} + \sum_{j=0}^{p} \gamma_y^j(s_t) m_{t-j} + \sigma_y(s_t)\varepsilon_t^y$$

1. Negative correlation between mean and volatility

	Bad Regime		Normal Regime	
$\alpha_y(s_t)$	-0.97	[-1.24,-0.65]	0.51	[0.43, 0.61]
$\sigma_y(s_t)$	2.77	[2.56, 3.03]	0.78	[0.72, 0.85]

2. Asymmetry of sensitivity to fundamentals

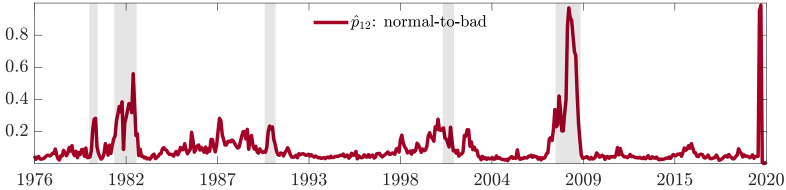
	Bad Regime		Normal Regime	
$\beta_y^0(s_t)$	-0.44	[-0.68,-0.16]	-0.21	[-0.47,-0.05]
$\gamma_y^0(s_t)$	**0.73**	[0.39, 1.09]	**0.17**	[0.0, 0.31]

Note: Numbers in brackets represent 95% confidence intervals.

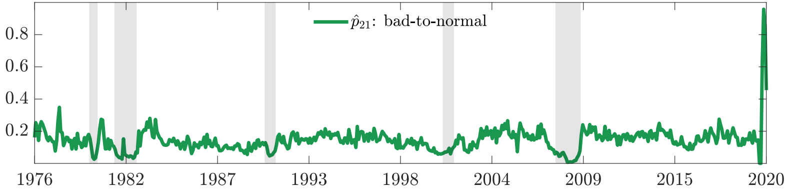
3. Asymmetry in regime transition probabilities

Endogenous Regime Transition Probabilities

- **normal-to-bad**: $\mathbb{P}(s_{t+1} = 2 | s_t = 1) = \frac{1}{1 + exp(a_{12} - b_{12} f_t - c_{12} m_t)}$



- **bad-to-normal**: $\mathbb{P}(s_{t+1} = 1 | s_t = 2) = \frac{1}{1 + exp(a_{21} - b_{21} f_t - c_{21} m_t)}$



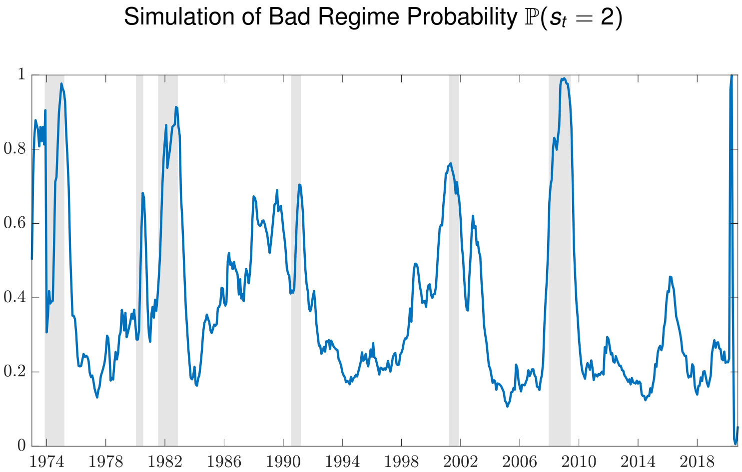
The Predictive Distribution of GDP Growth

$$p\left(\bar{\Delta} y_{t+1,t+H} | \mathcal{I}_t\right) = \int_{\theta} \int_{\epsilon_t^y} \left[\int_{s_{t-H+1:t}} p(\bar{\Delta} y_{t+1,t+H}, s_{t-H+1:t} | \mathcal{I}_t, \theta) ds_{t-H+1:t} \right] \times p(\epsilon_t^y | \mathcal{I}_t, \theta) p(\theta | \mathcal{I}_t) d\epsilon_t^y d\theta$$

- Sources of uncertainty:
  1. Parameter uncertainty $p(\theta|\mathcal{I}_t)$
  2. Shock uncertainty $p(\epsilon_t^y|\mathcal{I}_t, \theta)$
  3. Regime uncertainty $p(\bar{\Delta} y_{t+1,t+H}, s_{t-H+1:t}|\mathcal{I}_t, \theta)$
- Draw from $p\left(\bar{\Delta} y_{t+1,t+H}|\mathcal{I}_t\right)$ following Del Negro and Schorfheide (2013)
  - Challenge: $\mathcal{I}_t = \left\{\bar{\Delta} y_{t-H+1,t}, f_t, m_t, s_{t-H}\right\} \rightarrow$ real-time inference of s_t!
  - Uncertainty about s_t through direct simulation of the Markov-chain.

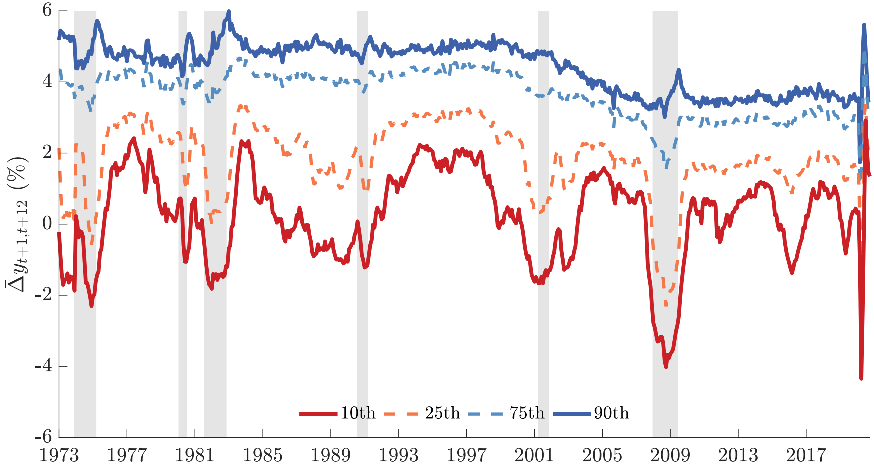
Details Direct Details Iterated

Simulated Regime Probability of “Bad Regime”



The Evolution of Growth-at-Risk

- MS model captures asymmetric dynamics of conditional quantiles



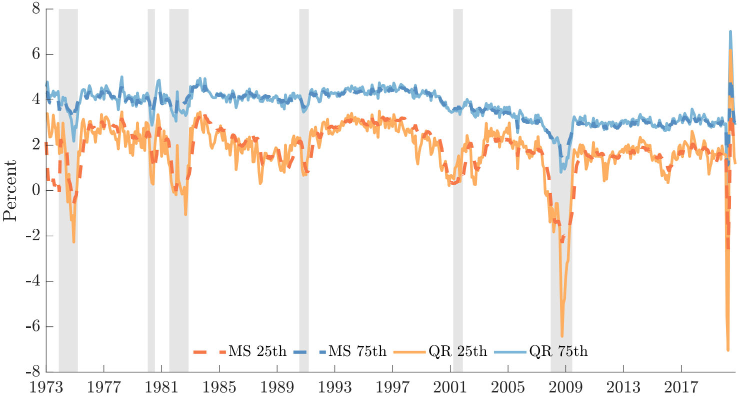
Iterated Model

MS and QR Capture Growth-at-Risk

- Follow QR framework of Adrian *et al.* (2019)

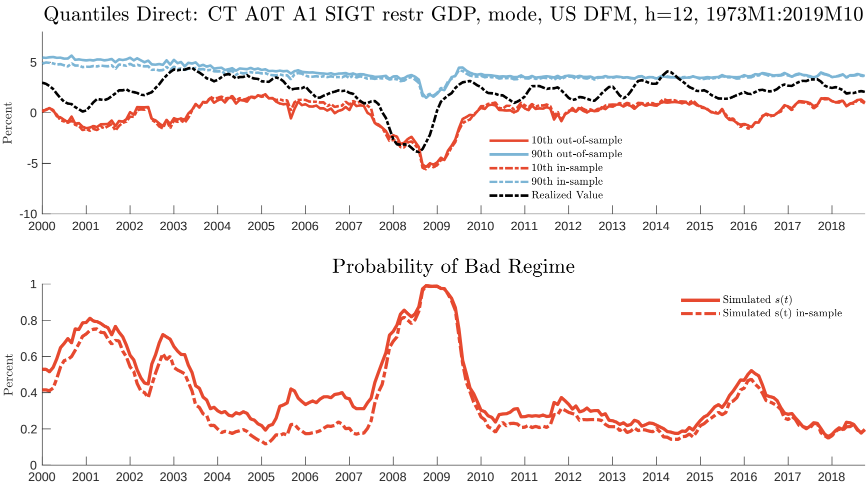
$$\widehat{\mathcal{Q}}_\tau(\bar{\Delta} y_{t+1,t+12}|x_t) \quad = \quad \hat{\alpha}_\tau + \hat{\beta}_\tau f_t + \hat{\gamma}_\tau m_t$$

- $\hat{\alpha}_\tau$, $\hat{\beta}_\tau$ and $\hat{\gamma}_\tau$ fold all the mechanims of GAR QR Estimation Results



MS Model Equivalent to QR

Out-of-Sample: Quantiles of Direct Approach

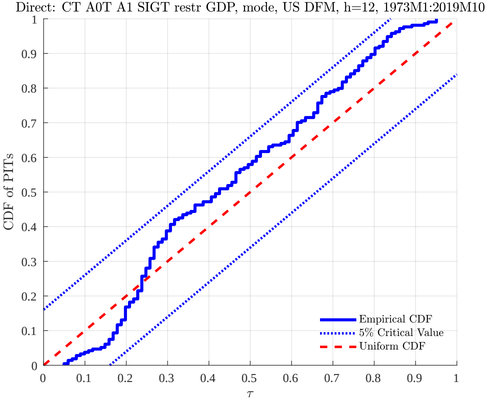


Iterated

Out-of-Sample: PITs CDF

- Model passes the Rossi and Sekhposyan (2019) test for correct calibration of predictive density.
- Test is based on CDF of Probability Integral Transforms (PITs):

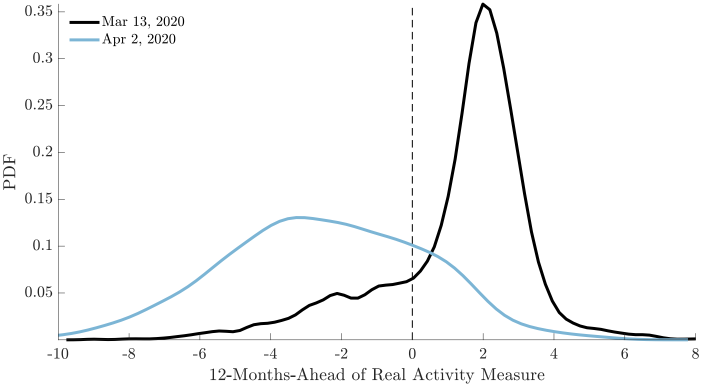
$$z_t \equiv F^{-1}\left(\bar{\Delta} y^*_{t+1,t+12} | x_t\right) = Prob\left(\bar{\Delta} y_{t+1,t+12} < \bar{\Delta} y^*_{t+1,t+12} | x_t\right)$$



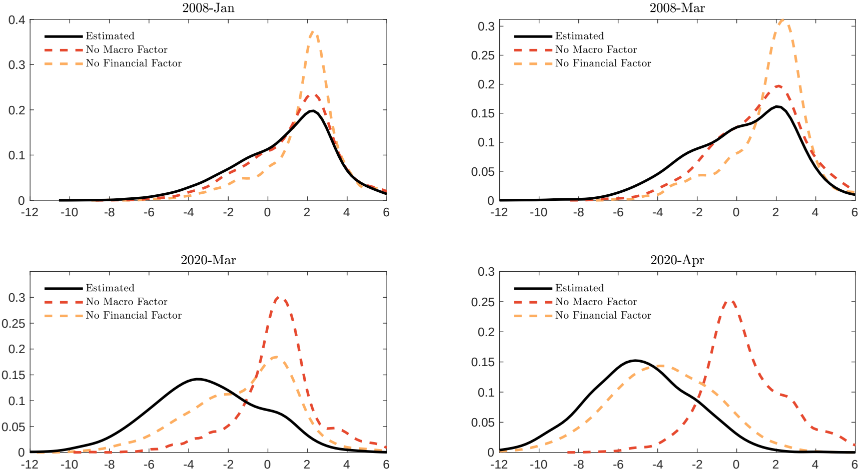
Iterated

Real-Time Risk Assessment - March 2020

- Two data vintages prior to major FOMC policy announcements:
  - **March-13**: Financial developments → fat left tail
  - **April-2**: Real developments → *switch to bad regime*
  - Probability of "bad" regime increased from 42% to 94%



Semi-Structural “Counterfactuals” - Direct Approach

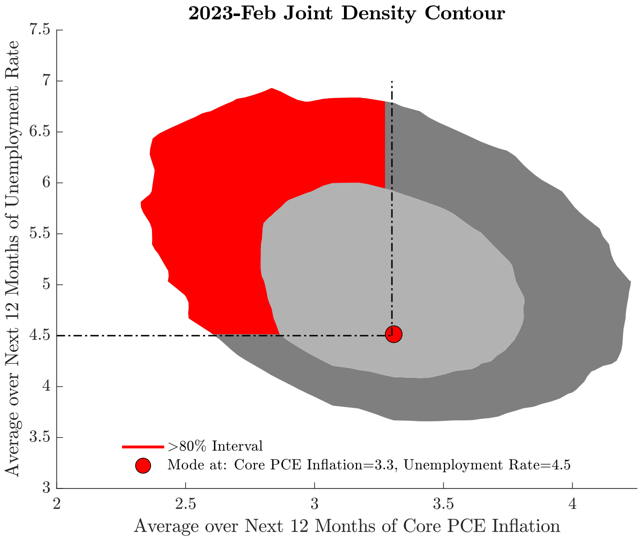


A Richer Application with the Iterated Approach

A Shock to Bank Lending Conditions and Joint Predictive Distributions

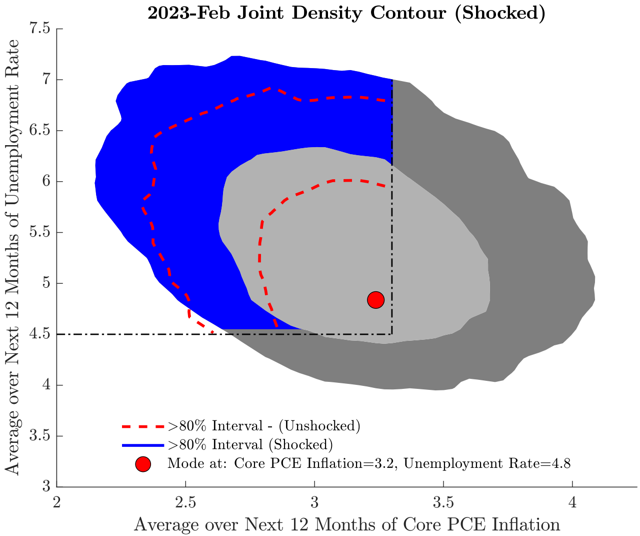
- **Estimation sample**: June 1991 to February 2023.
- **Variables**: Unemployment rate, core PCE inflation, shadow FFR, financial conditions index, and SLOOS.
- **Transformations**: All variables are in deviation from their long-run trend, except for unemployment rate which is in YoY changes.
- **Experiment**: Consider a shock to SLOOS of 0.5 SD over rest of year.
- **Identification**: Shock to SLOOS has no contemporaneous impact.
- **Caveat**: Model does not distinguish between shock to bank lending coming from supply vs. demand, focuses on average effect.

Joint Risks Absent Shock



Unconditional Joint Distribution of One-Year-Ahead Unemployment Rate and Inflation

Joint Risks After a Shock to Bank Lending Standards



Shocked Joint Distribution of One-Year-Ahead Unemployment Rate and Inflation

Taking Stock

- **MS models can capture growth-at-risk.**
- **Intuitive interpretation of macroeconomic risk:**

  Regime uncertainty AND distinct dynamics across regimes generate risk.
- **MS and QR models**: Similar risk dynamics, complementary tools for risk assessment.
- **MS advantages:** Intuitive interpretation of risk, transparency about GAR mechanism and possibility of structural analysis.

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Stock, J. H. and Watson, M. W. (1989). New indexes of coincident and leading economic indicators. *NBER macroeconomics annual*, **4**, 351–394.

Simulating the Predictive Density - Direct Model

- Write the model as an SVAR

$$A_0(s_t)Y_t = C(s_t) + A_1(s_t)Y_{t-1} + \Sigma(s_t)\varepsilon_t,$$

where $Y_t = [\bar{\Delta} y_{t+1,t+12}, f_t, m_t]'$ and $s_t = \{1, 2\}$.

- Define:

$$D(s_t) = A_0(s_t)^{-1}C(s_t), B(s_t) = A_0(s_t)^{-1}A_1(s_t), \Omega(s_t) = A_0(s_t)^{-1}\Sigma(s_t)$$

- Step 1: For $i = 1, \ldots, N^{draws}$:
  - Step 1a: Conditional on $Y_{t-12}, \ldots, Y_{t-1}$ and s_{t-12}, forecast $s^i_{t-11}, \ldots, s^i_t$
  - Step 1b: Draw ϵ^i_t
  - Step 1c: Compute $Y^i_t = D(s^i_t) + B_1(s^i_t)Y_{t-1} + \Omega(s^i_t)\varepsilon^i_t$
- Step 2: Compute quantiles for $\left\{Y^i_t\right\}_{i=1}^{Ndraws}$

Back

Simulating the Predictive Density - Iterated Model

- Write the model as an SVAR

$$A_0(s_t)Y_t = C(s_t) + A_1(s_t)Y_{t-1} + \Sigma(s_t)\varepsilon_t,$$

where $Y_t = [\bar{\Delta}y_t, f_t, m_t]'$ and $s_t = \{1, 2\}$.

- Define:

$$D(s_t) = A_0(s_t)^{-1}C(s_t), B(s_t) = A_0(s_t)^{-1}A_1(s_t), \Omega(s_t) = A_0(s_t)^{-1}\Sigma(s_t)$$

- Step 1: For $i = 1, \ldots, N^{draws}$:
  - Step 1a: Conditional on Y_t and s_t, forecast s^i_{t+1}
  - Step 1b: Draw ϵ^i_{t+1}
  - Step 1c: Compute $Y^i_{t+1} = D(s^i_{t+1}) + B_1(s^i_{t+1})Y_t + \Omega(s^i_{t+1})\varepsilon^i_{t+1}$
  - Step 1d: Repeat steps 1a to 1c for $t+2$ to $t+12$, compute $\bar{Y}^i_t = \frac{\sum_{j=1}^{12} Y^i_{t+j}}{12}$
- Step 2: Compute quantiles for $\left\{\bar{Y}^i_t\right\}_{i=1}^{Ndraws}$

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Quantile Regression: Estimation Results

$$\widehat{\mathcal{Q}}_\tau(\bar{\Delta} y_{t+1,t+12}|x_t) \quad = \quad \hat{\alpha}_\tau + \hat{\beta}_\tau f_t + \hat{\gamma}_\tau m_t,$$

- $\Delta y_{t+1,t+12}$ is calculated from our monthly GDP series
- $\hat{\alpha}_\tau$, $\hat{\beta}_\tau$ and $\hat{\gamma}_\tau$ fold all the mechanims of GAR
  - Lower quantile with similar growth than MS *bad regime*
  - Lower quantile more responsive to f_t and m_t
  - Asymetry in m_t

	Quantile Regression					
	25th Quantile		***Median***		***75th Quantile***	
α_τ	-0.88	[-1.00,-0.77]	0.24	[0.15, 0.33]	1.04	[0.98, 1.11]
β_τ	-0.63	[-0.74,-0.52]	-0.31	[-0.39,-0.23]	-0.13	[-0.18,-0.08]
γ_τ	0.47	[0.30, 0.63]	0.40	[0.28, 0.52]	0.32	[0.22, 0.43]

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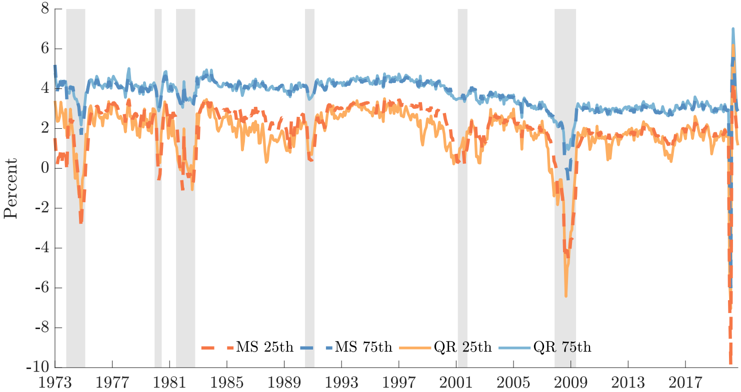
MS and QR Capture Growth-at-Risk

- Follow QR framework of Adrian *et al.* (2019)

$$\widehat{\mathcal{Q}}_\tau(\bar{\Delta} y_{t+1,t+12}|x_t) \quad = \quad \hat{\alpha}_\tau + \hat{\beta}_\tau f_t + \hat{\gamma}_\tau m_t$$

- Estimate exact same model in MS-VAR (switches only in GDP equation)



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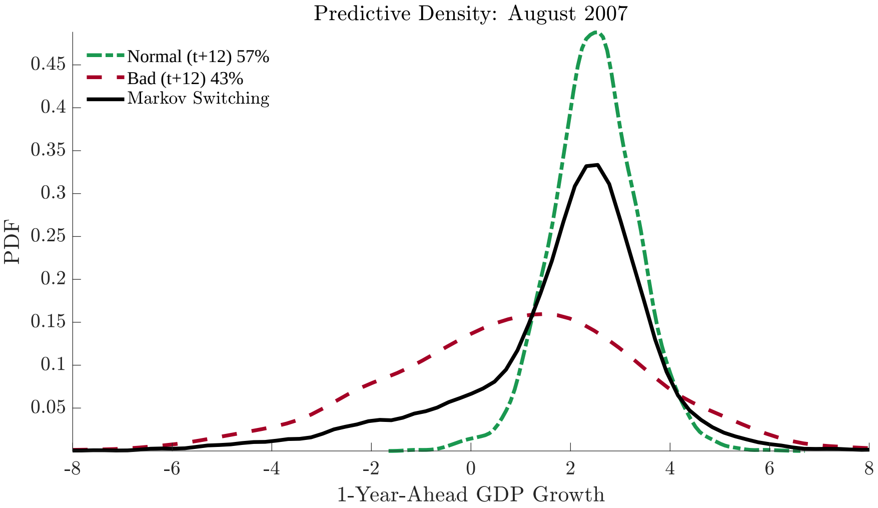
Intuition: Iterated Model



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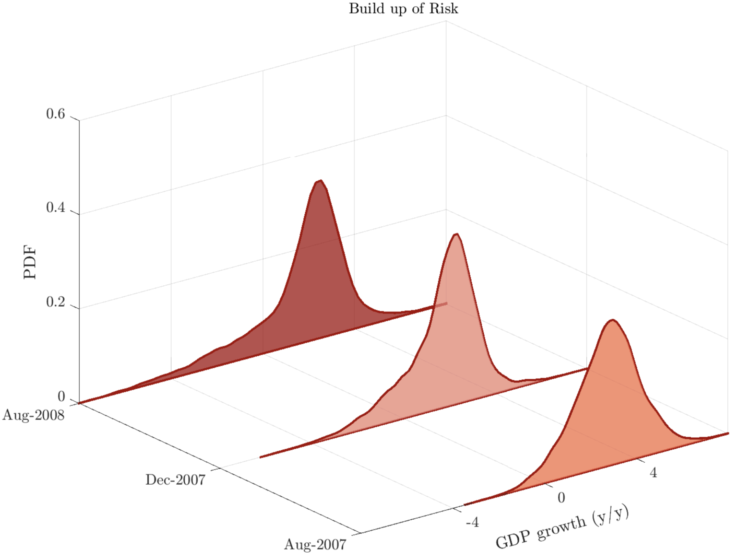
Tracking the Build up of Risk: Iterated Model



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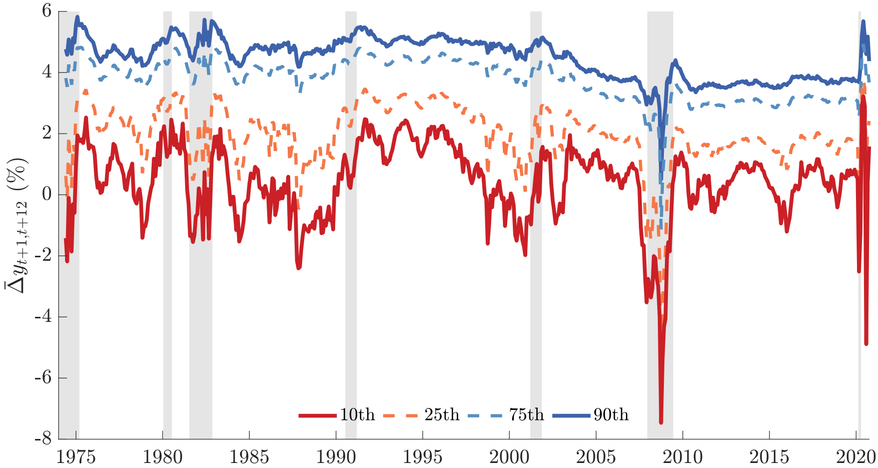
Growth-at-Risk Quantiles: Iterated Model



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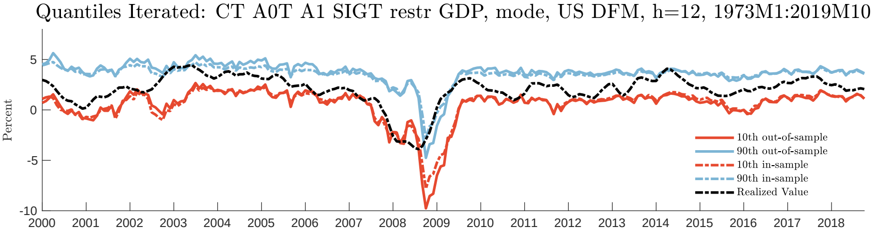
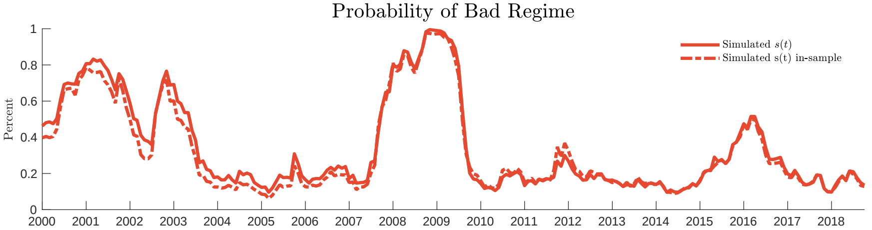
Out-of-Sample: Quantiles of Iterated Approach





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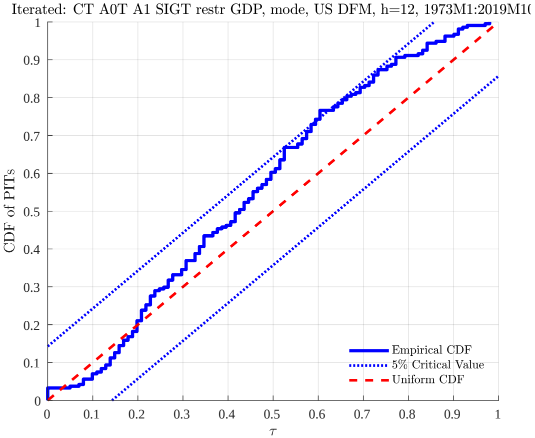
Out-of-Sample: PITs CDF - Iterated Approach

- Test is based on CDF of Probability Integral Transforms (PITs):

$$z_t \equiv F^{-1}\left(\bar{\Delta} y^*_{t+1,t+12} | x_t\right) = Prob\left(\bar{\Delta} y_{t+1,t+12} < \bar{\Delta} y^*_{t+1,t+12} | x_t\right)$$



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