State Dependence of Fiscal Multipliers: the Source of Fluctuations Matters

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Motivation: missing link

- Ramey and Zubairy (2018):

  "Other than the zero lower bound papers, $< \ldots >$ *there is only a limited literature analyzing rigorous models that produces fiscal multipliers that are higher during times of high unemployment. Thus, there is still a gap between Keynes′ original notion and modern theories".*

Fiscal multipliers and states of the world

- Empirical debate:

Auerbach and Gorodnichenko (2012, 2013)
Fazzari, Morley and Panovska (2015) vs Ramey and Zubairy (2018)
(State dependence) (No state dependence)

- Theoretical models:

Fiscal multipliers almost state-independent in workhorse models (Sims and Wolff, 2017):

$$\frac{dY}{dG}(s) \approx \frac{dY}{dG}(s'), \qquad s' \neq s$$

where $s, s' \in S$ are states of the world (away from ZLB)

This paper: main results

- **Theory** of state-dependent government spending and taxation multipliers, in a framework with interaction between **idle capacity** and **unsatisfied demand**
  - Cyclicality of fiscal multipliers depends on the **source of fluctuations**
  - **Spending multipliers** high in demand-driven recessions, low if recession supply driven
  - **Tax cut multipliers** high in supply-driven recessions, low if recession demand driven
  - **Spending austerity** effective in supply recessions or periods of excessive demand if the labor market is sufficiently rigid

- **Estimation** of state-dependent multipliers, **conditional** on the source of fluctuations
  - Use co-movement of economic activity and inflation to identify states; findings support theory

Standard approach vs. our novel approach

- **Standard approach**: production is equal to demand

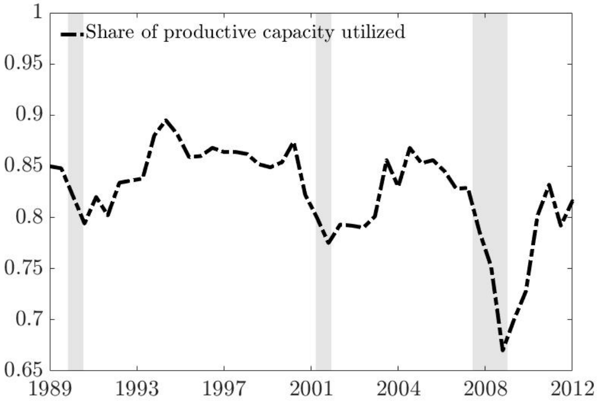
$$Y = C + G \tag{1}$$

- **Our approach**: presence of *idle capacity* and *unsatisfied demand*

- **Justification**: *Idle capacity* and *unsatisfied demand* are cyclical, affect optimal decisions of seller and buyers. They may play a role in the effect of fiscal policy

Evidence on idle capacity

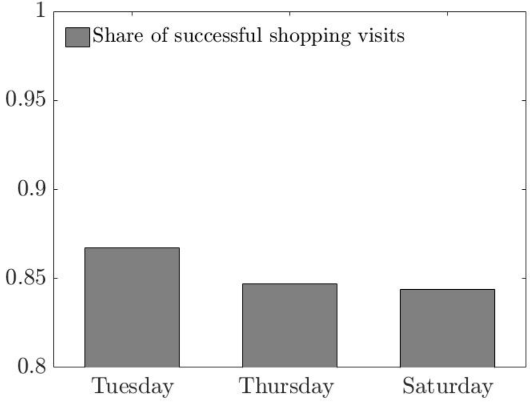
- ISM data: firms only utilize around **80%** of their **current** capacity



Source: Institute for Supply Management (ISM).

Evidence on unsatisfied demand

- Stockouts occur on **15%** of visits to shops (Taylor and Fawcett, 2001)
- Even more frequent at **25%** for online orders (Jing and Lewis, 2011)



Source: Taylor and Fawcett (2001).

Contribution to the literature

- **Theory of fiscal policy state dependence**: Christiano et al. (2011); Michaillat (2014); Canzoneri et al. (2016); Ziegenbein (2017); Jo and Zubairy (2022); Michaillat and Saez (2019)

  Our Contribution. We show that the *source of fluctuations* matters for cyclicality of fiscal multipliers; also, we *jointly* rationalise state dependence of both spending and taxation multipliers.

- **Empirics of fiscal policy state dependence**: Auerbach and Gorodnichenko (2012, 2013); Fazzari et al. (2014); Ziegenbein (2017); Ramey and Zubairy (2018); Barnichon and Matthes (2021)

  Our Contribution. Estimate *conditional* state-dependent fiscal multipliers; offer reconciliation of the empirical debate.

Roadmap

1 Framework
- Agents' optimisation problems
- Equilibrium types: flexprice vs. fixprice

2 Fiscal multipliers in a static model
- Analytical solutions for fiscal multipliers
- Derive cyclical properties of fiscal multipliers

Contribution 1

3 Fiscal multipliers in a quantitative dynamic model
- Develop a dynamic model with goods market search
- Features: long-term customer relationships, rigid prices
- Evaluate multipliers in shock-specific recessions

Contribution 2

4 Model-free econometric evidence
- Estimate multipliers in shock-specific recessions

Contribution 3

Framework: search-and-matching in the goods market

- Framework similar to Michaillat and Saez (2015)
- Matching function maps sales (y) to capacity (k) and purchasing visits (v), so that $y \leq \min\{k, v\}$:

$$\underbrace{y}_{\text{Sales}} = \Big(\underbrace{k^{-\delta}}_{\text{“Shop size”}} + \underbrace{v^{-\delta}}_{\text{“Queue length”}} \Big)^{-\frac{1}{\delta}}$$

- Goods market tightness (x):

$$x \equiv \underbrace{\frac{v}{k}}_{\text{“Shop congestion”}}$$

- Pr. of selling a product: $f(x) \equiv \frac{y}{k} = (1 + x^{-\delta})^{-\frac{1}{\delta}}, f' > 0$ Evidence
- Pr. of a successful visit: $q(x) \equiv \frac{y}{v} = (1 + x^{\delta})^{-\frac{1}{\delta}}, q' < 0$ Evidence
- Government spending affects v, and (supply-side) taxes affect k

Households shopping costs

- Households make v^c visits to shops, and there is cost of $\rho \in (0,1)$ of consumption good per visit
- Total sales (y^c) to households:

$$y^c = q(x)v^c = c + \rho v^c.$$

- One unit of consumption thus requires $\frac{1}{q(x)-\rho}$ visits, bringing total sales for one unit of consumption equal to:

$$1 + \rho \frac{1}{q(x) - \rho} = 1 + \frac{\rho x}{f(x) - \rho x} \equiv 1 + \gamma(x),$$

where $\gamma(x) \equiv \frac{\rho x}{f(x)-\rho x}$, $\gamma' > 0$ represents a 'congestion' wedge introduced by search-and-matching frictions

Households optimization

Consumption demand and labor supply

- Representative household gains utility from consumption of the produced good (c), non-produced good (m) that is in fixed supply ($\bar{m}$) and suffers disutility from supplying labour (l):

$$\max_{c,m,l} \left[\chi \frac{c^{1-\sigma}}{1-\sigma} + \zeta(m) - \frac{l^{1+\psi}}{1+\psi} \right] \qquad s.t.$$

$$p[1+\gamma(x)]c + m \leq wl + \Pi - T + \bar{m}.$$

- Today consider $\sigma = 1$ (solution for generic $\sigma \geq 0$ in the paper)
- Consumption and labour supply functions (normalise $\zeta'(\bar{m}) = 1$):

$$c(p,x) = \frac{\chi}{p[1+\gamma(x)]} \quad \text{and} \quad l(w) = w^{\frac{1}{\psi}},$$

where $\frac{\partial c}{\partial p} < 0$, $\frac{\partial c}{\partial x} < 0$ and $\frac{\partial l}{\partial w} > 0$

Firms optimization

Capacity, sales and labor demand

- Representative firm hires labour (n) that yields the following level of *current capacity* k:

$$k(n) = an^{\alpha}, \quad \alpha \in (0, 1].$$

- Due to search-and-matching frictions in the goods market, only a fraction $f(x)$ of current capacity is utilised:

$$y(x; n) = f(x)k(n) = f(x)an^{\alpha}.$$

- Profit maximisation given by:

$$\max_{n} \Pi = pf(x)an^{\alpha} - wn(1 + \tau)$$

- Labour demand function:

$$n(p, w, x; s) = \left[\frac{\alpha pf(x)a}{w(1 + \tau)}\right]^{\frac{1}{1-\alpha}},$$

where $\frac{\partial n}{\partial p} > 0$, $\frac{\partial n}{\partial x} > 0$ and $\frac{\partial n}{\partial w} < 0$.

Government

- Given its exogenous consumption of the produced good G and payroll tax rate τ, the government imposes a lump sum tax T on the consumer that ensures that balanced budget is run:

$$T = p[1 + \gamma(x)]G - wn\tau.$$

- Alternative fiscal instruments considered in the paper:
  $\underbrace{\text{public employment, consumption tax}}_{\text{Cyclicality just like gov. consumption}}, \underbrace{\text{labor income tax, sales tax}}_{\text{Cyclicality just like payroll tax}}$

- Will focus on cases where there's only either *demand-side fiscal policy* $(G \neq 0, \tau = 0)$, or *supply-side fiscal policy* $(G = 0, \tau \neq 0)$

Equilibrium: analytical conditions

- Goods market clearing:

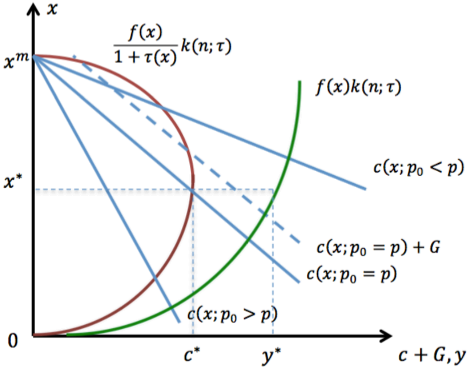
$$\frac{f(x)}{1+\gamma(x)}k(n;\tau) = c(p,x) + G$$

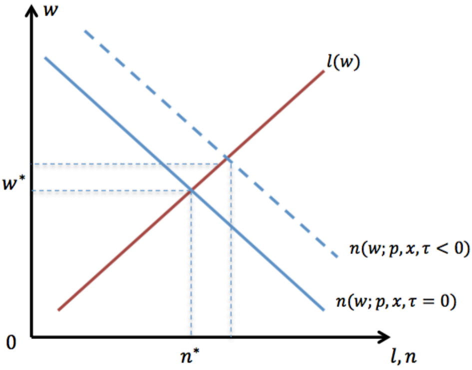
- Labour market clearing:

$$l(w) = n(p,x,w;\tau)$$

Equilibrium: visual representation

Impact response to expansionary fiscal policy: increase in G, and a fall in τ





Equilibrium determination

- Goods market clearing:

$$\frac{f(x)}{1+\gamma(x)}k(n;\tau) = c(p,x) + G$$

- Labour market clearing:

$$l(w) = n(p,x,w;\tau)$$

- **Indeterminacy**: *two* equilibrium conditions to hold for *three* variables (p, x and w), so infinitely many solutions

Closing the model: two polar cases

- **Competitive equilibrium**: fix tightness at the efficient level $(x = x^*)$, and let (p^*, w^*) clear the markets SP problem
  - Results for multipliers fully extend to other equilibria where tightness is fixed over the business cycle: Nash Bargaining, fixed markup pricing, as well as a generic Tightness Determination Mapping (TDM) More
- **Fixprice equilibrium**: fix the price $(p = p_0)$, let (x, w) clear the markets
  - Results for multipliers fully extend to other equilibria where tightness varies over the business cycle: rigid (Calvo-type) pricing, as well as a generic Frictional Mapping (FM) More

Fiscal multiplier

- Define GDP as $Z \equiv c + G$

- Demand-side fiscal multiplier given by:

$$\varphi^d(x) \equiv \frac{dZ}{dG} = \frac{dZ/Z}{d(G/Z)} = \frac{dc}{dG} + 1.$$

$$\frac{dc}{dG} = \frac{\partial c}{\partial p}\frac{dp}{dG} + \frac{\partial c}{\partial x}\frac{dx}{dG}. \tag{2}$$

- Supply-side fiscal multiplier given by:

$$\varphi^s(x) \equiv -\frac{dZ/Z}{d\tau} = -\frac{1}{c}\frac{dc}{d\tau}.$$

$$\frac{dc}{d\tau} = \frac{\partial c}{\partial p}\frac{dp}{d\tau} + \frac{\partial c}{\partial x}\frac{dx}{d\tau}. \tag{3}$$

Flexprice equilibrium multipliers

Proposition 1. *In a competitive equilibrium, the demand-side fiscal multiplier and the supply-side fiscal multiplier are equal and given by*:

$$\varphi^* = \frac{\alpha}{1+\psi} \in [0, 1].$$

- Note that $\epsilon^s = \frac{\partial \ln l}{\partial \ln w} = \frac{1}{\psi}$ and $|\epsilon^d| = |\frac{\partial \ln n}{\partial \ln w}| = \frac{1}{1-\alpha}$, so all that matters for the value of the multiplier are the relative elasticities of labour supply and labour demand
- Importantly, $\varphi^* \to 0$ as $\psi \to \infty$; and $\varphi^* = 1$ when $\alpha = 1, \psi = 0$
- Thus φ^* depends on labour market flexibility

Fixprice equilibrium multipliers

Fixed capacity fiscal multiplier

Lemma 3. *Define the fixed capacity fiscal multiplier $\theta(x)$ to be the demand-side fiscal multiplier under fixed labour supply in the economy, so that*

$$\theta(x) \equiv \frac{dZ}{dG}|_{\psi \to \infty}$$

then $\theta(x)$ has the following properties:

$$\theta(x) = \begin{cases} (-\infty, 0), & \text{if } x \in (x^*, x^m) \\ 0, & \text{if } x = x^* \\ (0, 1), & \text{if } x \in (0, x^*) \end{cases}$$

$$\theta'(x) < 0, \qquad x \in (0, x^m),$$

where x^m is given by $f(x^m) = \rho x^m$.

Demand-side fiscal multiplier (fixprice equilibrium)

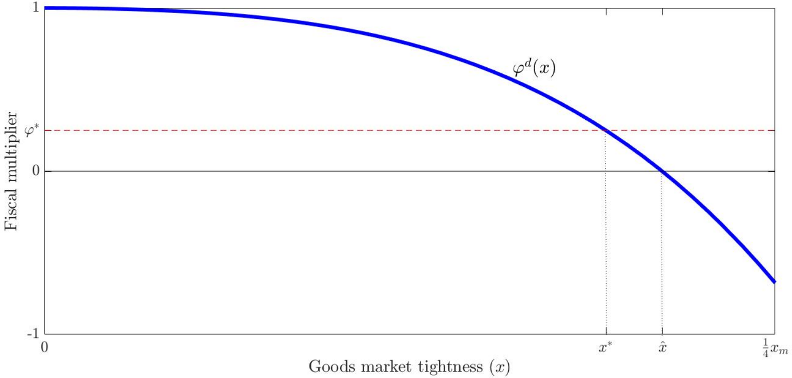
Proposition 2. *In a fixprice equilibrium, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} + \underbrace{\theta(x) \times (1 - \varphi^*)}_{\text{State-dependent component}}$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (-\infty, 1]$ and $\frac{d\varphi^d(x)}{dx} < 0, \forall x \in (0, x^m)$.

- $\frac{d\varphi^d}{dx} < 0$ so φ^d strictly falls in tightness
- Note that $\varphi^d(x^*) = \varphi^*$, so competitive and fixprice equilibrium multipliers can coincide
- Convex combination: $1 \times \varphi^* + \theta(x) \times (1 - \varphi^*)$

Demand-side fiscal multiplier (fixprice equilibrium)



Demand-side fiscal multiplier (fixprice equilibrium)

Corollary 1. *There always exists tightness* $\hat{x} \in [x^*, x^m)$ *such that* $\varphi^d(x) < 0, \forall x \in (\hat{x}, x^m)$*, and it is given by*:

$$\hat{x} = \theta^{-1}\left(-\frac{\varphi^*}{1-\varphi^*}\right),$$

and hence $\frac{d\hat{x}}{d\varphi^*} > 0$.

- Endogenous supply response does not eliminate the possibility of a negative demand-side multiplier
- There always exists a fixprice equilibrium that is sufficiently tight to make government spending crowd out private consumption more than one for one

Supply-side fiscal multiplier (fixprice equilibrium)

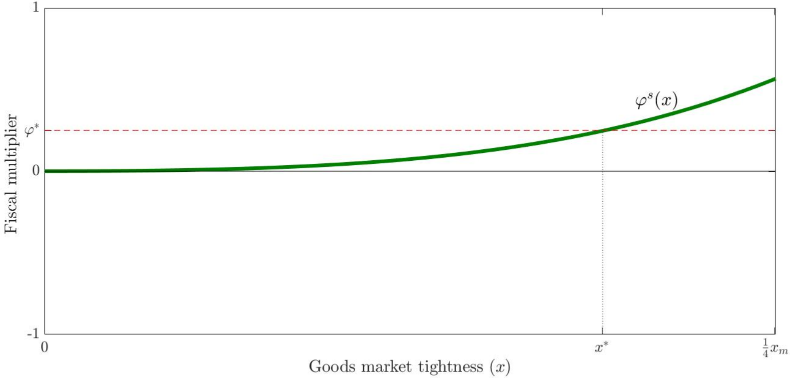
Proposition 3. *In a fixprice equilibrium, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \underbrace{\varphi^*}_{\text{State-invariant component}} - \underbrace{\theta(x) \times \varphi^*}_{\text{State-dependent component}},$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the competitive equilibrium multiplier. Hence, $\varphi^d(x) \in (0, +\infty)$ and $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x^m)$

- $\frac{d\varphi^s}{dx} > 0$, so moves in the same direction as tightness
- Again, note that $\varphi^s(x^*) = \varphi^*$, just like for the demand-side multiplier

Supply-side fiscal multiplier (fixprice equilibrium)



Relationship between the two multipliers

Corollary 2. *In a fixprice equilibrium, the demand-side and supply-side fiscal multipliers are related as*

$$\underbrace{\varphi^d(x)}_{\text{Demand-side multiplier}} = \underbrace{\theta(x)}_{\text{Fixed capacity multiplier}} + \underbrace{\varphi^s(x)}_{\text{Supply-side multiplier}},$$

so that the difference between the two is just the fixed capacity fiscal multiplier.

- Given the properties of $\theta(x)$, it follows that $\varphi^d(x) > \varphi^s(x)$ if $x < x^*$ and vice versa
- Is there any stimulative role for fiscal austerity?

Spending Austerity Threshold

Corollary 3. *Suppose* $\varphi^* < 0.5$*, then there always exists tightness* $\tilde{x} \in [x^*, x^m)$ *such that:*

$$-\varphi^d(x) > \varphi^s(x) > \varphi^d(x), \quad \forall x \in (\tilde{x}, x^m).$$

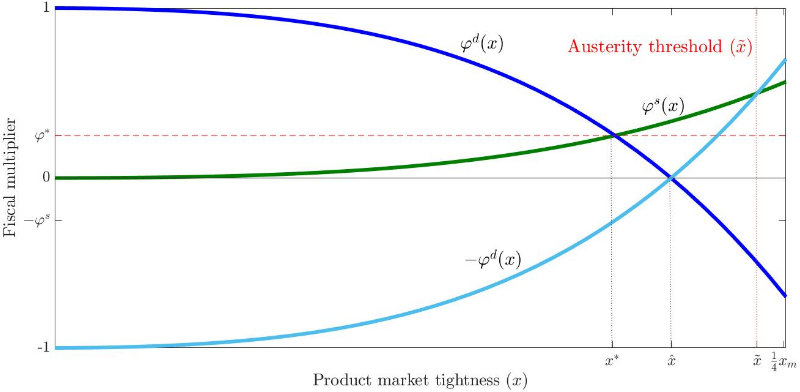
Furthermore, $\tilde{x}$ *is given by*:

$$\tilde{x} = \theta^{-1}\left(-\frac{2\varphi^*}{1-2\varphi^*}\right), \quad \varphi^* < 0.5$$

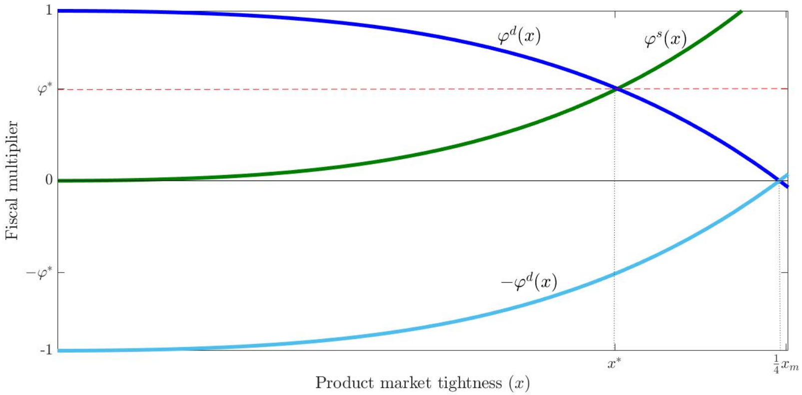
and hence $\frac{d\tilde{x}}{d\varphi^*} > 0$.

- If the labour market is sufficiently inelastic $(\varphi^* < 0.5)$ and the fixprice equilibrium is sufficiently tight $(x > \tilde{x} > x^*)$, then *spending austerity* is the policy with the highest multiplier

Inelastic labour market ($\varphi^* < 0.5$)



Elastic labour market ($\varphi^* > 0.5$)



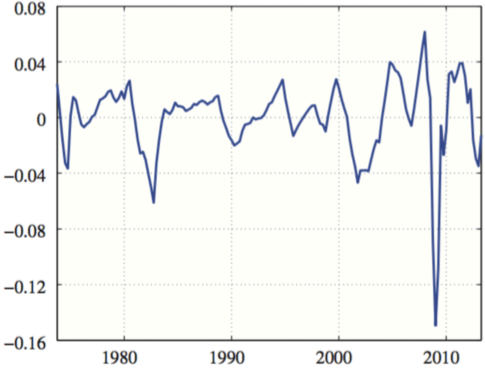
Cyclicality of fiscal multipliers

Corollary 4. *In a competitive equilibrium, both demand-side and supply-side multipliers are acyclical.*

Corollary 5. *In a fixprice equilibrium, the demand-side multiplier is countercyclical under demand-driven fluctuations, and procyclical under supply-driven fluctuations.*

Corollary 6. *In a fixprice equilibrium, the supply-side multiplier is countercyclical under supply-driven fluctuations, and procyclical under demand-driven fluctuations.*

What type of equilibrium? (US)



A. Cyclical component of product market tightness

Source: Michaillat and Saez (2015).

Quantitative dynamic model – overview

1. **Long-term customer relationships**: a fraction $\eta \in (0, 1]$ destroyed in any given period; new customer relationships governed by the matching function

$$\left[(k_t - (1-\eta)y_{t-1})^{-\delta} + v_t^{-\delta}\right]^{-\frac{1}{\delta}}, \qquad \delta > 0$$

   and goods market tightness now given by $x_t \equiv \frac{v_t}{k_t-(1-\eta)y_{t-1}}$

2. **Partial price rigidity**: let $\{p_t^*\}_{t=0}^{\infty}$ be a sequence of prices consistent with an equilibrium featuring efficient tightness; only a fraction $(1-\varepsilon) \in [0, 1]$ of firms get to set this price:

$$p_t = p_{t-1}^{\varepsilon}(p_t^*)^{1-\varepsilon}, \quad \varepsilon \in [0, 1].$$

Conditional state-dependent fiscal multipliers

- Use fully non-linear solution to our dynamic model under perfect foresight to construct spending and tax-cut multipliers in recession/expansion episodes generated by particular shocks
- Obtain impulse response to a preference/technology shock $\{GDP_j^{shock}\}_{j=0}^{H}$, where *shock* is one-time innovation to χ or a
- Obtain impulse response to simultaneous preference/technology and spending shock $\{GDP_j^{shock+G}\}_{j=0}^{H}$
- *Conditional* spending multiplier:

$$\varphi^G(shock) = \frac{\sum_{j=0}^{H}\left[GDP_j^{shock+\varepsilon^G} - GDP_j^{shock}\right]}{\sum_{j=0}^{H}\left[G_j^{\varepsilon^G} - g\right]}$$

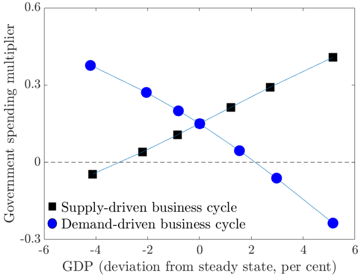
- Similarly, *conditional* tax cut multiplier:

$$\varphi^{\tau^i}(shock) = \frac{\left[GDP_H^{shock-\varepsilon^\tau} - GDP_H^{shock}\right]/\overline{GDP}}{\varepsilon^{\tau^i}}$$

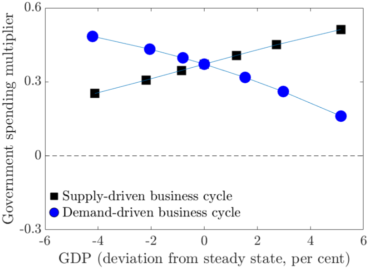
Conditional spending multipliers

- Government spending 1% of GDP

Impact multiplier



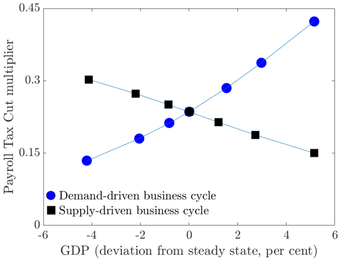
Cumulative 2-year multiplier



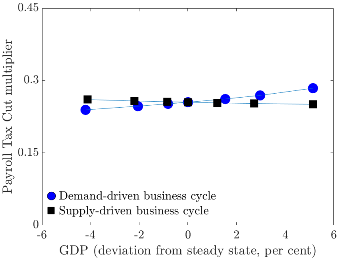
Conditional tax cut multipliers

- Cut payroll tax by 1%

Impact multiplier



Cumulative 2-year multiplier



Conditional state-dependent spending multipliers

- Extend the one-step IV procedure from Ramey and Zubairy (2018):

$$\sum_{s=t}^{t+H}\left(\frac{GDP}{GDP^*}\right)_s = \mathbf{1}\{U_{t-1} < \bar{U}\}\left[\alpha_H^E + \beta_H^E \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^E \mathbf{z}_{t-1}\right] +$$

$$\mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} < \tilde{\pi}_{t-1}\}\left[\alpha_H^{DR} + \beta_H^{DR} \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^{DR} \mathbf{z}_{t-1}\right] +$$

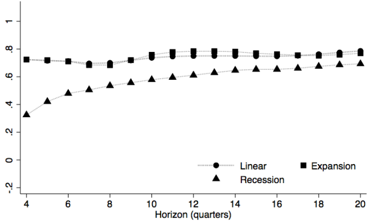
$$\mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} \geq \tilde{\pi}_{t-1}\}\left[\alpha_H^{SR} + \beta_H^{SR} \sum_{s=t}^{t+H}\left(\frac{G}{GDP^*}\right)_s + \gamma_H^{SR} \mathbf{z}_{t-1}\right] + \varepsilon_{t+H}$$

- Spending instrument: historical data on military spending news in US (1889-2015) (Owyang, Ramey and Zubairy, 2013)

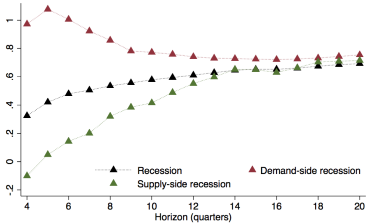
Econometric evidence: conditional state-dependent spending multipliers

US data (1889-2015)		2 year		4 year	
State	(1)	(2)	(3)	(4)	(5)
Linear	**0.70*****				
	(0.06)				
$\mathbf{1}\{U_t < \bar{U}\}$		**0.68*****	**0.68*****	**0.76*****	**0.76*****
		(0.10)	*(0.10)*	*(0.13)*	*(0.12)*
$\mathbf{1}\{U_t \geq \bar{U}\}$		**0.54*****		**0.65*****	
		(0.13)		*(0.08)*	
$\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \tilde{\pi}_t\}$			**0.86*****		**0.71*****
			(0.33)		*(0.12)*
$\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \tilde{\pi}_t\}$			**0.32*****		**0.63*****
			(0.11)		*(0.09)*
T	416	416	416	408	408

Government spending multipliers across horizons



Government spending multipliers in recessions and expansions across horizons



Government spending multipliers in demand-side and supply-side recessions across horizons

Conditional state-dependent tax cut multipliers

- Extend the procedure from Eskandari (2019):

$$\ln GDP_{t+H} - \ln GDP_{t-1} = \mathbf{1}\{U_{t-1} < \bar{U}\}\left[\alpha_H^E + \beta_H^E \tau_t + \gamma_H^E \mathbf{z}_{t-1}\right] +$$

$$\mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} < \tilde{\pi}_{t-1}\}\left[\alpha_H^{DR} + \beta_H^{DR} \tau_t + \gamma_H^{DR} \mathbf{z}_{t-1}\right] +$$

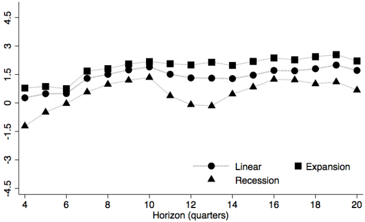
$$\mathbf{1}\{U_{t-1} \geq \bar{U}; \pi_{t-1} \geq \tilde{\pi}_{t-1}\}\left[\alpha_H^{SR} + \beta_H^{SR} \tau_t + \gamma_H^{SR} \mathbf{z}_{t-1}\right] + \varepsilon_{t+H}$$

- Use exogenous variation in US tax rates (1947-2007) from narrative accounts (Romer and Romer, 2010)

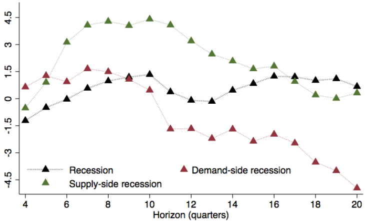
Conditional state-dependent tax cut multipliers

US data (1947-2007)	2 year			4 year	
State	(1)	(2)	(3)	(4)	(5)
Linear	**1.50**				
	(1.14)				
$\mathbf{1}\{U_t < \bar{U}\}$		**1.81**	**1.81**	**2.37****	**2.37****
		(1.17)	*(1.12)*	(0.99)	*(0.99)*
$\mathbf{1}\{U_t \geq \bar{U}\}$		**0.98**		**1.24**	
		(1.07)		*(0.87)*	
$\mathbf{1}\{U_t \geq \bar{U}; \pi_t < \tilde{\pi}_t\}$			**1.49**		**-1.98**
			(1.04)		*(2.75)*
$\mathbf{1}\{U_t \geq \bar{U}; \pi_t \geq \tilde{\pi}_t\}$			**4.29***		**1.80***
			(2.18)		*(1.00)*
T	240	240	240	240	240

Tax cut multipliers across horizons (US Romer-Romer narrative tax shocks, 1947-2007)

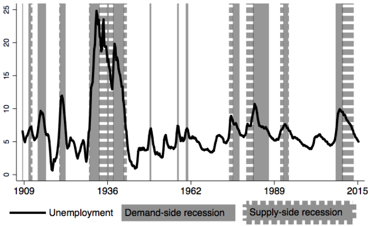


Tax cut multipliers in recessions and expansions across horizons

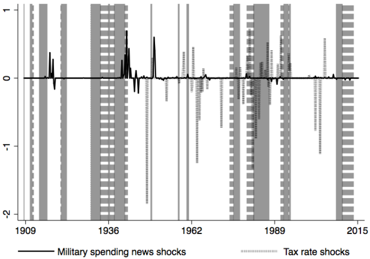


Tax cut multipliers in demand-side and supply-side recessions across horizons

Demand-side and supply-side recessions: a closer look



Demand- and supply-side recessions



Fiscal shocks and sources of recessions

Conclusion

- We develop a theory of state-dependent spending and taxation multipliers, in a framework with idle capacity and unsatisfied demand
- Key finding: the cyclicality of fiscal multipliers depends on the source of fluctuations
- Econometric estimation conditional on the source of fluctuations corroborates our theory on the state dependence of fiscal multipliers
- Provide a resolution to contrasting empirical findings

Thank you!

APPENDIX

(More) general cases

- Competitive equilibrium considered before involves pinning down tightness at the efficient level x^* and letting price and wage adjust fully flexibly in order to make sure it stays there following shocks
- However, this is not the only way to pin down tightness: first consider **Nash bargaining** and **fixed markup pricing** as two particular alternatives, then introduce a general **Tightness Determination Mapping (TDM)**
- Also, before we considered fixed price as a particular deviation from fully flexible pricing; now consider more general kind of frictions: generic price rigidity, informational frictions
- Then introduce a general **Frictional Mapping (FM)**

Nash bargaining

- Consumers' surplus from buying an additional unit at price $\tilde{p}$ after a match is made:
$$\mathcal{B}(\tilde{p}) = \frac{\chi}{c} - \tilde{p}.$$
- Firms' surplus from selling an extra unit at price $\tilde{p}$ after a match is made:
$$\mathcal{S}(\tilde{p}) = \tilde{p} - pf(x).$$
- Assuming that consumers' bargaining power given by $\beta \in (0, 1)$, solution to Nash bargaining given by:
$$(1 - \beta)\mathcal{S}(p) = \beta\mathcal{B}(p).$$

Nash bargaining

- Combining the solution to Nash bargaining with agents' optimality conditions:
$$\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}, \quad \frac{dx^L}{d\beta} < 0.$$
- Nash bargaining pins down tightness at $x = x^L$, which, as can be seen above, is indeed invariant to demand-side and supply-side shocks, as required for a long-run equilibirium
- The pair (p^L, w^L) now adjusts fully flexibly to ensure that all optimality and market clearing conditions are satisfied with $x = x^L$
- Hosios condition:
$$\beta = \frac{1}{1 + \frac{\gamma(x^*)}{1-f(x^*)}}.$$

Fixed markup pricing

- Assume that equilibrium prices are set at a fixed markup over the marginal cost:
$$p = \mu \times mc,$$
where mc is firms' marginal cost at the optimum, $\mu \geq 1$ is a markup *parameter*.
- From firms' profit maximisation condition:
$$p = \frac{1}{f(x)} mc.$$
- Hence the markup parameter pins down the equilibrium level of tightness according to:
$$f(x^L) = \frac{1}{\mu}, \quad \frac{dx^L}{d\mu} < 0.$$

Fixed markup pricing

- It is apparent that x^L is invariant to demand-side and supply-side shocks, so indeed qualifies for a long-run equilibrium
- The pair (p^L, w^L) now adjusts fully flexibly to ensure that agents' optimality and market clearing conditions are satisfied with $x = x^L$, pinned down by the markup parameter μ
- "Hosios" condition:
$$\mu = \frac{1}{f(x^*)}.$$
- Note that as we remove the matching cost $(\rho = 0)$, $f(x^*) \to 1$, and $\mu \to 1$, so converge to standard marginal cost pricing under perfect competition.

Tightness Determination Mapping

Definition 3. *A Tightness Determination Mapping (TDM) $\mathcal{M}$ is given by:*

$$\mathcal{M}: \quad \left\{\Omega^M, \Omega^S, \Omega^T\right\} \rightarrow x^L,$$

where $\Omega^M = \{\rho, \gamma, \psi, \alpha\}$ is the set of model structural parameters, $\Omega^S = \{\chi, a, G, s\}$ is the set of shock parameters, Ω^T is the set of parameters specific to the TDM and x^L is the resulting tightness.

Further, a TDM $\mathcal{M}$ is said to be ***shock invariant*** *if and only if*

$$\mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right) = \mathcal{M}\left(\Omega^M, \Omega^T\right),$$

so that the shock parameters do not affect the determination of tightness.

Flexible equilibrium

Definition 4. *A* ***flexible*** *equilibrium is a vector* $(p^L, w^L, \mathcal{M})$*, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with tightness pinned down at a level* $x^L = \mathcal{M}\left(\Omega^M, \Omega^S, \Omega^T\right)$.

- Competitive equilibrium is a special case of flexible equilibrium under $x^L = x^*$
- Nash bargaining is a particular TDM with $\Omega^T = \{\beta\}$, which pins down tightness according to $\frac{1-\beta}{\beta} = \frac{\gamma(x^L)}{1-f(x^L)}$ and then prices and wages adjust flexibly to ensure tightness stays at that level
- Similarly, fixed markup pricing is a TDM with $\Omega^T = \{\mu\}$, which pins down tightness according to $f(x^L) = \frac{1}{\mu}$
- Note that both are shock-invariant TDMs

Flexible equilibrium multipliers

Proposition 8. *In any flexible equilibrium generated by a shock-invariant TDM, the demand-side fiscal multiplier and the supply-side fiscal multiplier are equal and given by*:

$$\varphi^* = \frac{\alpha}{1+\psi} \in [0, 1].$$

- In any flexible equilibrium generated by a shock-invariant TDM (including the competitive equilibrium as a special case) both multipliers are equal, are invariant to both demand-side and supply-side shocks and lie strictly between 0 and 1 Back

Price rigidity

- Before we considered fixed prices a way to resolve indeterminacy, but can consider a more general case of rigid prices
- In particular, for a given flexible equilibrium $(p^L, w^L, \mathcal{M})$, can consider a **rigid price equilibrium**, where p is set according to:

$$p = (p_0)^\varepsilon (p^L)^{1-\varepsilon}, \quad \varepsilon \in (0, 1]$$

  where ε is the degree of price rigidity and p_0 is a parameter.
- Fixpirce equilibrium remains a special case under $\varepsilon = 1$

Demand-side fiscal multiplier (rigid price equilibirum)

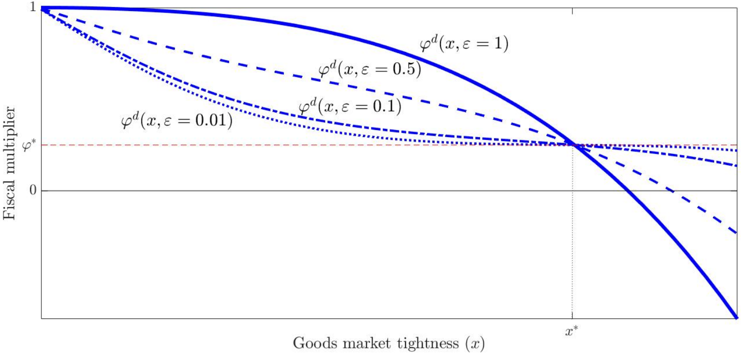
Proposition 9. *In a rigid price equilibrium* (p_0, x, w, ε)*, the demand-side fiscal multiplier* $\varphi^d(x)$ *is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\{1-(1-\varepsilon)g(x,x^L)\}\right]$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ *is the long-run equilibrium multiplier and the function* $g(x, x^L)$ *is given by:*

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}.$$

Demand-side fiscal multiplier (rigid price equilibrium)



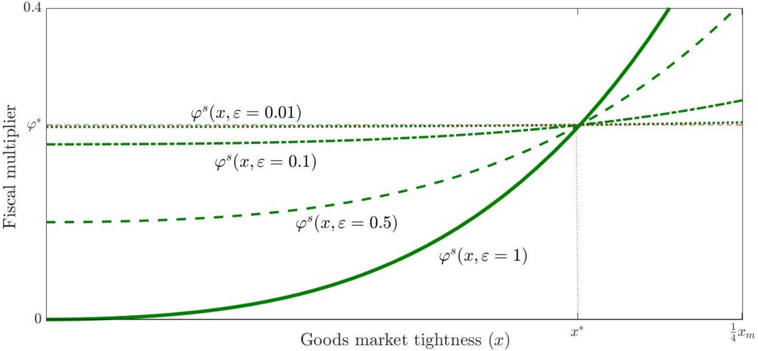
Supply-side fiscal multiplier (rigid price equilibrium)

Proposition 10. *In a rigid price equilibrium* (p_0, x, w, ε)*, the supply-side fiscal multiplier* $\varphi^s(x)$ *is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \varepsilon\varphi^*,$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ *is the long-run equilibrium multiplier. Hence,* $\varphi^d(x) \in (0, +\infty)$ *and* $\frac{d\varphi^d(x)}{dx} > 0, \forall x \in (0, x^m)$

Supply-side fiscal multiplier (rigid price equilibrium)



Frictional Mapping

Definition 5. *For a given flexible equilibrium $(p^L, w^L, \mathcal{M})$, a Frictional Mapping (FM) $\mathcal{T}$ is given by:*

$$\mathcal{T}: \quad \left\{p^L, \Omega^F\right\} \to p^F,$$

where Ω^F is the set of parameters specific to the FM and p^F is the resulting price.

Moreover, the Frictional Mapping $\mathcal{T}(p^L; \Omega^F)$ is said to be ***contractionary*** *if and only if*

$$\frac{d \ln p}{d \ln p^L} = \frac{d\mathcal{T}(p^L; \Omega^F)}{dp^L} \frac{p^L}{p} \in [0, 1).$$

Frictional equilibrium

Definition 6. *For a given flexible equilibrium* $(p^L, w^L, \mathcal{M})$*, a* ***frictional*** *equilibrium is a vector* $(p^F, w^F, \mathcal{T})$*, and associated allocations, such that the agents' optimality conditions and the market clearing conditions are satisfied with the price given by:*

$$p^F = \mathcal{T}(p^L).$$

- Rigid price equilibrium is a special case of a frictional equilibrium for $\mathcal{T}(z) = (p_0)^{\varepsilon}(z)^{1-\varepsilon}$, $\Omega^F = \{p_0, \varepsilon\}$, $\varepsilon \in (0, 1]$
- Further, the above frictional mapping associated with a rigid price equilibrium is indeed contractionary, since

$$\frac{d\mathcal{T}(z;\Omega^F)}{dz}\frac{z}{p^F} = (1-\varepsilon) \in [0, 1),$$

as $\varepsilon \in (0, 1]$

Demand-side fiscal multiplier (frictional equilibrium)

Proposition 11. *In a frictional equilibrium generated by a Frictional Mapping $\mathcal{T}(.)$, the demand-side fiscal multiplier $\varphi^d(x)$ is given by*

$$\varphi^d(x) = \varphi^* + \theta(x) \times \left[(1-\varphi^*)\{1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)} g(x, x^L)\}\right]$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier and the function $g(x, x^L)$ is given by:

$$g(x, x^L) = \frac{f(x) - \rho x}{f(x^L) - \rho x^L}.$$

Further, $\frac{d\varphi^d(x)}{dx}|_{x=x^L} < 0$ as long as $\mathcal{T}(.)$ is contractionary.

Supply-side fiscal multiplier (frictional equilibrium)

Proposition 12. *In a frictional equilibrium generated by a Frictional Mapping $\mathcal{T}(.)$, the supply-side fiscal multiplier $\varphi^s(x)$ is given by*

$$\varphi^s(x) = \varphi^* - \theta(x) \times \left(1 - \frac{\mathcal{T}'(p^L)p^L}{\mathcal{T}(p^L)}\right)\varphi^*,$$

where $\varphi^* = \frac{\alpha}{1+\psi}$ is the flexible equilibrium multiplier.
Further, $\frac{d\varphi^s(x)}{dx}|_{x=x^L} > 0$ as long as $\mathcal{T}(.)$ is contractionary.

Cyclicality of multipliers – flexible equilibria

Proposition 13. *In any flexible equilibrium generated by policy-invariant TDM both demand-side and supply-side multipliers are acyclical.*

Proof. Trivial – in any such flexible equilibrium both multipliers are given by φ^* and clearly acyclical.

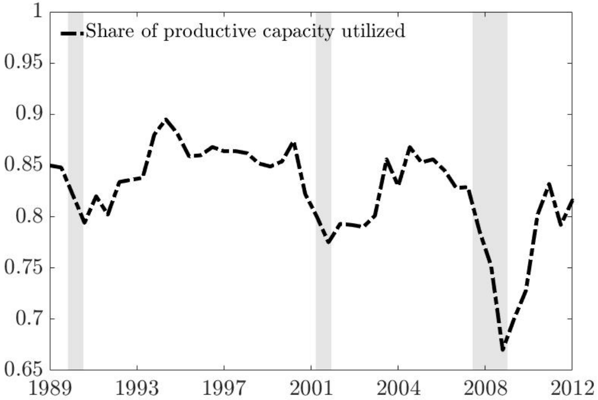
Cyclicality of multipliers – frictional equilibria

Proposition 14. *In any frictional equilibrium generated by a contractionary frictional mapping, in the local neighbourhood of the flexible equilibrium allocation, the demand-side multiplier is countercyclical under demand-driven fluctuations, and procyclical under supply-driven fluctuations*

Proposition 15. *In any frictional equilibrium generated by a contractionary frictional mapping, in the local neighbourhood of the flexible equilibrium allocation, the supply-side multiplier is procyclical under demand-driven fluctuations, and countercyclical under supply-driven fluctuations* Back

Goods market clearing? Firms not convinced

- ISM data: firms only utilize around **80%** of their **current** capacity

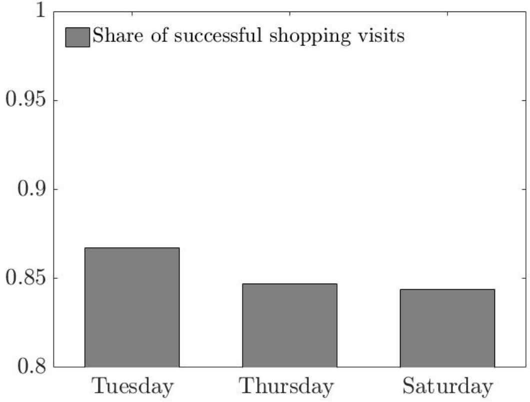


Source: Institute for Supply Management (ISM).

Back More

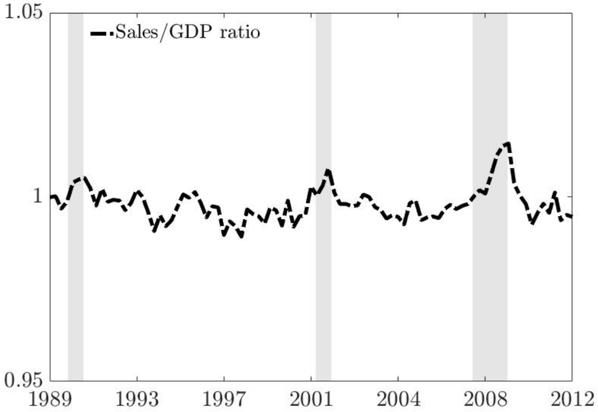
Goods market clearing? Households not sure either

- Stockouts occur on **15%** of visits to shops (Taylor and Fawcett, 2001)
- Even more frequent at **25%** for online orders (Jing and Lewis, 2011)



Source: Taylor and Fawcett (2001).

Accounting for inventories



Source: Bureau of Economic Analysis (BEA).

Back

Social planner's problem

- First consider the social planner's problem (tightness *not* taken as given):

$$\max_{c,m,l,v}\left[\chi\frac{c^{1-\sigma}}{1-\sigma}+\zeta(m)-\frac{l^{1+\psi}}{1+\psi}\right]\qquad s.t.$$

$$c+G+\rho v=\left(k^{-\delta}+v^{-\delta}\right)^{-\frac{1}{\delta}},$$

$$k=al^{\alpha},\, m=\bar{m}.$$

- Establishes *efficient tightness* x^*:

$$f'(x^*)=\rho,$$

as well as efficient levels of other variables (c^*, l^*, v^*)

Back