Natural Rate Chimera and Bond Pricing Reality

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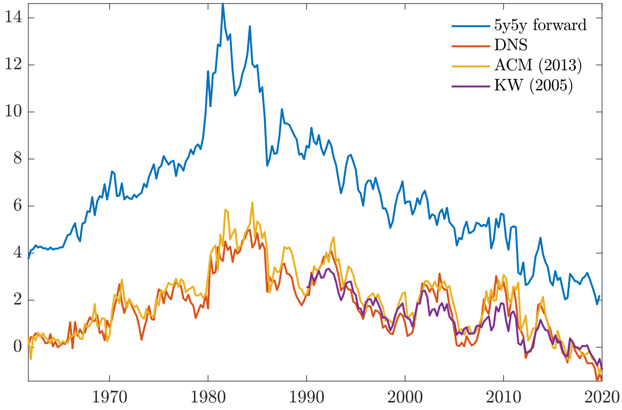
Downward trend in long-term yields - often attributed to falling term premia



Note: US data. *Source:* Yields constructed using Gürkaynak et al. (2007) parameters. Adrian et al. (2013) and DNS term premia are own calculations, while Kim and Wright (2005) are taken from FRED.

Natural rate as a (often neglected) driver of the yield curve

- Recall nominal long-term rate decomposition:

$$y_t(\tau) = \frac{1}{\tau} E_t(r_t + \ldots + r_{t+\tau-1}) + \frac{1}{\tau} E_t(\pi_t + \ldots + \pi_{t+\tau-1}) + TP_t(\tau)$$

- Long-term trend:

$$y_t^*(\tau) \equiv \lim_h E_t y_{t+h}(\tau) = r_t^* + \pi_t^* + TP_t^*(\tau), \text{ with } X_t^* = \lim_h E_t X_{t+h}$$

- Trend decline in bond yields reflects trend decline in:
  1. Inflation expectations and/or
  2. Real-rate expectations and/or
  3. Term premia
- ... with most of the finance literature assuming constant long-term equilibrium of (1) and (2), ignoring macro trends, leading to trending term premia (see e.g. Kim and Wright, 2005; Cochrane, 2007; Adrian et al., 2013; Joslin et al., 2014)

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- Some term structure models incorporate time-varying equilibria (see e.g. Dewachter and Lyrio, 2006; Dewachter et al., 2014; Christensen and Rudebusch, 2019) but do not link r^* to macro-trends.
- Bauer and Rudebusch (2020) show: (i) trend inflation π_t^* and the natural rate of interest r_t^* are *fundamental* determinants of the yield curve and that (ii) term premia exhibit business-cycle characteristics, and are less trending. They they use model-independent π^* and r^*-estimates (OSE) or model-based estimates (ESE), but without macro link

Macro: natural real rate as 'benchmark' for the actual real rate

- Macro models (Laubach and Williams (2003) and followers) infer natural real rate from its 'role' as benchmark for the actual real rate
- (Stylised and backward-looking) IS curve:

$$\tilde{x}_t = a\tilde{x}_{t-1} + \beta\left(r_{t-1} - r^*_{t-1}\right) + \varepsilon_t \text{ where } \tilde{x}_t \text{ is the output gap}$$

Equating the actual real rate r_t with the natural real rate $r*_t$ eventually closes the output gap.

What we do

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- and as benchmark real interest rate that closes the output gap

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We

- close the **semi-structural macro model** of Laubach and Williams (2003) with an **arbitrage-free term structure model**, thereby using cross-sectional information in yields to estimate the natural rate
- use a **Bayesian approach** to estimate the model for US and EA
- obtain simultaneously natural-rate and term premia estimates

Macro part: vintage Laubach/Williams

- **IS Curve:** $\tilde{x}_t = a_1 \tilde{x}_{t-1} + a_2 \tilde{x}_{t-2} + \frac{a_3}{2}\left(\tilde{r}_{t-1} + \tilde{r}_{t-2}\right) + \varepsilon_t^{\tilde{x}}$,
  with $\tilde{x}_t = x_t - x_t^*$ output gap, with $x_t^* = x_{t-1}^* + g_{t-1} + \varepsilon_t^{x^*}$, and $\tilde{r}_t = r_t - r_t^*$ real rate gap
- **Natural rate of interest:** $r_t^* = 4g_t + z_t$, with $g_t, z_t \sim I(1)$.
- **Ex ante real rate:** $r_t = i_t - E_t\pi_{t+1}$,
  with i_t short term nominal interest rate, and $E_t\pi_{t+1} = E_t(\pi_{t+1}^* + \tilde{\pi}_{t+1})$ model-consistent inflation expectation
- **Phillips curve:** $\tilde{\pi}_t = b_1\tilde{\pi}_{t-1} + b_2\tilde{x}_{t-1} + \varepsilon_t^{\pi}$
  with inflation gap $\tilde{\pi}_t = \pi_t - \pi_t^*$ and $\pi_t^* \sim I(1)$.

The term structure of interest rates: arbitrage-free Nelson-Siegel model

- Nominal bond yields $y_t(\tau)$ (where $y_t(1) \equiv i_t$) on a risk-free zero-coupon bonds with maturity τ are explained by level L_t, slope S_t and curvature C_t factors:

$$y_t(\tau) = \mathcal{A}(\tau) + L_t + \theta_s(\tau)S_t + \theta_c(\tau)C_t + \varepsilon_t^{\tau},$$

where $\theta_s(\tau) = \frac{1-\exp(-\lambda\tau)}{\lambda\tau}$ and $\theta_c(\tau) = \frac{1-\exp(-\lambda\tau)}{\lambda\tau} - \exp(-\lambda\tau)$.

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- $\mathcal{A}(\tau)$ rules out risk-less arbitrage
- **Stochastic trend in the level factor** $L_t = L_t^* + \tilde{L}_t$, with $L_t^* = i_t^* = \pi_t^* + r_t^*$ (long-run Fisher equation) and $\tilde{L}_t = a_L\tilde{L}_{t-1} + \varepsilon_t^L$
- Model-implied anchor: $i_t^* = \lim_{h\to\infty} E_t i_{t+h}$
- Slope S_t and curvature C_t are stationary around a constant mean
- Time-varying level of **'natural yield curve'**, but constant shape

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- Slope S_t and curvature C_t are stationary around a constant mean
- Time-varying level of **'natural yield curve'**, but constant shape
- Model-implied **term premium**: $TP_t(\tau) = y_t(\tau) - \frac{1}{\tau}\sum_{h=0}^{\tau-1} E_t i_{t+h}$

State space representation

The model can be summarized as

$$\zeta_t = \gamma + C\xi_t + Du_t \tag{1}$$

$$\xi_t = \mu + A\xi_{t-1} + Be_t, \tag{2}$$

where

$$\zeta_t = \begin{pmatrix} y_t(\underline{\tau}) & \dots & y_t(\bar{\tau}) & x_t & \pi_t \end{pmatrix}',$$

and

$$\xi_t = \begin{pmatrix} L_t^c & S_t & C_t & \pi_t^* & y_t^* & g_t & z_t & \tilde{\pi}_t & \tilde{y}_t & L_{t-1}^c & S_{t-1} & C_{t-1} & \tilde{\pi}_{t-1} & \tilde{y}_{t-1} \end{pmatrix}'.$$

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Note:

- Measurement error in all observed yields except $y_t(1) \equiv i_t$ as it enters the IS curve.
- Include survey information on long-horizon inflation expectations ($E_t^{surv}\pi_{t+\infty}$), short-horizon short-rate expectations ($E_t^{surv} i_{t+4}$) and – for the euro area – long-horizon long-rate expectations ($E_t^{surv} y_{t+\infty}(40)$).

Bayesian Estimation

- Gibbs sampler and Durbin and Koopman (2002) simulation smoother (100,000 draws, 90,000 burn in, keep every 10th's).
- Initialization is based on HP-Filter for trends, and OLS regressions for parameters
- λ is calibrated (otherwise MH needed), based on ML estimates of standard DNS model.
- We use conjugate priors. These are flat, except for σ_g^2, for which we assume that the standard deviation of the expected change in the trend growth over one century is only 0.6ppt.
- Reject draws that violate $a_3 < 0$ and $b_2 > 0$

Data

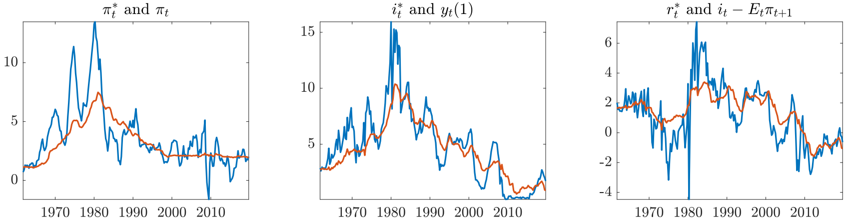
US data:

- Zero-coupon yield data constructed by Gürkaynak et al. (2007) to back out yields of maturities 1,2,...,8 and 12,16,...,40 quarters (16 yields)
- (log) quarterly real GDP (#GDPC1) and annual PCE inflation based on (#PCECTPI) both from FRED
- Surveys: Long-horizon inflation expectations (PTR), short-horizon short-rate expectations (Consensus Economics)
- Sample spans from 1961Q2 until 2019Q2

EA data:

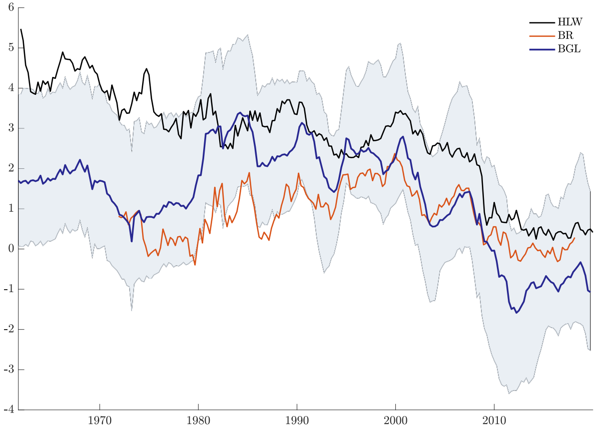
- OIS rates for maturities 1,2,4,8,..,40 quarters (13 yields)
- (log) quarterly real GDP and HICP inflation from the ECB
- Surveys: Long-horizon inflation expectations, short-horizon short-rate expectations, long-horizon long-rate expectations (all Consensus)
- Sample spans 1995Q1 until 2019Q2

US rates vs. trends



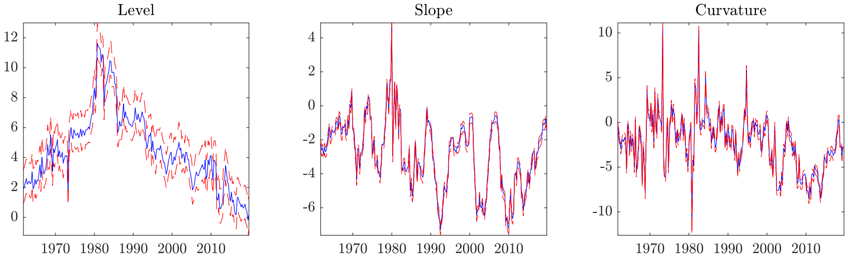
Note: Estimated trends in blue and observed data in red.

US natural rate estimates



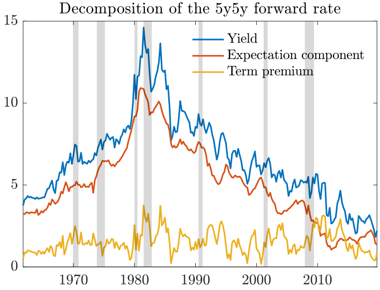
Note: 5% to 95% credibility bands depicted by blue-shaded area.

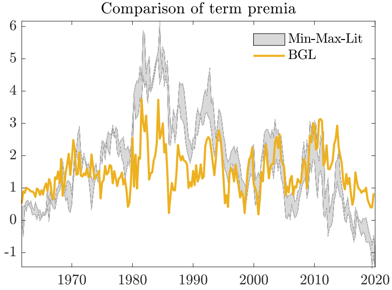
US yield curve factors



Note: 5% to 95% credibility bands in red.

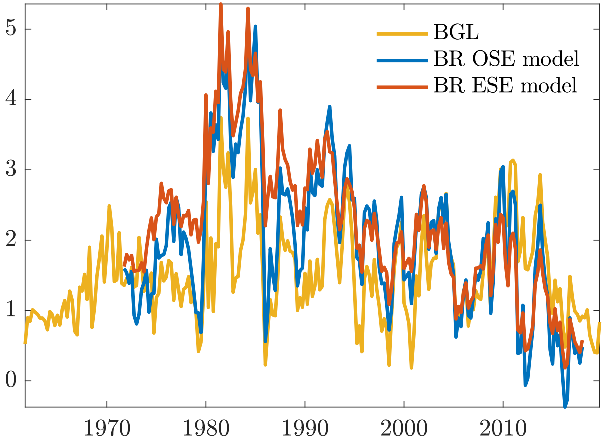
Decomposition of yields





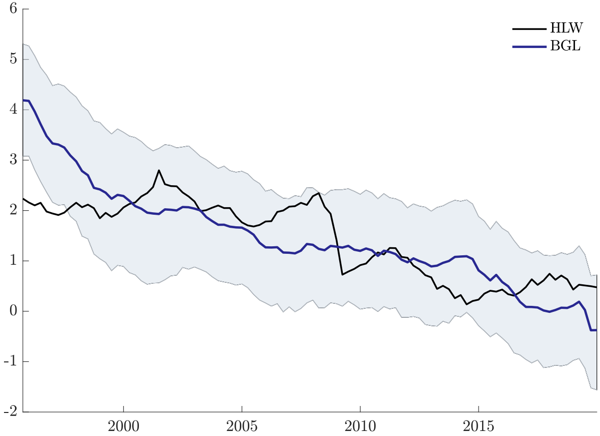
Note: NBER recessions in gray. RHS: min-max-range (grey area) contains: Kim and Wright (2005) (taken from FRED), Adrian et al. (2013) and a DNS model following Diebold and Li (2006) (all authors' calculations).

Term premium: comparison to Bauer and Rudebusch



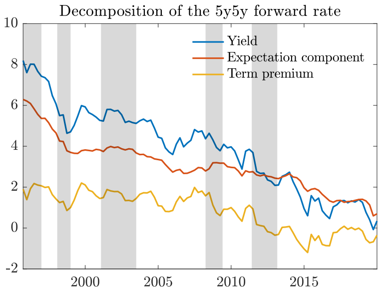
Note: BR OSE (ESE) denote Bauer and Rudebusch (2020) estimates of their Observed (Estimated) Shifting Endpoint model.

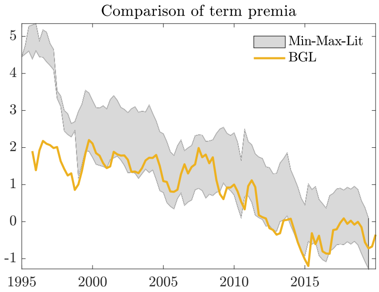
Euro area natural rate



Note: Estimated trends in blue and observed data in red.

Euro area term premia





Note: Shaded areas represent CEPR recessions. RHS: min-max-range of several estimates in the literature, including estimates from Geiger and Schupp (2018), and estimates from Adrian et al. (2013) and Diebold and Li (2006) (both own estimates).

Conclusion

- Integrated macro-finance model to jointly estimate the natural real rate r^* and bond risk premia
- Acknowledges dual role of r^* as time-varying anchor of the yield curve and benchmark real rate that closes the output gap
- Estimated term premia less trending than those of constant-mean models
- Estimated r^* with trend decline over last decade, high estimation uncertainty
- Follow-up work may feature QE, the lower bond on interest rates and include the pandemic period

Bibliography I

Adrian, Tobias, Richard K Crump and Emanuel Moench (2013), 'Pricing the term structure with linear regressions', *Journal of Financial Economics* **110**(1), 110–138.

Bauer, Michael D and Glenn D Rudebusch (2020), 'Interest rates under falling stars', *American Economic Review* **110**(5), 1316–54.

Christensen, Jens HE and Glenn D Rudebusch (2019), A new normal for interest rates? evidence from inflation-indexed debt, Federal Reserve Bank of San Francisco.

Cochrane, John H. (2007), 'Commentary on "Macroeconomic implications of changes in the term premium";', *Federal Reserve Bank of St. Louis Review* (Jul), 271–282.

Dewachter, Hans, Leonardo Iania and Marco Lyrio (2014), 'Information in the yield curve: A macro-finance approach', *Journal of Applied Econometrics* **29**(1), 42–64.

Dewachter, Hans and Marco Lyrio (2006), 'Macro factors and the term structure of interest rates', *Journal of Money, Credit and Banking* pp. 119–140.

Durbin, James and Siem Jan Koopman (2002), 'A simple and efficient simulation smoother for state space time series analysis', *Biometrika* **89**(3), 603–616.

Gürkaynak, Refet S, Brian Sack and Jonathan H Wright (2007), 'The us treasury yield curve: 1961 to the present', *Journal of monetary Economics* **54**(8), 2291–2304.

Joslin, Scott, Marcel Priebsch and Kenneth J Singleton (2014), 'Risk premiums in dynamic term structure models with unspanned macro risks', *The Journal of Finance* **69**(3), 1197–1233.

Kim, Don H and Jonathan H Wright (2005), 'An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates'.

Laubach, Thomas and John C Williams (2003), 'Measuring the natural rate of interest', *Review of Economics and Statistics* **85**(4), 1063–1070.