Bond Risk Premia in Consumption-based Models

Drew D. Creal
Chicago Booth

Jing Cynthia Wu
Chicago Booth & NBER

Motivation

Fed Chair Janet Yellen: large-scale asset purchases December, 2014, Press Conference

> *...we're reminding the public that we continue to hold a large stock of assets, and that is tending to push down **term premiums** in longer-term yields.*

Fed Chair Ben Bernanke: decomposition March, 2006, New York

> *To the extent that the decline in forward rates can be traced to a decline in the **term premium**... the effect is financially stimulative and argues for greater monetary policy restraint... However, if the behavior of long-term yields reflects current or prospective economic conditions, the implications for policy may be quite different-indeed, quite the opposite.*

Fed Chair Alan Greenspan: conundrum June, 2005, Beijing

> *That improved performance has doubtless contributed to lower inflation-related **risk premiums**, and the lowering of these **premiums** is reflected in significant declines in nominal and real long-term rates. Although this explanation contributes to an understanding of the past decade, I do not believe it explains the decline of long-term interest rates over the past year despite rising short-term rates.*

Term premium: two models & two channels

- Gaussian ATSM:
  - benchmark model
  - time-varying term premia via price of risk
- Consumption-based models with recursive preferences
  - time-varying term premia via SV

Goal of this paper: reconcile the two literatures

Literature

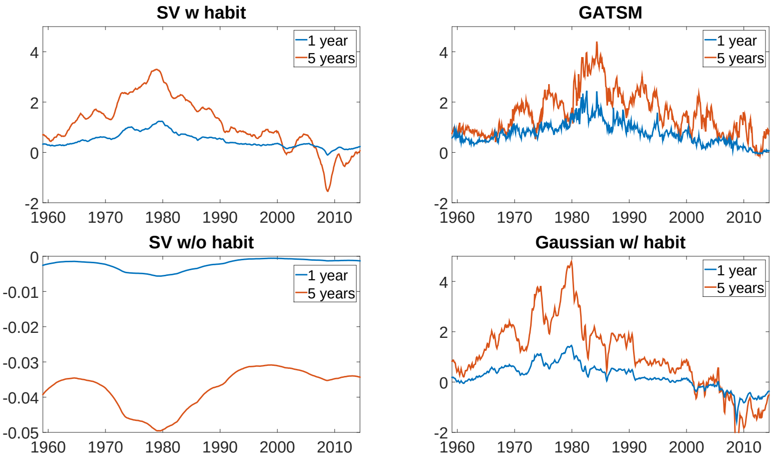
Contributions

- Introduce a new structural model with both channels
  - habit $\rightarrow$ time-varying price of risk
  - SV $\rightarrow$ time-varying quantity of risk
  - recursive preferences
- Our model has a reduced form of ATSM
  - inherits tractability
  - analytical bond prices
- Models with recursive preferences
  - a model solution doesn't always exist
  - we provide conditions for its existence

Empirical findings

- Our model matches empirical facts about bonds
  - realistic time variation for term premium
  - upward slope
  - mimics time series of level and slope
- Habit is the key for term premium
  - the price of expected inflation risk is the driving force
  - it comoves with the expected inflation itself
- Models with SV but not habit produce counterfactual implications for bonds
  - long run risk model
  - downward slope
  - term premia are economically insignificant, and negative

Term premia



Bottom line: habit is crucial to generate the patten in term premia

Outline

1. Model
2. Estimation
3. Results
4. Model solution

Model

Agent's problem

$$V_t = \max_{C_t} \left[(1-\beta) \left(\frac{C_t}{H_t} \right)^{1-\eta} + \beta \left\{ \mathrm{E}_t \left[V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\eta}{1-\gamma}} \right]^{\frac{1}{1-\eta}}$$

$$\text{s.t.} \quad W_{t+1} = (W_t - C_t)\, R_{c,t+1}$$

- H_t is habit
  - consumption to habit ratio enters the utility as in Abel (1999)
- β is the time discount factor
- γ measures risk aversion
- $\psi = \frac{1}{\eta}$ is the elasticity of intertemporal substitution

Stochastic discount factor

$$m_{t+1} = \vartheta \ln(\beta) + \vartheta \Delta \upsilon_{t+1} - \eta \vartheta \Delta c_{t+1} + (\vartheta - 1) r_{c,t+1}$$

- $\Delta \upsilon_{t+1} = (\eta - 1) \ln \left(\frac{H_{t+1}}{H_t} \right)$
- $\Delta c_{t+1} = \ln \left(\frac{C_{t+1}}{C_t} \right)$
- $r_{c,t+1} = \ln \left(R_{c,t+1} \right)$
- $\vartheta = \frac{1-\gamma}{1-\eta}$

Stochastic discount factor

$$m_{t+1} = \vartheta \ln(\beta) + \vartheta \Delta \upsilon_{t+1} - \eta\vartheta \Delta c_{t+1} + (\vartheta - 1) r_{c,t+1}$$

- $\Delta \upsilon_{t+1} = (\eta - 1) \ln\left(\frac{H_{t+1}}{H_t}\right)$
- $\Delta c_{t+1} = \ln\left(\frac{C_{t+1}}{C_t}\right)$
- $r_{c,t+1} = \ln(R_{c,t+1})$
- $\vartheta = \frac{1-\gamma}{1-\eta}$

$$m^{\$}_{t+1} = m_{t+1} - \pi_{t+1}$$

- π_{t+1} is inflation
- nominal variables have \$

Dynamics of the state vector

$$\Delta c_t = Z_c' g_t, \qquad \pi_t = Z_\pi' g_t,$$

where

$$\begin{aligned}
g_{t+1} &= \mu_g + \Phi_g g_t + \Phi_{gh} h_t + \Sigma_{gh}\varepsilon_{h,t+1} + \Sigma_{g,t}\varepsilon_{g,t+1} \\
\Sigma_{g,t}\Sigma_{g,t}' &= \Sigma_{0,g}\Sigma_{0,g}' + \sum_{i=1}^{H} \Sigma_{i,g}\Sigma_{i,g}' h_{it} \\
h_{t+1} &\sim \text{NCG}\left(\nu_h, \Phi_h, \Sigma_h\right) \\
\varepsilon_{h,t+1} &= h_{t+1} - \mathrm{E}_t\left[h_{t+1} | h_t\right]
\end{aligned}$$

- Volatility follows non-central gamma process of Creal and Wu (JoE 2015)
- Z_c and Z_π are selection vectors
- It's a companion form, nesting long-run risk & VARMA.

Long run risk

$$\begin{aligned}
\pi_{t+1} &= \bar{\pi}_t + \varepsilon_{\pi_1,t+1} & \varepsilon_{\pi_1,t+1} \sim \mathrm{N}\left(0, h_{t,\pi_1}\right) \\
\Delta c_{t+1} &= \bar{c}_t + \varepsilon_{c_1,t+1} & \varepsilon_{c_1,t+1} \sim \mathrm{N}\left(0, h_{t,c_1}\right) \\
\bar{\pi}_{t+1} &= \mu_\pi + \phi_\pi \bar{\pi}_t + \phi_{\pi,c}\bar{c}_t + \varepsilon_{\pi_2,t+1} & \varepsilon_{\pi_2,t+1} \sim \mathrm{N}\left(0, h_{t,\pi_2}\right) \\
\bar{c}_{t+1} &= \mu_c + \phi_{c,\pi}\bar{\pi}_t + \phi_c \bar{c}_t + \sigma_{c,\pi}\varepsilon_{\pi_2,t+1} + \varepsilon_{c_2,t+1} & \varepsilon_{c_2,t+1} \sim \mathrm{N}\left(0, h_{t,c_2}\right)
\end{aligned}$$

where $g_t = (\pi_t, \Delta c_t, \bar{\pi}_t, \bar{c}_t)'$

Difference from the literature: our volatility process guarantees positivity

Habit

$$\Delta v_{t+1} = \Lambda_1(g_t) + \Lambda_2(g_t)' \varepsilon_{g,t+1}$$
$$\Lambda_2(g_t) = -\eta \Sigma_{g,t}^{-1}(\lambda_0 + \lambda_g g_t)$$

- $\Lambda_2(g_t)$ is the risk sensitivity function
- $\lambda_g \neq 0 \Rightarrow$ **price of risk moves with g_t**
- We allow inflation to be non-neutral
  - Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013)

Relation to Wachter(2006)

If $\eta = \gamma$, $\Delta c_{t+1} = \bar{c} + \sigma_c \varepsilon_{c_1,t+1}$, $\varepsilon_{g,t+1} = \varepsilon_{c_1,t+1}$, and

$$\begin{aligned} \Lambda_{1t} &= (1-\phi)(\bar{\upsilon} - \upsilon_t) \\ \Lambda_{2t} &= \frac{1}{\bar{H}}\sqrt{\eta + 2(\upsilon_t - \bar{\upsilon})} + \eta\sigma_c \end{aligned}$$

then the SDF becomes the same as Wachter(2006).

Relation to Wachter(2006)

If $\eta = \gamma$, $\Delta c_{t+1} = \bar{c} + \sigma_c \varepsilon_{c_1,t+1}$, $\varepsilon_{g,t+1} = \varepsilon_{c_1,t+1}$, and

$$\begin{aligned} \Lambda_{1t} &= (1-\phi)(\bar{\upsilon} - \upsilon_t) \\ \Lambda_{2t} &= \frac{1}{\bar{H}}\sqrt{\eta + 2(\upsilon_t - \bar{\upsilon})} + \eta\sigma_c \end{aligned}$$

then the SDF becomes the same as Wachter(2006).

The differences are

- Our model is affine
  - analytical bond prices
  - tractability
- We allow expected inflation risk to be priced
  - It turns out to be the key driving factor

Relation to preference shock

If we define $\Upsilon_t \equiv H_t^{\eta-1}$, then the objective function becomes

$$V_t = \max_{C_t} \left[(1-\beta)\,\Upsilon_t C_t^{1-\eta} + \beta \left\{ \mathrm{E}_t \left[V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\eta}{1-\gamma}} \right]^{\frac{1}{1-\eta}},$$

where Υ_t is the time preference. The macro literature specifies

$$\Delta \upsilon_{t+1} = Z_\upsilon' g_{t+1}$$

- Z_υ is a selection vector
- latent preference factor
- Albuquerque, Eichenbaum & Rebelo (2014), Schorfheide, Song & Yaron (2014)
- no time-varying price of risk

Model solution

Log-linearize $r_{c,t+1}$ via Campbell & Shiller (1989)

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}$$

Real pricing kernel prices consumption good

$$1 = \mathrm{E}_t \left[\exp \left(m_{t+1} + r_{c,t+1} \right) \right],$$

Guess a solution

$$pc_t = D_0 + D_g' g_t + D_h' h_t$$

Solve the fixed point problem

$$\bar{pc} = D_0 \left(\bar{pc} \right) + D_g \left(\bar{pc} \right)' \bar{\mu}_g + D_h \left(\bar{pc} \right)' \bar{\mu}_h$$

Plug the solution $r_{c,t+1}$ into the SDF

Sources of risk premia

Pricing kernel

$$m^{\$}_{t+1} - \mathrm{E}_t\left[m^{\$}_{t+1}\right] \quad = \quad -\lambda^{\$,\prime}_{g,t}\Sigma_{g,t}\varepsilon_{g,t+1} - \lambda^{\$,\prime}_{h}\Sigma_{h,t}\tilde{\varepsilon}_{h,t+1}$$

where

$$\begin{aligned} \lambda^{\$}_{g,t} \quad = \quad & \gamma Z_c + Z_\pi & \leftarrow \text{power utility} \\ & +\kappa_1 \frac{\gamma - \eta}{1 - \eta} D_g & \leftarrow \text{recursive preferences} \end{aligned}$$

Sources of risk premia

Pricing kernel

$$m_{t+1}^{\$} - \mathrm{E}_t\left[m_{t+1}^{\$}\right] = -\lambda_{g,t}^{\$,\prime}\Sigma_{g,t}\varepsilon_{g,t+1} - \lambda_h^{\$,\prime}\Sigma_{h,t}\tilde{\varepsilon}_{h,t+1}$$

where

$$\begin{aligned} \lambda_{g,t}^{\$} &= \gamma Z_c + Z_\pi && \leftarrow \text{power utility} \\ &+\kappa_1 \frac{\gamma-\eta}{1-\eta} D_g && \leftarrow \text{recursive preferences} \\ &+\vartheta\eta\left(\Sigma_{g,t}\Sigma_{g,t}^{\prime}\right)^{-1}\left(\lambda_0 + \lambda_g g_t\right) && \leftarrow \text{habit formation} \end{aligned}$$

The price of risk only varies with g_t if $\lambda_g \neq 0$.

This channel remains the same if we shut SV.

Sources of risk premia

Pricing kernel

$$m^{\$}_{t+1} - \mathrm{E}_t\left[m^{\$}_{t+1}\right] = -\lambda^{\$,\prime}_{g,t}\Sigma_{g,t}\varepsilon_{g,t+1} - \lambda^{\$,\prime}_{h}\Sigma_{h,t}\tilde{\varepsilon}_{h,t+1}$$

where

$$\lambda^{\$}_h = \Sigma'_{gh}\left(\gamma Z_c + Z_\pi\right) \quad \leftarrow \text{power utility}$$

$$+\kappa_1\frac{(\gamma-\eta)}{(1-\eta)}\left(\Sigma'_{gh}D_g + D_h\right) \quad \leftarrow \text{recursive preference}$$

Prices of volatility risks are constant, as in the literature.

Bond prices

$$P_t^{\$,(n)} = \mathrm{E}_t\left[\exp\left(m_{t+1}^{\$}\right) P_{t+1}^{\$,(n-1)}\right]$$

yields

$$y_t^{\$,(n)} \equiv -\frac{1}{n}\ln\left(P_t^{\$,(n)}\right) = a_n^{\$} + b_{n,g}^{\$,\prime} g_t + b_{n,h}^{\$,\prime} h_t$$

where $b_{n,g}^{\$} = -\frac{1}{n}\bar{b}_{n,g}^{\$}$ and

$$\bar{b}_{n,g}^{\$} = \underbrace{(\Phi_g - \eta\vartheta\lambda_g)'}_{\Phi_g^{\mathbb{Q}^{\$}}}\bar{b}_{n-1,g}^{\$} + \bar{b}_{1,g}^{\$}$$

- The separation between Φ_g and $\Phi_g^{\mathbb{Q}^{\$}} \equiv \Phi_g - \eta\vartheta\lambda_g$ is the key
- Derive bond prices as in Creal & Wu (JoE 2015)
- Yields are affine functions of the state variables.
- Loadings are functions of (β, γ, ψ)

Short rate

Consumption-inflation representation

$$\begin{aligned} r_t^{\$} &= -\log\left(P_t^{\$,(1)}\right) \\ &= -\ln(\beta) + \eta \mathrm{E}_t\left[\Delta c_{t+1}\right] + \mathrm{E}_t\left[\pi_{t+1}\right] \\ & \quad -\eta\vartheta(\eta Z_c + Z_\pi)'(\lambda_0 + \lambda_g g_t) \\ & \quad +\text{Jensen's ineq.} \end{aligned}$$

- Line 2: time discount, expected consumption and inflation
- Line 3: risk adjustment

Term premium

$$tp_t^{\$,(n)} = y_t^{\$,(n)} - \frac{1}{n}\mathrm{E}_t\left[r_t^{\$} + r_{t+1}^{\$} + \ldots, + r_{t+n-1}^{\$}\right]$$

Difference between

- Buy an *n*-period bond
- Rolling over 1-period bond *n* times

Outline

1. Model

2. Estimation

3. Results

4. Model solution

Observation equation

Stack

$$y_t^{\$,(n)} \;=\; a_n^{\$} + b_{n,g}^{\$,\prime} g_t + b_{n,h}^{\$,\prime} h_t$$

for $n = n_1, n_2, ..., n_N$, and allow pricing errors

$$y_t^{\$} = A + B_g^{\$} g_t + B_h^{\$} h_t + e_t, \quad e_t \sim \text{i.i.d.}\,(0, \Omega)$$

where $A^{\$} = (a_{n_1}^{\$}, \ldots, a_{n_N}^{\$})'$, $B_g^{\$} = (b_{g,n_1}^{\$,\prime}, ..., b_{g,n_N}^{\$,\prime})'$,and $B_h^{\$} = (b_{h,n_1}^{\$,\prime}, ..., b_{h,n_N}^{\$,\prime})'$.

Least squares

Estimate $\theta^{\mathbb{Q}} = \left(\beta, \gamma, \psi, \Phi_g^{\mathbb{Q}^\$}\right)$ by minimizing the pricing errors

$$\min e_t' \Omega^{-1} e_t$$

- Some macro variables g_t and h_t are latent
- We approximate $p(g_t, h_t, \theta^{\mathbb{P}} | m_{1:T})$ by a Particle Gibbs sampler.

Data

Monthly data from Feb. 1959 to June 2014

Yields

- Fama-Bliss zero-coupon yields from CRSP
- maturities: 3m, 1y, 2y, 3y, 4y, 5y

Inflation + Population

- FRED database at St. Louis FRB
- CPI inflation
- Civilian population over 16

Consumption

- U.S. Bureau of Economic Analysis
- non-durables + services

Restrictions

- Four free parameters in λ_g
- $\lambda_0 = 0$
- $\Sigma_{0,g} = 0$
- Φ_h, Σ_h are diagonal
- $\Phi_{gh} = 0$ and $\Sigma_{gh} = 0$
- $\Omega = \omega^2 I$

Posterior distribution of macro factors

MCMC + particle filters → Particle Gibbs sampler.

For $j = 1, \ldots, M$

$$
\begin{aligned}
(g_{1:T}, h_{0:T})^{(j)} &\sim p\left(g_{1:T}, h_{0:T} | m_{1:T}, \theta^{\mathbb{P},(j-1)}\right) \\
\theta^{\mathbb{P},(j)} &\sim p\left(\theta^{\mathbb{P}} | m_{1:T}, g_{1:T}^{(j)}, h_{0:T}^{(j)}\right)
\end{aligned}
$$

- Draw the state variables using the particle filter, see Andrieu, Doucet, Holenstein (10).
- Use independence Metropolis-Hastings to draw the parameters $\theta^{\mathbb{P}}$.

Least squares

$$\min e_t' \Omega^{-1} e_t$$

where

$$e_t = y_t^{\$} - A^{\$}\left(\theta^{\mathbb{Q}}, \hat{\theta}^{\mathbb{P}}\right) + B_g^{\$}\left(\theta^{\mathbb{Q}}, \hat{\theta}^{\mathbb{P}}\right)\hat{g}_t + B_h^{\$}\left(\theta^{\mathbb{Q}}, \hat{\theta}^{\mathbb{P}}\right)\hat{h}_t$$

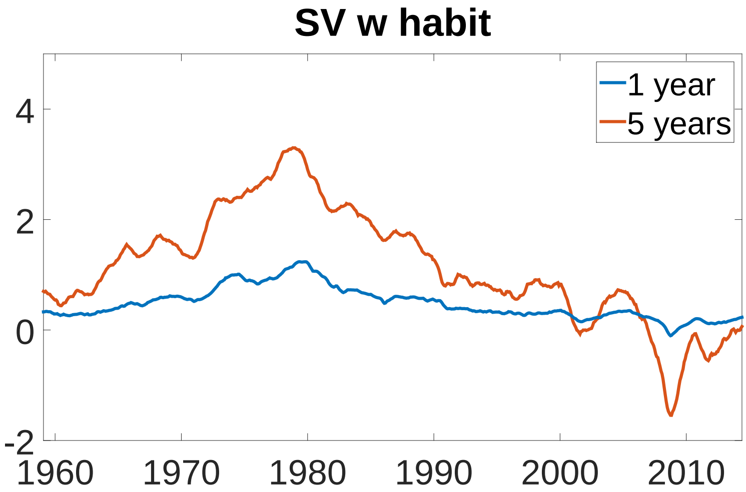
Structural parameters

		global		local	
Preference	ψ	1.02		0.70	
		(0.03)		(0.04)	
	β	0.9998		1.003	
		(0.0000)		(0.000)	
	γ	6.75		1.73	
		(2.02)		(0.16)	
Habit	$\Phi_g^{\mathbb{Q}^\$}$				
		0.993	0.018	0.994	-0.015
		(0.002)	(0.007)	(0.003)	(0.005)
		0.000	0.997	-0.005	0.996
		(0.001)	(0.000)	(0.002)	(0.000)
	λ_g	$1e^{-3}\times$			
		0.05	-0.12	-0.007	0.030
		0.00	0.18	0.001	-0.023

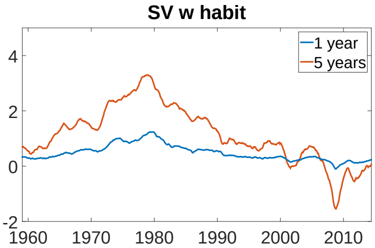
- Both global and local have similar implications for bonds.
- Key: $\Phi_g^{\mathbb{Q}^\$}$ are persistent
- Other structural parameters (γ, ψ, β) vary with different economic interpretations.

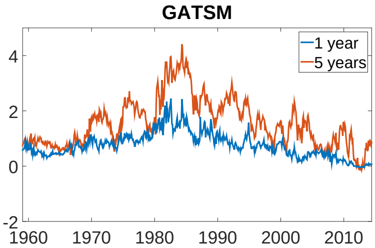
Outline

Term premia in the benchmark model



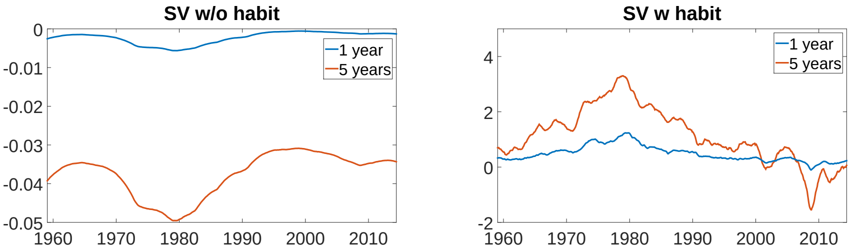
Comparison with GATSM





- Left: our benchmark
- Right: GATSM

Only quantity of risk

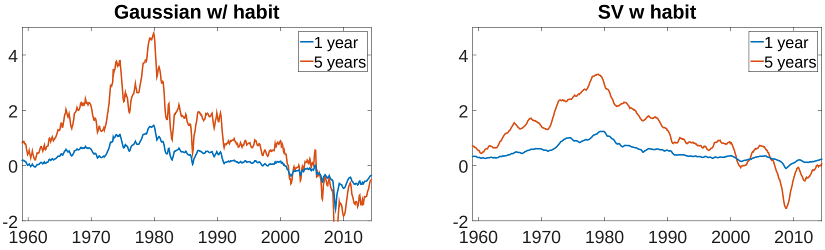


- Left: with SV, no habit (long run risk model)
- Right: our benchmark

Long run risk model produces counterfactual term premia

- economically insignificant
- negative

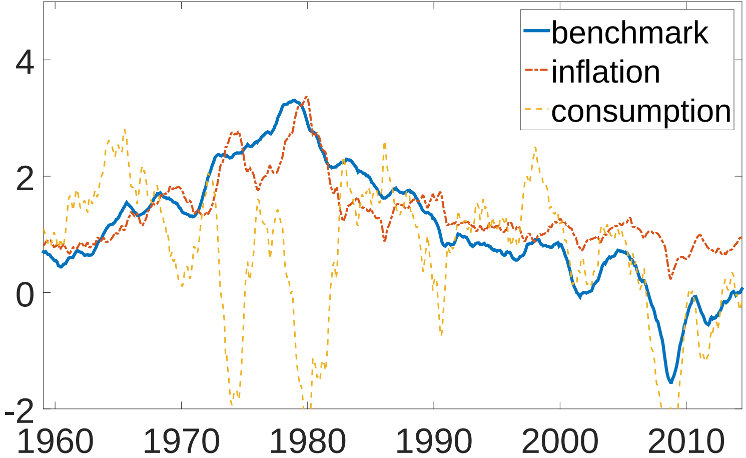
Only price of risk



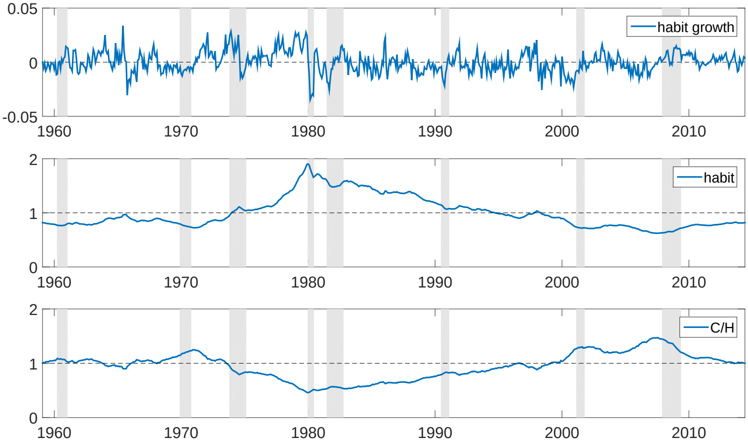
- Left: no SV, with habit
- Right: our benchmark

Bottom line: habit is crucial to generate the patten in term premia

Inflation vs. consumption

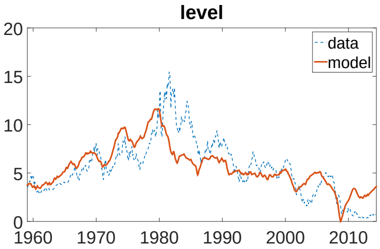


Habit



0.05
0
-0.05
habit growth
1960
1970
1980
1990
2000
2010
2
1
0
habit
C/H

Level and slope from benchmark model



- Level: average across maturities
- Slope: 5 year - 3 month

Slope

Unconditional slope has been the focus for the majority of the literature

		3	12	24	36	48	60	level	slope
data		4.94	5.33	5.54	5.72	5.88	5.98	5.57	1.04
SV w/ habit	global	4.91	5.27	5.63	5.85	5.92	5.84	5.57	0.93
	local	4.95	5.20	5.49	5.74	5.95	6.13	5.58	1.18
Gaussian w/ habit		5.08	5.25	5.47	5.69	5.89	6.09	5.58	1.01
SV w/o habit		5.64	5.63	5.61	5.59	5.57	5.56	5.60	-0.08

- SV seems to be flexible with g_t and h_t
- But there are more moments to match A, B_g, B_h
- There are only 3 free parameters (β, γ, ψ) to match all
- It's difficult to match both the average level, and slope

▸ Literature

Outline

1. Model

2. Estimation

3. Results

4. Model solution

Model solution

Log-linearize $r_{c,t+1}$ via Campbell & Shiller (1989)

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}$$

Guess a solution

$$pc_t = D_0 + D_g' g_t + D_h' h_t$$

Solve the fixed point problem

$$\bar{pc} = D_0(\bar{pc}) + D_g(\bar{pc})' \bar{\mu}_g + D_h(\bar{pc})' \bar{\mu}_h$$

Plug the solution $r_{c,t+1}$ into the SDF

Problem: a solution to the fixed point problem does not always exist.

General case

Assumption

The parameters $\theta \in \Theta^r$ must satisfy that for any real $\bar{pc}$,

1. *the loadings $D_h(\bar{pc}, \theta)$ are real,*
2. *the expectation $1 = E_t\left[\exp\left(m_{t+1} + r_{c,t+1}\right)\right]$ exists for $D_h\left(\bar{pc}, \theta\right)$.*

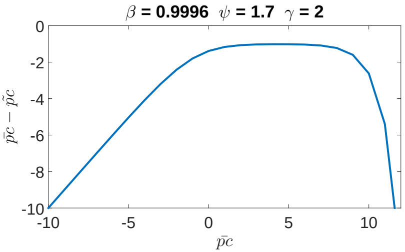
Proposition

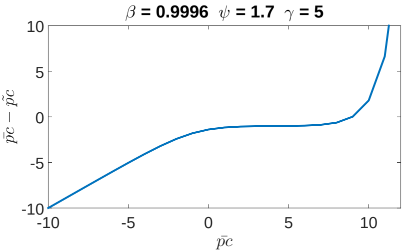
Given Assumptions, there is a value $\bar{\beta}(\psi, \gamma, \theta^{\mathbb{P}}, \theta^{\lambda})$ such that if $\beta < \bar{\beta}$, then there exists a real solution for the fixed point problem.

Sketch of proof

Define $\tilde{pc}\left(\bar{pc}\right) = D_0\left(\bar{pc}\right) + D_g\left(\bar{pc}\right)' \bar{\mu}_g + D_h\left(\bar{pc}\right)' \bar{\mu}_h$

The fixed point problem has a solution if $\bar{pc} - \tilde{pc}\left(\bar{pc}\right) = 0$



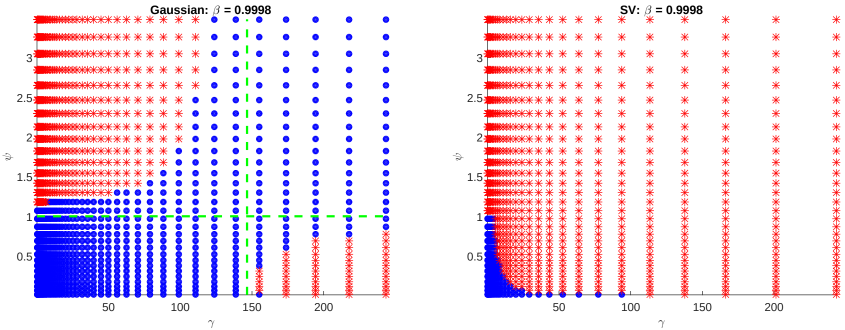


Special case: Gaussian

Corollary

1. *If* $Z_1^{\infty\prime}\mu_g^* \leq 0$ *and* $\beta \leq 1$, *then* $\frac{1-\gamma}{1-\psi} > 0$ *guarantees the existence of a solution.*
2. *If* $\beta \leq 1$, *then there is a value* $\bar{\gamma}(\theta^{\mathbb{P}}, \theta^{\lambda})$ *such that* $\frac{\bar{\gamma}-\gamma}{1-\psi} > 0$ *guarantees a solution.*
3. *For any* ψ, $\bar{\beta}$ *is monotonic in* γ: *for* $\psi > 1$, *then* $\frac{d\bar{\beta}}{d\gamma} > 0$; *for* $\psi < 1$, *then* $\frac{d\bar{\beta}}{d\gamma} < 0$.

Numerical illustration: part 2



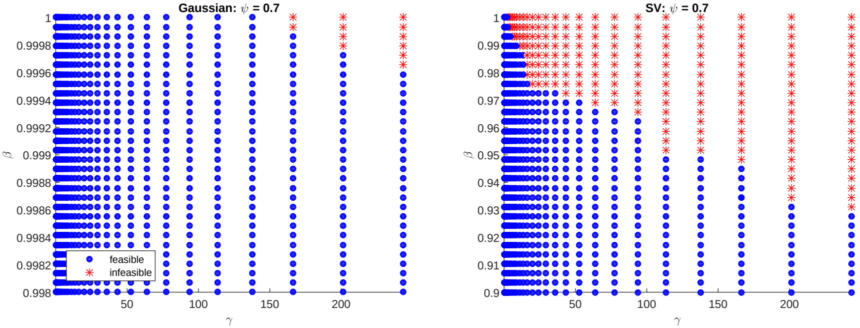
- Gaussian: $\bar{\gamma} = 146.5$
- benchmark: for $\psi = 0.97$,$\gamma < 4.8$
- benchmark: for $\psi = 0.52$,$\gamma < 6.9$

Special case: Gaussian

Corollary

1. *If* $Z_1^{\infty\prime}\mu_g^* \leq 0$ *and* $\beta \leq 1$, *then* $\frac{1-\gamma}{1-\psi} > 0$ *guarantees the existence of a solution.*
2. *If* $\beta \leq 1$, *then there is a value* $\bar{\gamma}(\theta^{\mathbb{P}}, \theta^{\lambda})$ *such that* $\frac{\bar{\gamma}-\gamma}{1-\psi} > 0$ *guarantees a solution.*
3. *For any* ψ, $\bar{\beta}$ *is monotonic in* γ*: for* $\psi > 1$, *then* $\frac{d\bar{\beta}}{d\gamma} > 0$; *for* $\psi < 1$, *then* $\frac{d\bar{\beta}}{d\gamma} < 0$.

Numerical illustration: part 3



- Gaussian: for $\gamma = 244$, $\beta < 0.9996$
- benchmark: for $\gamma = 244, \beta < 0.93$

What we have learned

- A small γ might not mark the success of a model, but simply to satisfy the constraint
- The separation of regions might cause numerical problems for estimation, frequentist or Bayesian
- SV models encounter more problems

Conclusion

- Build a new structural model with two forces for term premia
  - habit $\rightarrow$ time-varying prices of risk
  - SV $\rightarrow$ time-varying quantity of risk
- Empirical results:
  - Our model
    - captures realistic dynamics for risk premia
    - upward slope
  - Habit is the driving force for term premia
    - the price of expected inflation risk is the key, which comoves with the expected inflation itself
  - Models with SV but not habit produce counterfactual implications
    - downward slope
    - term premia are economically insignificant, and negative
- Provide conditions guaranteeing a solution for models with recursive preferences

Literature:

Consumption-based models

- recursive preferences: *Piazzesi Schneider (07), Le Singleton (10)*
- recursive preferences + SV: *Bansal Yaron (04), Bansal Gallant Tauchen (07), Bansal Shaliastovich (13),*
- habit formation: *Wachter (06)*
- recursive preferences + SV in ICAPM: *Campbell Giglio et al. (14)*
- recursive preferences + preference shocks: *Albuquerque Eichenbaum Rebelo (14) Schorfheide Song Yaron (14)*

DSGE models

- habit formation: *Rudebusch Swanson (08)*
- recursive preferences: *Rudebusch Swanson (08), van Binsbergen et al. (12), Dew-Becker (14)*
- solution methods: *Caldara et al. (12)*

Literature:

Term premium

- ATSM: *Duffee (02), Ang Piazzesi (03), Wright (11), Bauer Rudebusch Wu (12)*

Model solution

- *Hansen Scheinkman (12), Campbell Giglio et al. (14)*

Back

Comparison with estimation in the literature

$$\Phi_g^{\mathbb{Q}^\$} \equiv \Phi_g - \eta\vartheta\lambda_g$$

- Dynamics of macro variables: Φ_g
- Cross section of yields: $\Phi_g^{\mathbb{Q}^\$}$
- Term premia: the difference between $\mathbb{P}$ and $\mathbb{Q}$

In models without habit, $\Phi_g = \Phi_g^{\mathbb{Q}^\$}$

- If we extract macro dynamics from macro data (*ours*), then
  - Macro factors retain their interpretation
  - Φ_g is estimated from the macro dynamics
  - and determines the slope is downward
- If we extract macro dynamics primarily from yields (*literature*), then
  - $\Phi_g^{\mathbb{Q}}$ is estimated from the cross section of yields
  - and then determines the dynamics of the factors
  - macro factors mimic level, slope and curvature of yields

Habit allows $\Phi_g \neq \Phi_g^{\mathbb{Q}}$

Macro factors and yields

Regression R^2s of macro factors on yields

	our estimates	inversion	
		w/o p.e	w/ p.e.
expected inflation	57%	100%	98%
expeted growth	31%	100%	96%
expected inflation vol	48%	100%	36%
expected growth vol	31%	100%	72%

Back