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A MODEL WITH EXPLICIT EXPECTATIONS FOR BELGIUM

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ABSTRACT

This paper presents a new quarterly macroeconometric model of the Belgian economy. It is intended to contribute to existing analytical work covering the specific transmission mechanisms of the euro area monetary policy in the Belgian economy. It also contributes to the forecast exercises and to their risk analysis. Finally it is also used to analyse the consequences of specific Belgian shocks. The model is small-scale and based on recent macroeconomic theory. The model's dynamics not only allow for the lagged adjustments from economic agents due to transaction costs to be taken into consideration, but also for agents to anticipate future developments and policy reactions. In simulations, expectation formation can be assumed either to be model consistent or to be generated by VAR-based extrapolations. On the basis of a few diagnostic simulations it is shown that in the long run the model converges to its steady state, defined by the underlying economic theory.

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0 <u>INTRODUCTION</u>

This paper presents a new quarterly econometric model for the Belgian economy[1]. It has been developed as part of a larger project within the European System of Central Banks (ESCB). The transfer of national monetary policy decisions to ESCB level has created the need for a new range of specific analytical tools such as the development of econometric models for the euro area economy. Against this background it was decided that a new multi-country model should be built, in the sense that national models would be linked to each other as well as to models representative of the larger non-euro area countries such as the European pre-in countries and non-European Union countries. For this model to be useful in the monetary policy debate at the ESCB-level, it should meet a number of specific objectives and contain specific characteristics.

The main objectives of the model are twofold. First, it is intended to be used to understand the transmission mechanisms of monetary policy in the eurozone. Since these transmission mechanisms may, at least partly, depend on the specific individual financial and economic structures of the participating countries, it is useful to distinguish the effects of a common monetary policy in each of the participating countries. This implies that such a multi-country model should reflect characteristics specific to each country. The National Bank of Belgium therefore decided to develop its own contribution to the ESCB multi-country model, making use of its knowledge and experience acquired from modelling the Belgian economy. Secondly, the Belgian model is used in both national and euro area forecast exercises. These forecasts are based on both judgmental analysis and model simulations. The model also contributes to the risk analysis accompanying the forecast.

The need to develop a new model of the Belgian economy provided the opportunity to incorporate certain characteristics reflecting recent theoretical and empirical developments. One of the main motivations underlying our contribution to the ESCB modelling project (this contribution was left to the free choice of the NCBs) is based on our belief that a gap has emerged between traditional econometric macromodels and modern macroeconomic theory. Traditional macroeconometric models assume a set of basic long-term economic relationships. The parameters of those relationships and their short-term dynamics are econometrically estimated on the basis of past data in an attempt to obtain

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[1] An earlier, more disaggregated quarterly model, constructed in the National Bank of Belgium, was presented to the public at a conference held in Brussels on 16th November 1995.

the best possible statistical fit. Since they are usually not based on formal theories of optimal planning over time, the numerical values of their estimated coefficients are in fact a mixture of parameters reflecting fundamental structural relationships and the formation of agents' expectations. As argued in the Lucas critique (1976), these estimated relationships are not necessarily valid when the policy regime changes and expectations are regime-dependant, in which case such models may be unreliable as a basis for policy evaluation. In more general terms, this model emphasises the essential role that market agent's expectations play in the analysis of monetary policy. Indeed, central bank credibility and the effectiveness of monetary policy is essentially related to the response of agents' expectations to diverse economic and financial shocks.

The paper is organised as follows. Section 1 discusses the theoretical foundations of the model and its steady state properties. Section 2 explains the methodology concerning the derivation and estimation of Polynomial Adjustment Costs. Section 3 investigates some of the model's properties under different expectation formations hypotheses. The final section provides some conclusions.

1 THEORETICAL STRUCTURE OF THE MODEL

The two main groups of private agents in the model are households and firms. Households maximise utility, subject to an intertemporal budget constraint, but at least some of them are limited in their ability to borrow. Firms minimise costs of production subject to a Cobb-Douglas technology. Goods and labour markets are imperfectly competitive.

1.1 Households

1.1.1 Consumption

Households maximise their expected lifetime utility

$$\text{Max } E_0 \sum_{s=0}^{\infty} \varphi^s \, U(C_{t+s}) \tag{1}$$

subject to the asset accumulation constraint :

$$FW_{t+s} = (1+r_{t+s-1}) \, FW_{t+s-1} + YL_{t+s} - C_{t+s} \tag{2}$$

where C_{t+s} is consumption at time t+s, FW_{t+s} is end-of-period asset holdings which earn real return r_{t+s}, and YL_{t+s} is the after-tax labour income "sensu lato", i.e. inclusive of transfer payments. Future utility is discounted at rate $\varphi = 1/(1+\Theta)$ where Θ represents the subjective rate of time preference .

We also need an additional condition to prevent the consumer from choosing a path with an exploding debt, while allowing him to be temporarily indebted. This is the so-called No-Ponzi-Game condition implying that assets holdings should asymptotically be non-negative :

$$\lim_{s \to \infty} (1+r)^{-(s-1)} \, FW_{t+s} \geq 0 \tag{3}$$

From non-satiation, the condition holds as an equality.

The optimal solution is given by the intertemporal Euler equation :

$$U'(C_{t+s}) = (1+r) \, \varphi \, U'(C_{t+s+1}) \tag{4}$$

To provide a closed-form solution we assume that the instantaneous utility function exhibits constant relative risk aversion where the elasticity of substitution between consumption at any two points in time is constant and equal to σ , that is :

$$U(C) = (\sigma/(\sigma-1)) \, . \, C^{(\sigma-1)/\sigma} \tag{5}$$

On this assumption and provided that the stability condition $(1+r)^{\sigma-1} \, \varphi^{\sigma} < 1$ holds, we obtain the following consumption function :

$$C_t = \Omega \, TW_t \tag{6}$$

$$\Omega = 1 - (1+r)^{\sigma-1} \, \varphi^{\sigma} \tag{7}$$

where Ω, the propensity to consume out of total wealth (TW), depends on the real rate of interest and on the elasticity of intertemporal substitution. Ω is constant in the particular case of logarithmic utility ($\sigma = 1$).

Total wealth contains two components: human wealth (HW) which is defined as the sum of discounted future labour income and non-human wealth. The latter is the value of asset holdings at the beginning of the period plus accrued return:

$$TW_t = HW_t + (1+r)FW_{t-1} \tag{8}$$

$$HW_t = \sum (1+r)^{-s} \, YL_{t+s} \tag{9}$$

In the steady-state of the model, the desired level of consumption derived from this life cycle model depends on the market value of financial and real assets and on human wealth, defined as the present value of expected future wage income net of taxes and inclusive of transfer payments.

Households are supposed to be risk averse. This risk aversion causes future income flows to be discounted at a rate above the market interest rate in the spirit of Blanchard's (1985) model of perpetual youth. As a consequence, Ricardian equivalence does not hold since the present value of future tax changes does not completely match current adjustments in tax payments. Expected future income is overdiscounted at around 25 p.c. a year. This discount rate equals the sum of a real interest rate augmented by a factor reflecting households' aversion to future income uncertainty. The mark-up corresponds to a time horizon of some 20 years. Indeed, the present value of an income flow 20 years ahead represents only 1 p.c. of today's human wealth. This also means that the first 10 years count for more than 90 p.c. in the present value sum of future incomes. This 20-year time horizon also seems quite reasonable for a representative household since, in Belgium, most mortgage loans are contracted for a 20 year period.

The consumption function can now be written as (all variables are in real terms):

$$C_t = \Omega \left((1+r) \, FW_{t-1} + E_t \, HW_t \right) \tag{10}$$

$$E_t \, HW_t = \sum (1+r)^{-s} \, YL_{t+s} \ , \ s = 0, \dots, 80 \tag{11}$$

Estimation is based on a log-linear approximation of this consumption function. Furthermore, in addition to wealth, desired aggregate consumption is also assumed to depend on the rate of employment, approximating the effect of counter cyclical variations in the perceived uncertainty about future income. The dynamic consumption equation also reflects heterogeneity among households. We distinguish two types of consumers along the lines of Campbell and Mankiw (1989). Type-one consumers are forward-looking and, in the long run, behave as the individual consumer described above. In the short-term, they act under dynamic frictions. Type-two consumers are liquidity constrained and therefore consume their current income. The dynamic consumption function is a weighted average of these two types of consumer behaviour. The estimated coefficient implies that liquidity constrained consumers represent on average around 22 p.c. of the population.

1.1.2 _Households' non-human wealth_

1.1.2.1 Market value of housing

Households' housing investment is explained in two steps. First, households decide on the desired proportion of housing stock (HHOS) in their non-human wealth allocation. Since the stock of housing is quite sticky in the short run, this desired value actually explains movements of prices in the secondary housing market. Secondly, the long-term ratio of housing investment to aggregate consumption is a function of the relative expected returns and prices.

The optimal proportion of housing in total wealth depends on an expected risk premium defined as the difference between the real return on housing (RETKRH) and the mortgage rate (RMT), corrected for anticipated inflation (INFQE). As is often the case in empirical studies of portfolio allocation, a transaction variable, such as real labour income, needs to be added to the equation. Indeed, through the mortgage market, human wealth can be mobilised for immediate investment in housing assets which does not apply to other assets such as shares. Moreover the level of mortgage debt (LMT) may affect the supply of lending by banks. In steady state, real labour income and the real value of mortgage liabilities all grow at the steady state growth rate of the economy ($\zeta + n$, see Section 1.6). Furthermore the real excess return is stationary and therefore along the equilibrium growth path, the share of housing in wealth remains constant:

$$HHOS/FW = \chi_1 \, (yl - (\zeta + n) \, T) + \chi_2 \, (RETKRH-RMT-INFQE) \, FW/HHOS$$
$$+ \chi_3 \, (lmt-pcd - (\zeta + n) \, T) \tag{12}$$

Since housing prices are much more volatile compared to housing numbers or volume, this last equation tends to explain the movements of market prices. It should therefore be supplemented with an explanation of the supply of new houses coming from residential investment. The ratio of housing investment (IHR) to consumption (PCR) is given by relative prices of new dwellings (IHXN) versus consumption (PCD) and returns.

$$ihr = pcr + \tau_1 \, (RMT-RETKRH) + \tau_2 \, (ihxn - pcd) \tag{13}$$

1.1.2.2 Market value of the productive capital stock

In order to calculate households' non-human wealth, an evaluation of the market value of the private non-residential capital stock is needed. Here we follow a discrete-time version of the approach used in Multimod (IMF, 1998).

The real market value of the capital stock existing at time t is the discounted sum of the stream of its after-tax real income:

$$K_t Q_t / PCD_t = E_t \left\{ \sum_{s=0}^{\infty} (1+r)^{-s} \, {}_t YK_{t+s} \right\} \tag{14}$$

where $_t YK_{t+s}$ is the real net income in period t+s of capital existing in period t and r is a real discount rate.

The after-tax income of the "total" capital stock in period t+s is the value of its marginal product multiplied by the capital stock less taxes on companies (OTN) and deflated by the consumption price to obtain real value :

$$YK = (YVAD * \partial YVAR / \partial K * K - OTN) / PCD \tag{15}$$

where YVAR and YVAD are respectively real private value-added and its deflator (both are defined below).

Due to the Cobb-Douglas technology used in the supply-side, the marginal product of capital multiplied by the capital stock is a constant proportion (1-α) of output. So that

$$YK = ((1-\alpha) \, YVAD \, YVAR - OTN) / PCD \tag{16}$$

"Total" capital income in period t+s can be seen to contain two components: income generated by capital existing in period t and revenue from new capital installed between periods t and t+s. Since capital is homogeneous, the share of the period-t capital stock in total capital income in t+s is the ratio of the period-t capital alive in t+s to the total capital stock in t+s. Given a depreciation rate of δ, the period-t capital still existing in t+s is

$$_tK_{t+s} = (1-\delta)^s K_t \tag{17}$$

If, in each period, the growth rate of the capital stock is γ, then the total capital stock in period t+s is

$$K_{t+s} = (1+\gamma)^s K_t \tag{18}$$

Dividing (17) by (18) gives the share of income in period t+s received by capital existing in period t:

$$_tK_{t+s} / K_{t+s} = (1-\delta)^s / (1+\gamma)^s \tag{19}$$

The real market value of the period-t capital stock can then be written as:

$$K_t Q_t / PCD_t = E_t \left\{ \begin{array}{l} \sum_{S=0}^{\infty} (1+r)^{-s} . ((1-\delta)/(1+\gamma))^s . \\ ((1-\alpha) . YVAD_{t+s} . YVAR_{t+s} \\ - OTN_{t+s}) / PCD_{t+s} \end{array} \right\} \tag{20}$$

1.1.2.3 Money demand

A demand function for high-powered money is needed as a component of the private sector's non-human wealth. Cash balances, which are dominated assets in portfolio allocation, reduce transaction costs associated with acquiring goods and services. Let the transaction cost function have the form:

$$TC_t = \theta (P_t Y_t)^\nu M_t^{1-\nu} \qquad \theta>0 , \nu>1. \tag{21}$$

This function implies decreasing returns to hold money. The opportunity cost of holding one unit of money during period t is the discounted value of interest foregone so that the rental price of a unit of cash balances is $r_t/(1+r_t)$. Equalising marginal return and marginal cost gives:

$$M_t = [(1-\nu)\theta]^{1/\nu} (r_t/(1+r_t))^{-1/\nu} P_t Y_t \tag{22}$$

which is a fairly standard money demand function.

1.1.2.4 Net foreign assets position and government bonds

Net foreign assets (NFA) result from the accumulation of current account balances:

$$NFA = (1+r)\,NFA_{t-1} + XTN - MTN + TWN \tag{23}$$

where XTN and MTN are export and import values respectively and TWN represents capital transfers.

Government bonds are determined by the government budget constraint which says that debt (GDN) equals previous period debt minus budget surplus (GLN):

$$GDN = GDN_{t-1} - GLN \tag{24}$$

1.2   Supply-side

1.2.1   Factor demands and output price

Producers generate private value-added (YVAR) by means of a Cobb-Douglas production function in labour expressed in hours (LH) and capital (K) with constant returns to scale and exogenous labour augmenting technical progress:

$$YVAR = (e^{\zeta T}\,LH)^{\alpha}\,K^{1-\alpha} \tag{25}$$

where α is the share of labour and ζ is the rate of technological progress.

Minimising total production cost subject to (25) gives the three basic structural equations of the supply-side: the aggregate private value-added deflator, YVAD, which serves as unit of account in the model, optimal labour demand and the optimal capital stock.   Factor demands depend on output, relative factor costs (hourly wage cost WRH, and capital cost CC0) and the rate of technical progress ζ.   Denoting logarithms by lower case characters, the log-linearised equilibrium (denoted by *) factor demands are given as:

$$lh^* = (\alpha - 1).(wrh - cc0) + yvar - \alpha \zeta T \tag{26}$$

$$k^* = -\alpha.(cc0 - wrh) + yvar - \alpha \zeta T \tag{27}$$

The reason for using private sector value added rather than GDP, lies in the importance of identifying as accurately as possible the rate of technical progress, which is an essential parameter determining the steady-state properties of the model. Moreover, since GDP contains public wages and pensions, working with GDP may result in too high a labour factor share and therefore the elasticity of labour demand w.r.t. wage costs may be biased downwards in absolute value and the response of prices to wages could be biased upwards.

Average hours per worker is an increasing function of the extent of full-time working, of conventional working time, and is also cyclical around a trend.

Since the ratio of long-run equilibrium investment (IOR) to target capital equals the sum of the depreciation rate and the steady state growth rate of output (the latter being the sum of the rate of technical progress, ζ, plus the rate of population growth, n), the following steady state investment rate equation holds:

$$ior^* - k^* = \ln(\delta + \zeta + n) \tag{28}$$

Given these cost-minimising factor demand functions, firms set equilibrium prices to equalise marginal revenue and marginal cost so that the target price is a mark-up over marginal cost:

$$YVAD^* = \varepsilon \, MC = \varepsilon \, WRH^\alpha CC0^{(1-\alpha)} \, e^{-\alpha \zeta T} \tag{29}$$

or in logarithms and for a constant mark-up (c):

$$yvad^* = c + \alpha (wrh - \zeta T) + (1-\alpha) cc0 \tag{30}$$

However, due to the degree of openness of the Belgian economy there are good reasons for the mark-up to be non constant. Following Dombrecht and Moës (1998), the mark-up depends on the price elasticity of demand, which is related to the market share of domestic producers and hence to the relative price of national versus foreign goods.

When foreign prices (P*) are high, market shares of domestic firms are sufficiently large for them to raise their own prices. Foreign prices are therefore introduced into the price equation in addition to the degree of capacity utilisation (DUC) as a demand pressure variable:

$$yvad^* = \phi p^* + (1-\phi)[\alpha(wrh - \zeta T) + (1-\alpha)cc0] + \mu\, duc \qquad (31)$$

1.2.2 _Wages_

The government regularly intervened in the wage bargaining process. According to the most recent legislation, the principle of automatic indexation of wages to a "health" index is maintained but nominal wages should not grow faster than the weighted average wage growth in France, Germany and The Netherlands.

Nevertheless, despite its discretionary nature, wage formation seems to be adequately described as the result of bargaining between unions and firms. Following a "right to manage" model, unions and firms bargain over the wage level taking into account the labour demand curve (26). Thereafter firms set goods prices and employment. Consequently, equilibrium wage setting depends on the tax wedge (TW), the relative price of output (YVAD) in terms of the consumption price level (PCD), the unemployment rate (URX) and trend labour productivity:

$$wrh^* = \zeta T + \varpi_1\, tw - \varpi_2\, URX + \varpi_1\, yvad + (1-\varpi_1)\, pcd \qquad (32)$$

In the short run they also respond to apparent productivity changes and to the health index.

1.3   **Foreign trade**

The specification of foreign trade is based on an imperfect substitution approach: neither real imports (MTR) nor real exports (XTR) are perfect substitutes for domestic and international goods respectively. Export volume is related to a world demand variable (WDR) and competitors' export prices (CXBEF) relative to domestic export prices (XTD). Import volume depends on final demand (Y) and the output prices of foreign competitors (CMBEF) relative to domestic prices (YVAD). In a monopolistic competition environment,

domestic exporters enjoy, up to a certain degree, a status of price-makership. They set their export prices with reference to domestic output prices (YVAD) and foreign competitors' export prices. Import prices reflect some 'pricing to the market' behaviour by international competitors and therefore respond not only to foreign prices, including energy prices (PEI), but also to the prices of domestic producers. All foreign prices are expressed in domestic currency and therefore incorporate exchange rate movements.

$$xtr^* = wdr - x_1 (xtd - cxbef) \qquad\qquad (33)$$

$$mtr^* = y - m_1 (cmbef - yvad) \qquad\qquad (34)$$

$$xtd^* = xd_1 \, yvad + (1-xd_1) \, cxbef \qquad\qquad (35)$$

$$mtd^* = \varepsilon \, yvad + mt_2 \, pei + (1 - \varepsilon - mt_2) \, cmbef \qquad\qquad (36)$$

1.4   Main prices

The model treats several price measures. The most important prices among them are:

- the private consumption deflator (PCD), which is modelled by adding indirect tax rates (IT) to an average of private value-added, import and public consumption (GCD) deflators:

$$pcd = it + p_1 \, yvad + p_2 \, mtd + (1 - p_1 - p_2) \, gcd \qquad\qquad (37)$$

- the private investment deflator (IOD) is an average of domestic labour costs, relative to autonomous productivity growth, and competitors' prices on the import side:

$$iod = \lambda(wrh - \zeta T) + (1 - \lambda)cmbef \qquad\qquad (38)$$

- all other prices are derived from the value added deflator or from domestic costs and import prices.

Figure 1 summarises the price formation mechanism.

Figure 1 - The price formation mechanism

THE PRICE FORMATION MECHANISM
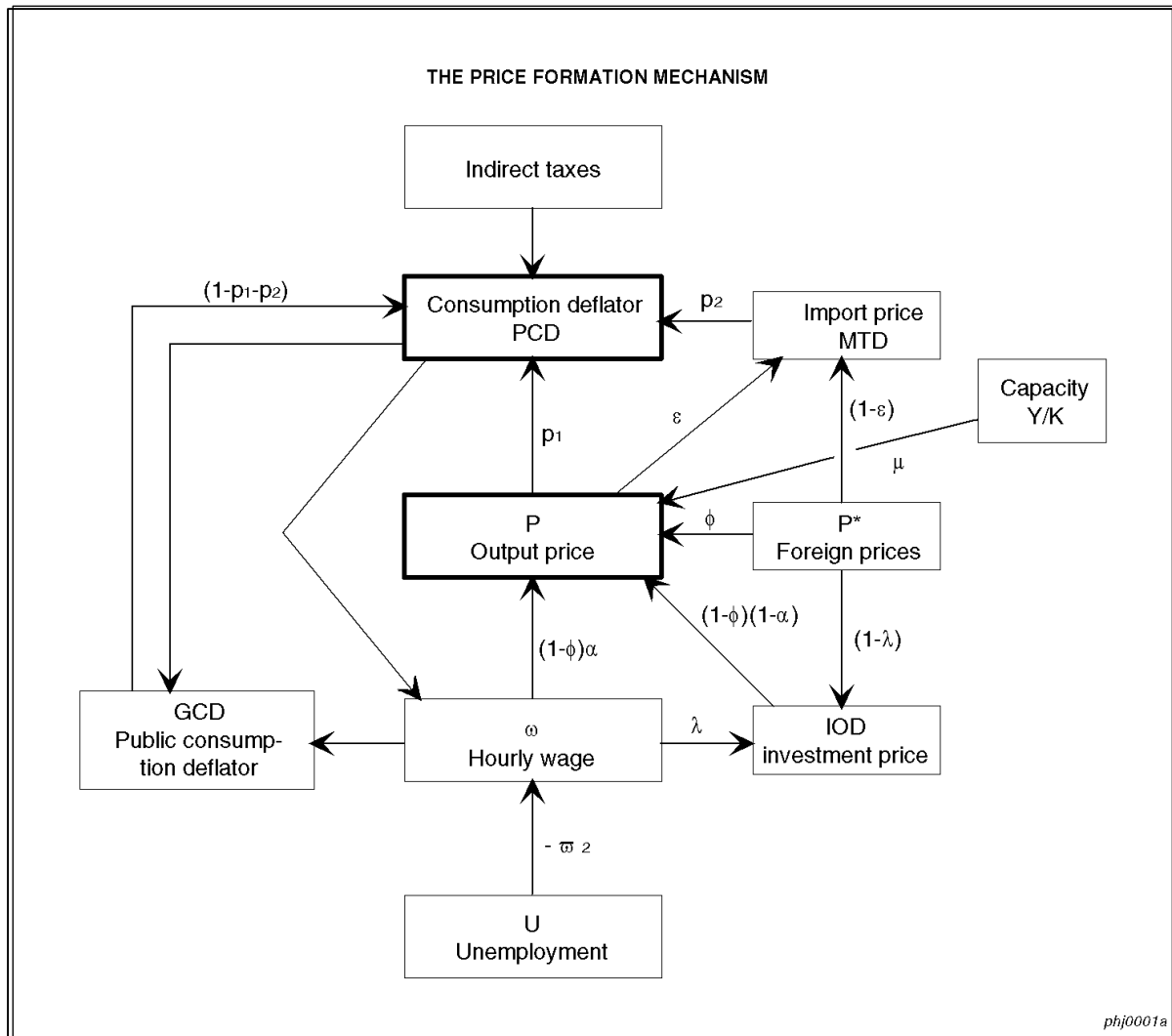
1.5 Government

Many variables in this sector are either exogenous in real terms or defined through technical relations. Current expenditure is divided into interest payments on government debt and different types of primary expenditure categories. The allocation of the outstanding debt over long-term BEF, short-term BEF and foreign currency debt is taken as given and representative interest rates are applied to each corresponding debt category. The weighted sum of these representative rates is in turn used to estimate the implicit rate on government debt. In modelling primary expenditure, the following main items are distinguished:

- government wages and pensions are indexed on the "health" CPI and, in addition, partly follow real wages in the private sector according to a demonstration effect;

- government consumption of goods and public investment are exogenous in real terms and their deflators are linked to the private value-added deflator and to the price of energy;

- most transfers to households are exogenous in real terms but are indexed to the "health" CPI. Unemployment benefits are the only business cycle sensitive component. In the long run total transfer payments to households are sufficiently adjusted, according to a fiscal policy feedback rule, to ensure that the debt to GDP ratio settles down to its steady state value.

General government receipts have been split into direct taxes on households' earned income, taking into account its progressiveness, direct taxes on companies, capital income tax, social security contributions and indirect taxes. In each case, implied tax rates are explained by official rates.

1.6   Steady-state

The focus on model consistent expectations necessitates more attention being given to equilibrium properties than is the case in traditional macro models. Solving forward-looking models requires the imposition of terminal conditions that pin down agents' expectations beyond the simulation horizon. The model's steady state growth rates[2] are the natural candidates for determining such end points. These steady state growth rates can be summarised as follows :

Exogenous variables	Growth factor
- working-age population and labour supply :	$1+n$
- technical progress increases with the labour efficiency index :	$1+\zeta$
- foreign prices inflate at the rate of steady-state inflation in the eurozone[3] :	$1+\pi^*$

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[2]   While the steady-state *growth rates* are known and are invariant to shocks affecting the economy - other than shocks affecting directly the steady state growth rate itself-, the steady-state *levels* are conditional to their history in the simulations.

[3]   In simulating the model, this long-term steady-state inflation is supposed to correspond to the monetary authorities' inflation target.

<u>Endogenous Variables</u>

- various ratios, rates of growth, and rates of return are constant :   1
- all "real" variables increase at the same rate as the working
  population measured in efficiency units: $(1+n)(1+\zeta)$
- employment increases at rate n if the ratio of wage to the cost
  of capital follows the rate of technical progress : $1+n$
- rate of growth of wages equals the rate of growth of prices plus
  productivity growth : $(1+\zeta)(1+\pi^*)$
- domestic and foreign inflation rates converge: $(1+\pi^*)$

When trends are added without theoretical justification as in the exports and money demand equations, those trends are expressed in a logistic form to ensure that they die out in the long run and do not prevent the economy from reaching its steady state.

2   **DYNAMICS**

2.1   Theoretical considerations

In a first stage, equilibrium equations were estimated and subjected to coefficient restrictions from static economic theory according to section 1.   They take the following form:

$$y_t^* = \theta_0 + \sum_{j=1}^{p} \theta_j Z_{j,t} \tag{39}$$

where y_t^* is the decision variable [4]of interest and Z_j are its p explanatory variables.   These equilibrium paths for the decision targets describe the relationships between variables when all dynamic adjustments have been accomplished.

Of course, the current state of variables should not necessarily reflect equilibrium at all points in time. It is therefore necessary to embed the equilibrium conditions into dynamic equations describing their law of motion towards their equilibrium paths.   Many macroeconomic models incorporate deviations from equilibrium in *unrestricted* error correction equations :

$$\Delta y_t = c_0 - \mu\left(y_{t-1} - y_{t-1}^*\right) + a(L)\Delta y_{t-1} + \sum_{j=1}^{p} b_j(L)\Delta Z_{j,t} \tag{40}$$

where $a(L)$ and $b(L)$ are *unrestricted* polynomials in the lag operator added arbitrarily. Such equations may deliver nice empirical fits of the data but they are not suitable for a coherent analysis of responses by rational agents reacting to news about future events. Indeed, dynamic behaviour does not solely originate from delayed responses due to the costs of adjusting variables, but also from movements induced by changes in agents' expectations about future events.   To answer policy-related questions appropriately, agents' expectations need to be identified. By treating expectations explicitly in estimating dynamic equations, this should permit us to identify frictions that impede dynamic adjustments and expectations separately.

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[4]   In what follows, the terms 'decision variable', 'target' and 'equilibrium level' are used as equivalents.

In order to rationalise the introduction of dynamics in macro models, the intertemporal optimisation problem of households and firms is subjected to costs related to the adjustment of decision variables. Each type of agent chooses its decision variable, y, to minimise a combination of expected disequilibrium and adjustment costs: being out of equilibrium is sub-optimal but acting to reach equilibrium is costly as well. The most popular representation of such costs assumes the following quadratic cost function:

$$L_t = E_t \left\{ \sum_{i=o}^{\infty} \beta^i \left[\left(y_{t+i} - y_{t+i}^* \right)^2 + b \left((1-L) \ y_{t+i} \right)^2 \right] \right\} \tag{41}$$

where b represents the cost of adjustment relative to the cost of being out of equilibrium. The first-order conditions for minimising this criterion are given by the second order Euler equation.

$$E_t \left\{ (1 - \lambda_b L)(\lambda_f - L^{-1}) y_t - (1 - \lambda_b)(\lambda_f - 1) y_t^* \right\} = 0 \tag{42}$$

and the terminal condition

$$\lim_{i \to \infty} \left\{ \beta^i \left(y_{t+i} - y_{t+i}^* + b(1-L) y_{t+i} \right) y_{t+i} \right\} = 0 \tag{43}$$

where the two characteristic roots are on both sides of the unit circle, $0 < \lambda_f^{-1} < \lambda_b < 1$. The Euler equation can be estimated directly but this strategy has proved to be very sensitive to specification errors in the dynamic responses of agents, in particular in the presence of serial correlation in the residuals[5]. Furthermore, this equation is only valid for stationary variables, since I(1) variables are by definition cointegrated with each of their lags and leads. Another solution is to derive an error correction model from (42) by making use of an auxiliary model to forecast future values of the target, y^*. This forecast can be generated by a univariate autoregressive model as described in Nickell (1985), or can be a VAR on the determinants of the target. The latter solution gives the following dynamic equation:

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[5] The equation is estimated by GMM after substituting forward realisations of y for its expected values. This introduces a moving average component in the error term of the Euler equation.

$$\Delta y_t = c_0^{'} - \mu\left(y_{t-1} - y_{t-1}^{*}\right) + \sum_{j=1}^{p} b_j^{'}(L)\Delta Z_{j,t} \tag{44}$$

In contrast to (40) the lag polynomial in the dependent variable a(L) now equals zero and a rational expectations forecast assumption imposes cross-equation restrictions between the dynamic coefficients $b_j^{'}(L)$ in the agent's decision rule (44) and the dynamic coefficients in the agent's forecast model of the target. Therefore, contrary to $b_j(L)$ in (40), $b_j^{'}(L)$ in (44) is not unrestricted. The dynamics in (44) come from the auxiliary model used to forecast y^*. When estimating (44), subject to the restrictions implied by the forecasting model, the latter are almost always rejected. This rejection is mainly due to a lack of free parameters to estimate. A richer dynamic specification is therefore desirable.

Such richer dynamics can be introduced when using Polynomial Adjustment Costs (PAC). This generalisation of quadratic adjustment costs is due to Tinsley (1993). Only a brief description of his approach is presented hereafter[6]. Consider the following loss function:

$$C_t = E_t\left\{\sum_{i=0}^{\infty} \beta^i\left[b_0\left(y_{y+i} - y_{t+i}^{*}\right)^2 + \sum_{k=1}^{m} b_k\left((1-L)^k y_{t+i}\right)^2\right]\right\} \tag{45}$$

As in (41), the first squared term represents the disequilibrium cost while the second represents adjustment costs and β is a fixed discount factor. This decision rule loosers the assumption that it is costly to adjust only the *level* of the decision variable (k=1) and introduces costs in modifying *differences* in the variable: the rate of growth of y corresponds to k=2, the rate of acceleration to k=3, etc. Minimisation of this loss function yields the Euler equation

$$E_t\left\{y_t - y_t^{*} + \sum_{k=1}^{m} b_k\left[(1-L)\left(1-\beta L^{-1}\right)\right]^k y_t\right\} = 0 \tag{46}$$

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[6]   For a more exhaustive treatment of what follows, see Tinsley (1993)

or in notation analogous to (42)

$$E_t \left\{ \begin{array}{l} \left(1 - \alpha_1\beta L^{-1} - ... - \alpha_m\beta^m L^{-m}\right)\left(1 - \alpha_1 L - ... - \alpha_m L\right)y_t \\ -\left(1 - \alpha_1\beta - ... - \alpha_m\beta^m\right)\left(1 - \alpha_1 - ... - \alpha_m\right)y_t^* \end{array} \right\} = 0 \qquad (47)$$

A solution to this Euler equation well-suited for estimation is given by the following decision rule :

$$\Delta y_t = c_0 - A(1)\left(y_{t-1} - y_{t-1}^*\right) + \sum_{i=1}^{m-1} a_i^* \Delta y_{t-i} + \sum_{i=0}^{\infty} S_i(\beta, a, m) E_{t-1}\Delta y_{t+i}^* + e_t \qquad (48)$$

where S_i is the multiple-root discount factor, which is analogous to the inverse of the unstable root in the case of costs affecting only the level. They are non-linear functions of the discount rate β and of the m parameters of the lag polynomial A(.), written compactly as 'a'. Expectations of changes in the target, $E_{t-1} \Delta y_{t+i}^*$, are provided by an auxiliary VAR as in (44). Since the extent of these frictions (the size of m) is estimated rather than imposed by an a priori choice of a particular adjustment cost function, the empirical goodness-of-fit of the dynamic model equations is far better than those obtained from usual Rational Expectations models and is comparable with time series models. In particular, high residual autocorrelation which is generally present in empirical tests of decision rules based on level adjustment costs, is strongly reduced.

Optimal adjustment today $?y_t$ depends on three factors : (i) the deviation of the last period's level from its equilibrium $y_{t-1} - y_{t-1}^*$; (ii) past changes in y^7 ; (iii) a weighted forecast of future changes in equilibrium or target levels Δy_{t+i}^* for which the forecast weights S_i are declining in time since they are functions of the discount factor β (i.e. forecasts far in the future are less important than the forecast for tomorrow). It is the introduction of multiple lagged changes in y that enables a better fit to be found for the dynamic behaviour of most macroeconomic variables than fits obtained in former empirical implementations of rational expectations.

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[7]   These lagged terms are not present if agents only minimize the costs associated with changing the level of y which was the assumption made in earlier applications of rational expectations models as estimated from (42) and (44). The parameters a_i^* are the coefficients of the lag polynomial A*(L) implicitly defined by $A(L) \equiv 1 - L + A(1)L - A^*(L)(1-L)L$.

2.2 Estimation

Estimation of (48) requires a three-step process since its coefficients are complicated nonlinear functions of both the parameters in the forecast model and the parameters in the adjustment cost polynomial. First, coefficients in the definition of the targets y* are estimated in a cointegration framework or imposed from theoretical restrictions. Then a forecasting VAR model for Δy^* is estimated. Finally the coefficients a_i^* are estimated. Since the dynamic equation (48) is linear in variables, its nonlinear coefficient restrictions present in the forward weights S_i can be imposed with an iterative Least Squares procedure that, at each iteration, restricts the coefficients in S_i to values determined by estimates of the adjustment coefficients from the prior estimation[8]. In all cases, the value of β has been fixed to 0.95[9].

Households' decisions (consumption, share of housing and residential investment) and firms' decisions (labour demand, investment, and prices) have been modelled in the polynomial adjustment costs framework. As an illustration of the results, the firms' pricing decision rule is given:

$$\Delta yvad_t = -\underset{(0.019)}{0.075} \bullet \left(yvad_{t-1} - yvad_{t-1}^*\right) + \underset{(0.092)}{0.297} \bullet \Delta yvad_{t-1} + \underset{(0.090)}{0.176} \bullet \Delta yvad_{t-2}$$
$$+ E_{t-1} \sum_{i=0}^{\infty} \left(S_i \bullet \Delta yvad_{t+i}^*\right) \tag{49}$$

This equation contains a significant error-correction term (standard deviations are given in parentheses) and two lags in output price inflation, meaning that inflation is sticky. In addition, it is augmented with expectations of the target for which the sum of weights equals 0.446[10]. Grouping all lags and leads gives the following more compact notation:

$$\Delta yvad_t = -0.075 \bullet \left(yvad_{t-1} - yvad_{t-1}^*\right) + 0.473 \bullet lags_2\left(\Delta yvad_{t-i}\right)$$
$$+ 0.446 \bullet leads_{\infty}\left(E_{t-1}\Delta yvad_{t+i}^*\right) \tag{50}$$

---

[8] The order of adjustment costs, m, is chosen empirically by testing for the number of significant lags of the dependent variable in an unrestricted ECM.

[9] Results are not very sensitive to small variations in β, e.g. from 0.95 to 0.98.

[10] All diagnostic tests for the dynamic specification of the equations are given in Appendix A.

Table 1 - Compact view of equations

	Order of adjustment cost (m)	Mean lead of expectations of targets	LR test for REH[1]	Additional dynamics
<u>Households</u>				
Consumption	5	7.8	0.26	Liquidity constrained
Housing market value	4	3.4	0.92	
Investment in dwellings	3	3.1	0.81	
<u>Firms</u>				
Labour	4	6.7	0.74	
Investment	3	8.3	0.49	Accelerator + cash flow
Prices	3	5	0.53	

[1] LR test (p-value) of excluding Var determinants of expected target changes $\Delta y^{*\,e}$. A p-value of 0.05 or less indicates a rejection of REH restrictions with at least a 95% level of confidence.

Table 1 summarises the results obtained for the six equations mentioned above. Column 1 gives the order of adjustment costs, m, ranging from 3 to 5. Column 2 reports the mean lead of expectations of the targets. This is a compact measure of how far ahead agents tend to look as well as how quickly a variable adjusts to its target. In principle, agents plan over an infinite future, but the effective length of the planning period is determined by the extent of the frictions. Actually, a quick adjustment is associated with a short expectation horizon. Figure 2 shows the weight of future targets in current decisions made by households and firms. A long mean lead is reflected in a rather flat curve. The sum of the forward terms is given by the integral under the curve. Column 3 of Table 1 provides a Likelihood Ratio test of the rational expectations overidentifying restrictions on the coefficients of the agents' Var forecast model. If the additional regressors are statistically significant, it implies that the p-values are low, which means that households or firms do not have rational expectations as defined by the Var's forecasts in their dynamic adjustment equations. As shown by their p-values, these restrictions are never rejected at conventional levels of significance[11].

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[11] In the unrestricted equation used in the LR test, the same lags of the variables included in the VAR are introduced as additional regressors.

Figure 2 - Importance of future targets in current decisions

IMPORTANCE OF FUTURE TARGETS IN CURRENT DECISIONS

WORKING PAPER No.4 - MARCH 2000
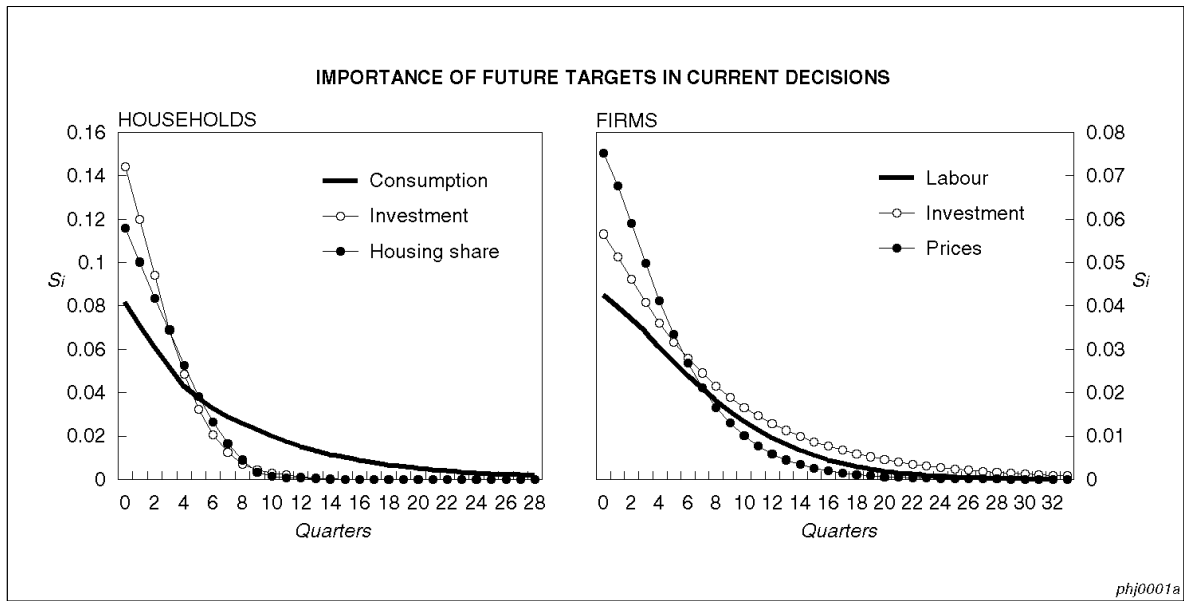
3 DIAGNOSTIC SIMULATIONS

3.1 *Preliminary remarks*

In the model described above, all exchange rates and interest rates are exogenous. Endogenisation of these variables requires an analysis of output, price and interest rate determination at the eurozone level. We leave such an extension for future research. Furthermore, since this model is also destined to be linked to the sub-models of the other EMU participating countries, an extra eurozone-segment will be developed to endogenise typical eurozone variables in the global Multi-Country Model. Moreover in the meantime, the question arises as how to obtain meaningful simulation results or how to interpret such results with the current stance of the Belgian stand-alone model.

Exogeneity of exchange rates implies that model simulations (especially those of shocks originating outside the eurozone area) may incorporate specific competitiveness effects with respect to non eurozone countries that would be absent in a model containing a particular exchange rate theory. The model described above contains both nominal and real interest rates. Since the model explains domestic inflation and inflation expectations π^e, keeping both nominal and real interest rates exogenous would be inconsistent. For long-term simulations such as those presented hereafter, it makes sense to keep the real short term interest rate (rr) constant since it is codetermined by the steady state growth rate of the economy which, in this model, is exogenous and constant. The nominal short term rate (r) is then obtained through Fisher's identity as

$$r_{t,t+i} \;=\; rr_{t,t+i} \,+\, \pi^e_{t,t+i} \tag{51}$$

The nominal long term interest rate can be obtained as a weighted average of expected future values of the short nominal rate plus a term premium, where the latter is considered to be constant. Its movements therefore basically depend on the movements of expected future inflation rates.   Alternatively, nominal interest rates can be kept constant, in which case real interest rates are determined by the path of inflation expectations.   In traditional macroeconometric backward-looking models, it is current practice to use observed inflation as an indicator for expected future inflation rates,

implying that the ex-ante real rate is replaced by an ex-post real rate[12]. Since current inflation is in general more variable than inflation expectations, such practice may imply an overestimation of nominal or real interest rate volatility. On the other hand, in pure rational expectations models, $\pi^e_{t,t+i}$ is obtained from model consistent expectations, implying immediate inflation jumps in response to shocks and therefore long rate volatility in excess of what is normally observed. In the construction of the model, considerable effort has therefore been made to obtain equations with satisfactory statistical properties. As demonstrated in Section 2, the use of Polynomial Adjustment Costs in equations derived from optimisation behaviour permits the presence of significant adjustment lags even under the assumption that agents base their decisions on forward-looking expectations. The introduction of frictions (also) implies slower responses of financial variables to anticipated events. The next section explains how inflation expectations are modelled.

3.2 A hybrid model of inflation expectations

Private sector consumption price inflation expectation is assumed to be common to both households and firms. Originally it was modelled using a separate VARX prediction model, involving consumer price inflation, changes in unit labour costs and import price inflation with the short term interest rate considered to be exogenous. However, the empirical results seemed to indicate that inflation is mainly generated by an autoregressive process, the contribution of the other determinants (i.e. other than past inflation) being rather limited. Moreover, using such a VARX implies potential lasting divergences between expected inflation and the monetary authorities' inflation "target". If monetary policy is credible, it should succeed in preventing inflation rates from diverging persistently from its target. Rational agents should then incorporate this target when formulating their inflation expectations, the weight given to it depending on the degree of credibility. Therefore inflation expectations should take into account some measure of the central bank's inflation "target" as a nominal anchor. Following Black, Macklem and Rose (1998), expectations are based on both backward- and forward-looking components and furthermore also depend on a (very simple) measure of the perceived inflation target[13]. In backward-looking simulations, the forward-looking component is replaced by an autoregressive equation.

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[12]   In the case where the real interest rate is exogenous, this implies that the central bank reacts one for one to observed inflation which is unrealistic.

[13]   This target is measured by applying a HP filter on observed inflation during the sample period. For the future this target is chosen to meet the steady state level of inflation. For comparative simulations, the exact choice of the target does not matter. However for forecasting it does.

3.3   *Simulations*

The purpose of the following two simulations is to investigate model properties. They are not intended to provide a comprehensive account of the model's multipliers. Both the diagnostic simulations presented begin in 1998Q1 and end in 2038Q4. The choice for such a long simulation period - 41 years- is dictated by two requirements. First, solving models with model-consistent-expectations requires a terminal date sufficiently far into the future to avoid the simulation results being affected by the choice of the terminal date. Second, simulating the model over a long period allows the long-term solution of the model to be inspected. Each simulation exercise has been performed under two different assumptions concerning expectation formation. The model is firstly solved under full model consistent expectations. Thereafter, expectations are assumed to be based on the small VAR's used in estimation, i.e. according to the same limited information auxiliary forecasting models as those used by the econometrician.

Simulation results are reported relative to a base constructed over the future which is residual- or add-factor-free, i.e. the base line scenario is not calibrated to match observed data in the first quarters. As such, the base does not represent a forecast in the usual meaning of the word. In the simulation with forward-looking expectations the shock is announced in advance and therefore perfectly anticipated. On the other hand, in VAR-based expectation simulation the shock is recognised only when it actually occurs since by definition the information set is then limited to the predetermined variables. All the simulations are conditional on the assumption of constant short term and long term *real* interest rates.

3.3.1   *A temporary fiscal shock*

This first shock treats the effects of a change in the relative size of government as measured by the ratio of government spending to total output. In period t-1 it is announced that from the next period onwards, the ratio of government spending on goods, services and investment to base GDP will be increased by 1 percentage point. This increase is announced as temporary, lasting two years, after which that ratio will return to its baseline level. This shock implies an increase in the level of government purchases by 30 % during 2 years. When the model functions normally a government reaction function on transfers to households enforces a pre-specified trajectory to bring the public debt to GDP-ratio to a

steady state level, thereby preventing explosive paths for the government debt. This reaction function will therefore tend to start compensating the increased government expenditures by reducing transfers paid to households. To avoid such a compensation affecting the short-term simulation results, the government reaction function was turned off during the first five years of the simulation. This leaves the government debt and deficit ratios free to worsen during this period. The results are summarised in table 2.

Table 2 - Fiscal Shock

year:	-1	0	1	2	5	10	long-term
consumption	0.03	0.12	0.12	0.07	0.01	-0.05	-0.03
	0.00	*0.07*	*0.05*	*0.04*	*0.21*	*0.07*	*-0.02*
investment of cies	0.12	2.14	1.95	-0.82	-0.12	-0.09	0.00
	0.00	*1.98*	*2.02*	*-0.53*	*-0.03*	*-0.01*	*0.00*
private output	0.00	1.00	1.07	-0.04	0.03	0.00	-0.01
	0.00	*1.00*	*1.04*	*-0.06*	*0.06*	*0.01*	*-0.01*
output deflator	0.05	0.13	0.15	0.08	0.05	0.01	0.00
	0.00	*0.10*	*0.18*	*0.09*	*0.10*	*0.07*	*-0.01*
CPI inflation[1]	0.02	0.06	0.06	0.00	0.00	-0.01	0.00
	0.00	*0.04*	*0.09*	*0.02*	*0.01*	*-0.02*	*0.00*
wage inflation[1]	-0.02	0.35	-0.04	-0.29	-0.04	-0.01	0.00
	0.00	*0.25*	*-0.10*	*-0.16*	*-0.05*	*-0.03*	*0.00*
unemployment[1]	-0.07	-0.19	-0.25	-0.18	-0.03	0.00	0.00
	0.00	*-0.14*	*-0.34*	*-0.35*	*-0.11*	*0.00*	*0.00*
trade balance[1,2]	-0.03	-0.56	-0.65	-0.01	0.03	0.03	0.01
	0.00	*-0.52*	*-0.65*	*-0.05*	*-0.05*	*-0.03*	*0.00*
gov.financ.cap[1,2]	0.01	-0.80	-1.09	-0.15	-0.01	-0.03	0.00
	0.00	*-0.82*	*-1.07*	*-0.11*	*0.00*	*-0.02*	*0.00*
gov. debt[1,2]	-0.01	-0.31	0.33	1.63	1.62	1.51	0.01
	0.00	*-0.26*	*0.39*	*1.67*	*1.51*	*1.41*	*0.01*

Model consistent : normal font ; Var : italic.
[1]   Absolute difference from base.
[2]   As % of GDP.

While in the long run, the temporary fiscal expansion does not affect either real or nominal variables, short run effects induced by the excess demand from the public sector are not negligible under both types of expectations formation. In the model, excess demand influences inflation through two main channels. First, changes in unemployment affect wages through wage bargaining between firms and unions: lower unemployment tends to raise the real wage. Second, mark-ups depend upon capacity utilisation. However, as a consequence of the presence of costs in adjusting employment, labour productivity initially improves, but decelerates thereafter. In the model, these short-term effects dominate the wage formation so that wages do not follow the temporary decrease

in unemployment. Once these fluctuations have been absorbed, the unemployment effect is the driving force in both simulations. The short-term effect on wage inflation is somewhat more pronounced under model consistent expectations because forward-looking firms anticipate the temporary nature of the excess demand. Firms' *expected* output is then smoother and employment fluctuates less, thereby enhancing labour productivity and wages relatively to a greater degree. In addition, the high degree of openness of the Belgian economy implies that a large share of the fiscal impulse leaks abroad via higher imports. This reduces the pressure on production capacities and therefore on prices (Figure 3). The short run real output effects of the fiscal stimulus imply absence of Ricardian equivalence. As explained in Section 1.1 two basic model characteristics are responsible for this result. First, the presence of liquidity constrained consumers, who spend all their current income, implies that these consumers do not take account of future tax increases in their current spending behaviour. Second, forward-looking, i.e. non-liquidity constrained consumers, have finite life expectations and therefore discount future expected labour incomes, including future expected tax increases, at a rate above the market interest rate.

Figure 3 - Fiscal Shock

FISCAL SHOCK

Model consistent —— Var

phj0001a
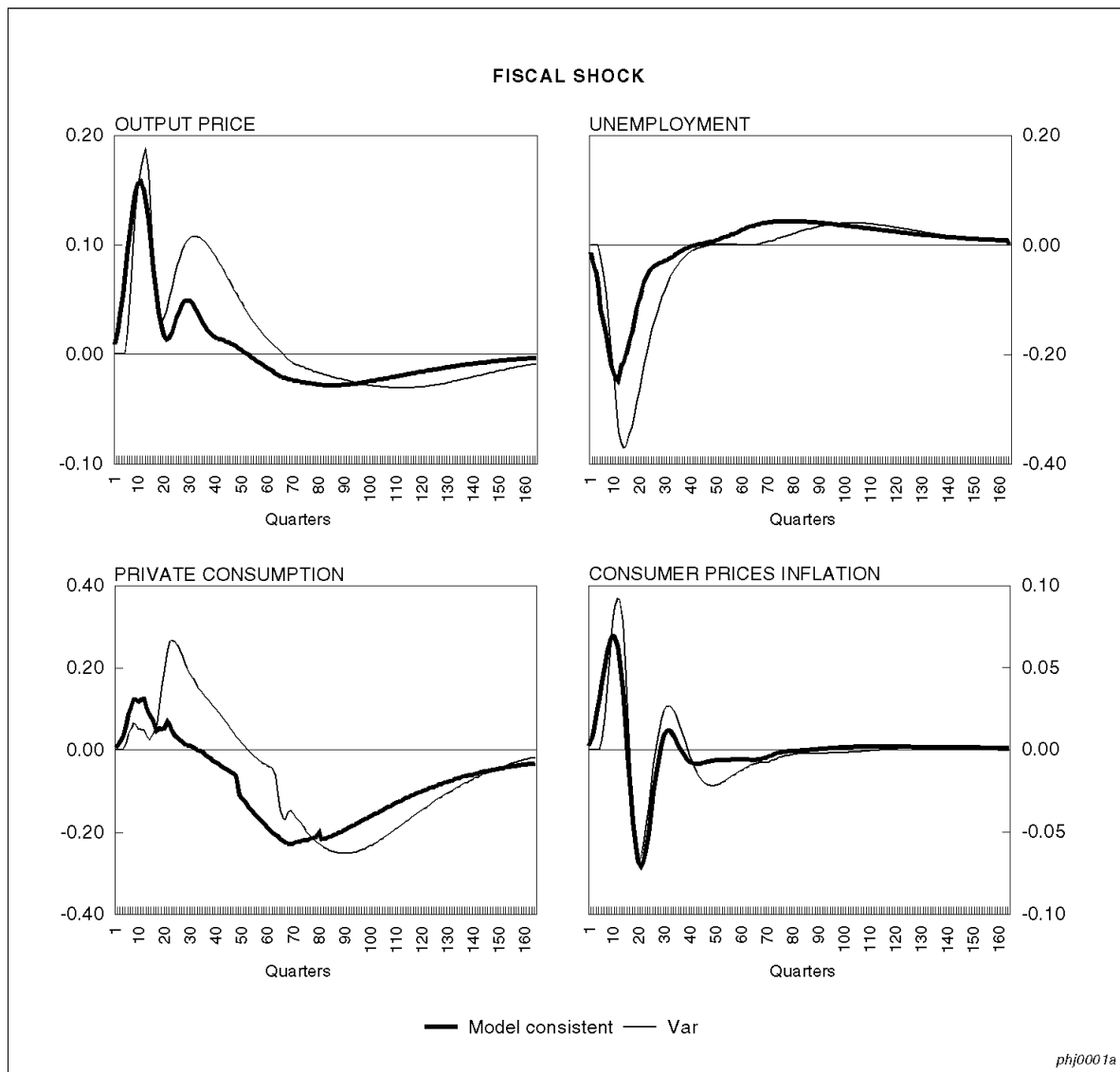
3.3.2 *A permanent labour supply shock*

In this simulation the variable LFN is increased by 1 % throughout the simulation period. This shock is equivalent to about 44,000 additional potential employed persons in 1999. On impact unemployment rises, exerting downward pressure on wages and prices. Competitiveness gains in both in the import and export markets stimulate domestic output and therefore employment. Moreover the simulation results show that the increase in demand for labour employment falls short of the initial increase in labour supply. The unemployment rate is therefore permanently higher by about 0.6 percentage points (see Table 3). This result reflects a lack of sufficient market flexibility to absorb effectively all of the changes in labour supply.

Table 3 - Labour Supply Shock

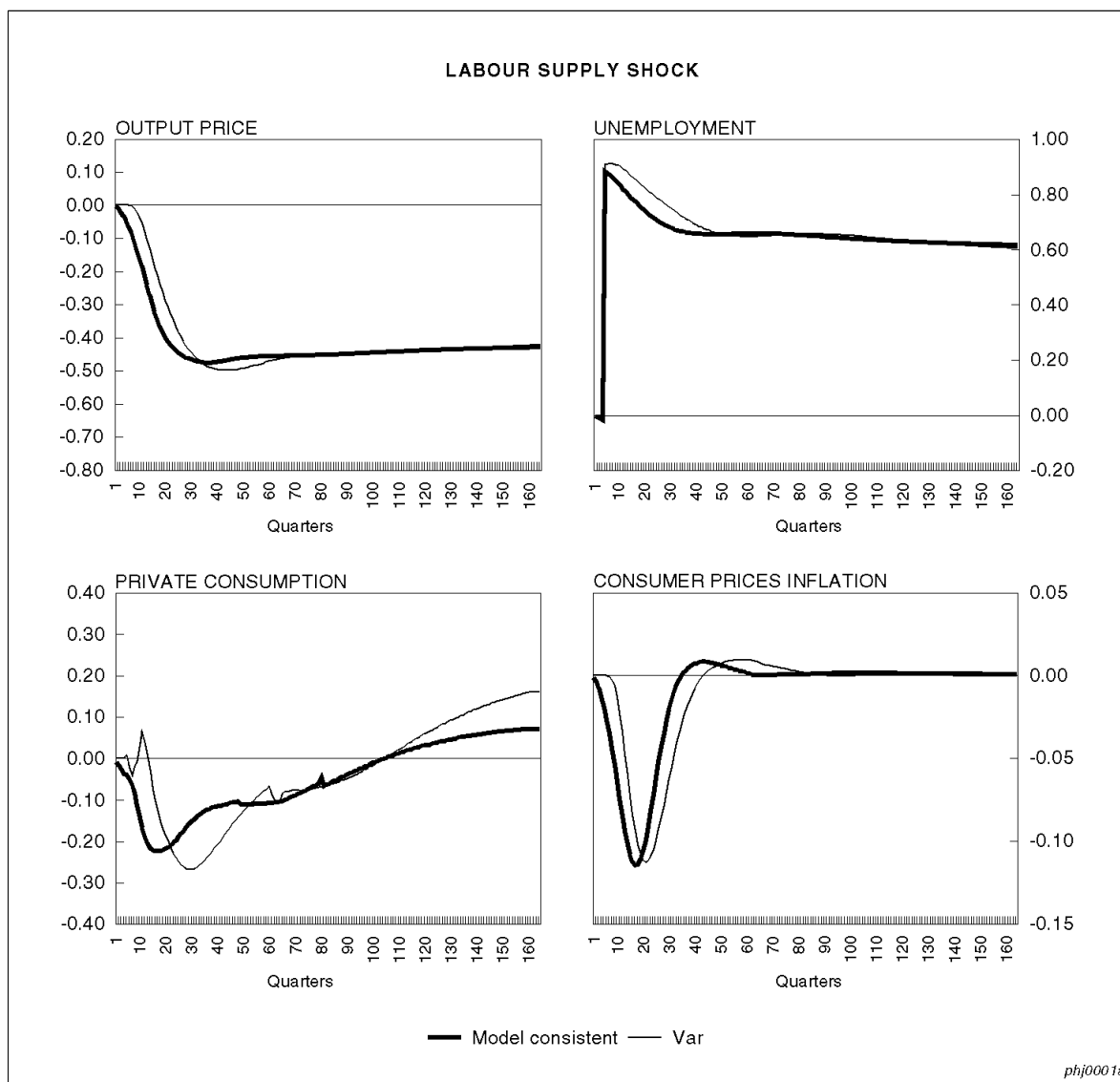| year:	-1	0	1	2	5	10	long-term
consumption	-0.04	-0.08	-0.19	-0.22	-0.17	-0.11	0.07
	0.00	*-0.02*	*0.04*	*-0.10*	*-0.26*	*-0.17*	*0.17*
investment of cies	-0.05	-0.09	-0.11	-0.08	0.10	0.09	0.14
	0.00	*0.01*	*0.10*	*0.01*	*-0.06*	*0.03*	*0.17*
private output	-0.01	0.00	-0.01	0.04	0.18	0.21	0.27
	0.00	*-0.01*	*0.03*	*0.02*	*0.12*	*0.21*	*0.30*
output deflator	-0.03	-0.11	-0.21	-0.32	-0.46	-0.47	-0.43
	0.00	*-0.01*	*-0.08*	*-0.19*	*-0.42*	*-0.50*	*-0.42*
CPI inflation[1]	-0.01	-0.04	-0.08	-0.11	-0.03	0.01	0.00
	0.00	*0.00*	*-0.03*	*-0.08*	*-0.08*	*0.00*	*0.00*
wage inflation[1]	-0.03	-0.27	-0.34	-0.15	-0.02	0.01	0.00
	0.00	*-0.23*	*-0.30*	*-0.19*	*-0.05*	*0.01*	*0.00*
unemployment[1]	-0.02	0.86	0.82	0.78	0.69	0.66	0.62
	0.00	*0.91*	*0.89*	*0.86*	*0.77*	*0.67*	*0.61*
trade balance [1,2]	0.02	0.06	0.13	0.18	0.21	0.21	0.14
	0.00	*0.00*	*-0.01*	*0.07*	*0.25*	*0.25*	*0.11*
gov.financ.cap[1,2]	0.00	-0.04	-0.05	-0.04	-0.03	-0.03	0.00
	0.00	*-0.04*	*-0.05*	*-0.05*	*-0.04*	*-0.03*	*0.00*
gov. debt[1,2]	0.01	0.06	0.17	0.30	0.49	0.52	0.05
	0.00	*0.04*	*0.09*	*0.19*	*0.50*	*0.59*	*0.04*

Model consistent : normal font; Var: italic.

[1]   Absolute difference from base.
[2]   As % of GDP.

Due to the backward-looking nature of wage indexation in Belgium, wage costs evolve quite similarly in the simulations with VAR and model consistent expectations. The main difference between both types of expectations lies in the composition of final demand. In both cases, the jump in unemployment, which increases the perceived risk concerning future expected labour income, has a discouraging effect on consumption. But as the decrease in wages is anticipated by forward-looking consumers, it induces a more rapid negative response on consumption (figure 4). Also, when firms have model consistent expectations, their pricing decision in response to an expected reduction in costs is far quicker. The trade balance therefore improves immediately under model consistent expectations whereas it takes two years in the VAR for such an improvement to occur. Under both assumptions about expectations formation, it takes around 15 years for real and nominal variables to return to their baseline *growth rates*.

Figure 4 - Labour Supply Shock

LABOUR SUPPLY SHOCK

OUTPUT PRICE

UNEMPLOYMENT

PRIVATE CONSUMPTION

CONSUMER PRICES INFLATION

Model consistent —— Var

phj0001a

4   <u>CONCLUSIONS</u>

The new quarterly macroeconometric model for Belgium presented in this paper, has been developed as a response to the need for new analytical instruments to contribute to the monetary policy debate in the euro area.   In this respect it serves two main purposes.   First, it contributes to the analysis of the transmission mechanisms of the common euro area monetary policy in the Belgian economy.   Secondly, it contributes to forecast exercises.   Besides its role in the European monetary policy debate, the model can also be used to analyse the consequences of asymmetric domestic shocks, under the assumption that these shocks affect neither euro area monetary policy, nor euro area wide aggregates.

The main features of the model can be summarised as follows.   First it is a fairly compact model.   It distinguishes of course the main sectors, such as households, firms, government and external transactions, although the private sector has not been further desegregated into sub-sectors such as industry, services, etc.   Secondly, the underlying theory is as close to recent theoretical macroeconomic developments as possible taking into account the specific structures of the Belgian economy.   Thirdly, the model's dynamics do not only allow for agent's lagged adjustments due to transaction costs, but also for agents to anticipate to future expected events or policy reactions.   The influence of forward-looking behaviour is fairly widespread in the model.   In this respect, not only financial variables react to anticipations, but also agents' real behaviour, such as consumption, factor demands and price formation.   In general therefore, economic agents' short-term responses to shocks are governed in part by lagged adjustment as well by anticipating adjustment.   Agents' short-term behaviour is basically anticipative, but due to transaction costs, it does not adjust immediately.   Anticipations themselves can in theory be based on several alternative assumptions.   If this model is thought to be fairly realistic in the sense that it reflects agents' behaviour reasonably well and if economic agents are fairly rational, then it would be natural to assume that expectations should be model consistent.   On the other hand, if it is assumed that agents form their expectations on the basis of rather limited information, it is more useful to model expectations on the basis of a VAR prediction model.   Both hypotheses are present in the model and simulations can be run under both assumptions.   Fourthly, long-term simulations have shown that under both assumptions concerning expectation formation, the model converges to its steady state, defined by the underlying macroeconomic theory.

The development of the model itself has, however, far from reached completion. Much remains to be done. The model's quarterly database has to be converted to the new ESA 95 data and the equations have then to be re-estimated and subjected to diagnostic simulations. Furthermore, as the model will continue to be used intensively, unrealistic properties may reveal themselves and improvements may certainly become necessary. The current version of the model as presented in this paper therefore certainly cannot be considered a final one. Furthermore, the model focuses on the behaviour of national agents and policies and ignores price, output, interest rates and exchange rate reactions in the eurozone. However, in reality, these eurozone variables do react to common area wide shocks, implying that the impact of such shocks on the Belgian economy also depends on the reaction of the area-wide aggregates. This observation underlies the choice to report only a very restricted number of simulations in this paper. The construction of a small structural forward looking eurozone model to supplement the Belgian model is therefore one of our priorities for future research.

APPENDIX A: THE MODEL'S EQUATIONS

This appendix contains a list of equations in which numbers in italic are standard deviations. A complete list of model variables is provided in Appendix B. Where no estimation details are given, the equation was not estimated. This is of course the case for identities, but in addition to these a large number of minor model variables have simply been assumed to grow in line with GDP or some other aggregate.

HOUSEHOLDS

Consumption

Target

Due to the presence of expected real human wealth, the long run equation is estimated by GMM which results in:

$$pcr^* = (1-0.213) \, E_t(hw) + 0.213 \, (fw - pcd) + 0.0687 \, RAGG_t - 0.002 \, URX_t$$

$$\quad\quad\quad\quad\quad (0.177) \quad\quad\quad (0.037) \quad\quad (0.335)$$

1981:2 - 1996:4 , Pvalue(Jtest) = 0,048

(small case letter denotes log of the variable and standard deviations are mentioned in italic)

Dynamics

$$\Delta pcr_t = (1 - 0,228).(-0,082 . (pcr_{t-1} - pcr^*_{t-1}) - 0,011 \, lags_4 \, \Delta pcr_{t-i} + 0,614 \, leads_\infty (E_{t-1}\Delta pcr^*_{t+i}))$$

$$\quad\quad + 0,228 \, \Delta(yl_t - pcd_t) - 0,196 - 0,017 \, d842 + 0,014 \, d872_t$$

$$\quad\quad (0,119) \quad\quad\quad (0,095) \quad (0,006) \quad\quad (0,006)$$

$R^2 = 0.41$; Ser = 0.0056 ; dw = 1.91 ; lm(4) = 6.51 (0.16) ; JB = 0.10 (0.98) ; Arch(4) = 1.97 (0.75); Mean lead = 7.8.

Housing
Target

Let h^* be the optimal share of housing, HHOS*/FW. The wealth term on the right-hand side is lagged for reason of simultaneity:

h^*_t = 1.00 *(0.05)* $(yl_t - (\zeta+n) T)$

      + 0.66 *(0.09)* $(FW_{t-1}/HHOS_{t-1})$. (RETKRH-RMT_real)/100

      - 0.27 *(0.02)* $(lmt_t - pcd_t - (\zeta+n) T)$

Dynamics

$\Delta(hhos_t - pcd_t)$ = -0.115 . $(1 - h^*_{t-1} / h_{t-1})$ + 0548 lags$_3$ $\Delta(hhos_{t-i} - pcd_{t-i})$

        + 0.476 leads$_\infty$ $(E_{t-1}\Delta(hhos^* - pcd)_{t+i})$ - 0.003 + 0.02$(d901 - d901_{t-1})$

R^2=0.58 ; Ser = 0.011 ; dw = 2.12 ; Lm(4)=4.17 (0.38); JB=0.13 (0.93); Arch(4)=5.26 (0.26); Mean lead = 3.4; 1982:2-1997:4.

Investment in dwellings

Since house prices are much more volatile compared to the demand, in volume, for houses, the former equation tends to explain the movements of market prices. It should therefore be supplemented with an explanation of the supply of new houses coming from residential investment. The ratio of housing investment to consumption is given by relative prices and returns:

Target

ihr_t^* = pcr_t - 1.36 * (RMT-RETKRH)/100 -1.05 * $(ihxn_t - pcd_t)$ + 3.88 - .32*d8089

Dynamics

$\Delta(ihr_t)$ = -0.144 $(ihr_{t-1} - ihr^*_{t-1})$+ 0.414 lags$_2$ $\Delta(ihr_{t-i})$ + 0.549leads$_\infty$ $(E_{t-1}\Delta ihr^*_{t+i})$ - 0.381

    +dummies

R^2=0.62 ; Ser = 0.032 ; dw = 2.26 ; Lm(4)=3.86 (0.42); JB=1.29 (0.52); Arch(4) = 4.50 (0.30) ; Mean lead = 3.1; 1981:2 - 1997:4.

Housing stock

The real capital housing stock itself is calculated using the permanent inventory method, by cumulating real investment in houses assuming a lifetime of 60 years and afterwards applying a correction factor, μ, to correct for registration rights:

$$HHOQ_t = (1-1/240)*HHOQ_{t-1} + \mu\ IHR_t$$

Price of houses on the secondary market

$$IHXO\ = 1511,584\ HHOS_t\ /HHOQ_t$$

Market value of the capital stock

The present value of after-tax income beyond t is replaced by the next period discounted value of Q/PCD. In the discount rate, the real short term interest rate is augmented with a risk premium, estimated at 0.07.

$$Q_t = (0.42.YVAN_t - OTN_t)/KRE_t$$
$$+ \left[(1-\delta)/(1+rr_t+0.07)(1+\Delta \ln(KRE_t)\right]E_t \left\{Q_{t+1}.(KRE_{t+1}/KRE_t).(PCD_t/PCD_{t+1})\right\}$$

M0

Consumption seems to perform better than any other transaction variable and the unit transaction elasticity has been imposed :

Equilibrium

$$\log(M0^*) = \log(PCR)+ \log(PCD)- 2.26\ (1/(1+e^{-0.025T})) - 1.35\ (STI\_real/100)$$

Dynamics

$$\Delta \log(M0_t) = -0.140\ (0.042)\ \{\ \log(M0_{t-1}) - \log(M0^*_{t-1})\ \}+0.238\ (0.093)\ \Delta \log(M0_{t-1})$$
$$-0.059\ (0.01)\ d944 - 0.049\ (0.01)\ d924 -0.025(0.01)\ d853-0.85\ (0.27)$$

R² = 0.57 ; Ser = 0.0094 ; dw = 2.09 ; Lm(4) = 1.55 (0.81); JB = 2.95 (0.22) ; Arch(4) = 2.75 (0.59), 1981:2-1997:4.

Consumption credit

$$\Delta \ (CCRES/PCN) = 0.04 \ (0.01) - 0.05 \ (0.02) \ (CCRES_{t-1}/PCN_{t-1}$$
$$-(-0.05 \ (0.02) \ RCC_{t-1} \ -0.27 \ (0.12)*D93END_{t-1})) \ -0.07d924+0.02d824$$

R² = 0.72 ; Ser = 0.0066 ; dw = 2.23 ; *Lm(4) = 15.4 (0)*; JB = 1.21 (0.54) ; Arch(4) = 7.99 (0.09); 1981:1-1996:4.

Mortgage loans

$$\Delta \log(LMT/PCD) = 0.28 \ (0.10) \ -0.09 \ (0.02) \ (\log(LMT_{t-1}/PCD_{t-1})$$
$$-(\ 0.50 \ (0.09) \log(FW_{t-1} \ /PCD_{t-1}) + 0.22 \ (0.08) \log(IHN_{t-1}/PCD_{t-1})) \ )$$
$$+0.11 \ (0.09) \ \Delta \log(LMT_{t-1}/PCD_{t-1}) - .54 \ (0.23) \ \Delta \log(PCD)+0.03*d824$$

R² = 0.72 ; Ser = 0.006 ; dw = 2.0 ; *Lm(4) = 15.2 (0)*; JB = 0.63 (0.72) ; Arch(4) = 1.55 (0.81); 1981:1-1996:4.

Financial wealth

$$FW = Q*KRE+HHOS+GDN+NFA+M0-CCRES-LMT$$

<u>**SUPPLY-SIDE**</u>

Private value added is related to GDP by the following relation:

$$YVAR = (1-rati\_yvar)*YER - GCR2 - GCR3 - GCR4 -ZVR$$

Here GDP is first corrected for the proportion of VAT, customs taxes and imputed rent of banking services, then public wages and pensions are subtracted and ZVR is a statistical discrepancy reflecting the difference between the sum of sectoral value-added and GDP.

Factor demands

<u>Targets</u>

$$\log(LPN \cdot HOURS) = 4.952 + (0.52-1) \cdot \log(WRH/CC0) + \log(YVAR) - 0.52 \cdot 0.003875 \cdot T$$
$$\qquad\qquad (0.007) \qquad\qquad\qquad\qquad\qquad (0.00019)$$
$$\qquad\qquad -0.026\ d8889$$
$$\qquad\qquad (0.009)$$

$$\log(KRE) = 3.121 - 0.52 \cdot \log(CC0/WRH) + \log(YVAR) - 0.52 \cdot 0.003875 \cdot T$$
$$\qquad\quad (0.036) \qquad\qquad\qquad\qquad\qquad (0.00019)$$
$$\qquad\quad +0.186\ \log(DUC) - 0.042\ d8889$$
$$\qquad\quad (0.009) \qquad\qquad (0.020)$$

<u>Dynamic factor demands</u>

Let LNHH denote total hours worked , LPN*HOURS. Then,

$$\Delta(lnhh_t) = -0.04\,(lnhh_{t-1} - lnhh^*_{t-1}) + 0.554\ \text{lags}_3\,(\Delta lnhh_{t-i}) + 0.365\ \text{leads}_\infty(\,E_{t-1}\Delta lnhh^*_{t+i})$$
$$\qquad + 0.21 + 0.015\ d874 - 0.008\ d924 + 0.02\ \Delta yvar_t$$

$R^2 = 0.647$; Ser=0.0025 ; dw = 2.05 ; Lm(4) = 3.10 (0.54) ; JB = 3.83 (0.14); Arch(4) = 0.78 (0.94); Mean lead = 6.7; 1981:2-1997:4

Since the ratio of long-run equilibrium investment, ior^*, to target capital equals the sum of the depreciation rate and the steady state growth rate of output, the dynamic investment equation can be rewritten as :

$$\Delta ior_t = -0.056\,(ior_{t-1} - ior^*_{t-1}) + 0.216\ \text{lags}_2(\Delta ior_{t-i}) + 0.492\ \text{leads}_\infty(\,E_{t-1}\Delta ior^*_{t+i})$$
$$\qquad +1.13\ (0.26)\ \Delta yvar_t + 0.34\ (0.17)\ \Delta\log(\textstyle\sum_{l=0\ \text{to}\ -3} GON_{t+l}\,)$$
$$\qquad +0.173 + 0.122(d841 - d841(-1)) + 0.078(d884 - d88(-1)) - 0.075(d924 - d924(-1))$$

$R^2 = 0.823$; Ser=0.018 ; dw = 1.78 ; Lm(4) = 6.06 (0.19); JB = 1.68 (0.41); Arch(4) = 0.74 (0.95); Mean lead = 8.3; 1981:3-1997:4.

Average hours

$$\Delta hours_t = -0.30\,(hours_{t-1} - (\,durcon_{t-1} + etpl_{t-1} + 0.013\,(yvar_{t-1}/kre_{t-2}))) + 0.75\,\Delta durcon_t$$

Private value added deflator

   Target

$$yvad^* = 0.35\,awmpcd + (1-0.35)\,[0.52\,(\,wrh - \zeta T\,) + (1-0.52)\,cc0\,] + 0.22\,duc$$

   Dynamics

$$\Delta yvad_t = -0.075\,(yvad_{t-1} - yvad^*_{t-1}) + 0.473\,lags_2(\Delta yvad_{t-i}) + 0.446\,leads_\infty(\,E_{t-1}\Delta yvad^*_{t+i})$$
$$+\,0.14 - 0.015\,(d821 - d821(-1)) + 0.009\,d844 - 0008\,(d832 - d832(-1)) + 0.008\,d901$$

$R^2 = 0.67$; Ser$=0.0038$; dw $= 1.85$; Lm(4) $= 6.12$ (0.19) ; JB $= 2.31$ (0.31) ; Arch(4) $= 4.73$ (0.36); Mean lead $= 5$; 1982:1-1997:4.

Wages

   Equilibrium

$$\log(WRH^*) = -7.64 + \zeta\,T + 0.13\,\log(1+TW1) - 0.13\,\log(1-TW2-TW3) - 0.0067\,URX$$
$$+\,0.13\,\log(YVAD) + (1-0.13)\,\log(PCD)$$

   Dynamics

$$\Delta\,wrh_t = -0.32\,\textit{(0.09)}\,(\,wrh_{t-1} - wrh^*_{t-1}) + 0.50\,\textit{(0.37)}\,\Delta\,pcd2_t + (1-0.50)\,\Delta\,pcd2_{t-1}$$
$$+\,0.35\ \ \textit{(0.21)}\,[\Delta(yvar_t - lpn_t - hours_t) + \Delta(yvar_{t-1} - lpn_{t-1} - hours_{t-1})]$$
$$-\,0.30\,\textit{(0.10)}\,\Delta\,wrh_{t-1} - 2.48 + 0.04(d911 - d911_{t-1}) + 0.04(d921 - d921_{t-1}) + 0.05d933$$

$R^2 = 0.60$; Ser$=0.017$; dw $= 2.12$; lm(4)$=1.46$ (0.83); JB$= 0.67$ (0.71); Arch(4)$= 2.40$ (0.66) ; 1981:3-1996:4.

PRICES

Cost of capital

It is defined as a real interest rate multiplied by the price of investment which is estimated as

$$iod = 0.63 \, (wrh - \zeta T) + (1-0.63) \, cmbef$$

Private Consumption Deflator

The private consumption deflator is an average of the value-added, import and public consumption deflators increased by VAT rate and excise duties on consumption goods and services.

$\log(PCD^*/(1+GR\_VATCONS)/(1+GR\_EXCISE)) =$
$\quad 0.80 \, (0.06) \log(YVAD) + 0.08 (0.02) \, (0.79 (0.19) \log(MTD) + 0.21\log(PEI))$
$\quad + (1-0.80-0.08) \log(GCD)$

$\Delta\log(PCD_t/(1+GR\_VATCONS_t)/(1+GR\_EXCISE_t)) =$
$\quad -0.102 \, (0.048) \, \{\log(PCD_{t-1}/(1+GR\_VATCONS_{t-1})/(1+GR\_EXCISE_{t-1})$
$\quad - (0.80 \log(YVAD_{t-1}) +$
$\quad 0.08 \, (0.79\log(MTD_{t-1})+0.21\log(PEI_{t-1})) + (1-0.80-0.08) \log(GCD_{t-1})) \, \}$
$\quad +0.248 \, (0.085) \, \Delta\log(PCD_{t-1})$
$\quad +0.343 \, (0.079) \, \Delta\log(PCD_{t-3})$
$\quad +0.325 \, (0.065) \, \Delta\log(PCD^*_t)$
$\quad +0.009 \, (0.002) \, (d811_t-d811_{t-1}) \, -0.004 \, (0.002) \, (d921-d921_{t-1}) -0.014 \, (0.006)$

$R^2=0.818$; Ser=0.0026 ; dw=1.82 ; Lm(4)=6.66 (0.15) ; JB=0.86 (0.64) ; Arch(4)=8.90 (0.06); 1981:1-1996:4

"Health" Index

$\log(PCD2/(1+GR\_VATCONS)) = 0.69 \, (0.03) \log(YVAD) + 0.14 \, (0.01) \log(MTD)$
$\quad\quad\quad +(1-0.69-0.14) \log(GCD)$

$$\Delta\log(PCD2_t/(1+GR\_VATCONS_t)) =$$

$$-0.212 \; (0.061) \; \{\log(PCD2_{t-1}/(1+GR\_VATCONS_{t-1})) - (0.69 \log(YVAD_{t-1}) +$$

$$0.14\log(MTD_{t-1}) + (1-0.69-0.14) \log(GCD_{t-1})) \}$$

$$+0.283 \; (0.094) \; \Delta\log(PCD2_{t-2})$$

$$+0.199 \; (0.089) \; \Delta\log(PCD2_{t-3})$$

$$+0.377 \; (0.084) \; \Delta\log(PCD2^*_t)$$

$$+0.010 \; (0.003) \; (d811_t-d811_{t-1})-0.019 \; (0.006)$$

$R^2=0.69$; Ser=0.0037 ; dw=1.86 ; Lm(4)=9.60 (0.05) ; JB=0.73 (0.69) ; Arch(4)=5.50 (0.23); 1981:1-1996:4

Price of building houses

$$\log(IHXN^*/(1+GR\_VATLOG_t)) =$$

$$0.66 \; (0.03) \; (wrh-(yvar-lpn-hours)) + (1-0.66) \; mtd +0.074 \; d8083$$

$$\Delta\log(IHXN_t/(1+GR\_VATLOG_t)) =$$

$$-0.069 \; (0.021) \; *(\log(IHXN_{t-1}/(1+GR\_VATLOG_{t-1}))-\log(IHXN^*_{t-1}))$$

$$+0.55 \; (0.08) \; \Delta\log(IHXN_{t-1}/(1+GR\_VATLOG_{t-1}))$$

$$+0.04 \; (0.03) \; \Delta\log(IHXN^*_t) + 0.43 \; (0.13)$$

$$-0.88 \; (0.07) \; \Delta\log(1+GR\_VATLOG_t)$$

$$+0.31 \; (0.11) \; \Delta\log(1+GR\_VATLOG_{t-1})$$

$$-0.15 \; (0.07) \; \Delta\log(IHXN_{t-4})$$

$R^2=0.929$; Ser = 0.0044 ; dw = 1.80 ; Lm(4) = 3.85 (0.42) ; JB = 5.02 (0.08); Arch(4) = 1.92(0.75); 1981:2-1996:4

FOREIGN TRADE

Export volume

The demand elasticity is set to one which is close to its freely estimated value. The competitiveness indicator is based on the Belgian competitor's export prices. A logistic trend reflects a structural loss of market share induced by non-price

competitiveness. A structural dummy reflects methodological changes in foreign trade statistics from 1993 onwards. This dummy is not necessary to get sensible results but it delivers a higher price elasticity

$$xtr^* = -0.60 (xtd-cxbef) + 1 \ wdr$$
$$- 0.99 (1/(1+e^{-0.02T})) +0.065 \ d93end - 0.207 \ duc$$

$$\Delta xtr_t = -0.468 \ (0.085) \ [\ xtr_{t-1} - xtr^*_{t-1} \ ]$$
$$+ 0.584 \ (0.225) \ \Delta wdr_t - 0.315 \ (0.098) \ \Delta(xtd_t - cxbef_t)$$
$$+0.045 \ (0.009) \ (d851-d851_{t-1}) - 0.058 \ (0.010) \ (d871-d871_{t-1})$$
$$+0.065 \ (0.009) \ (d924-d924_{t-1}) +0.032 \ (0.014) \ d973$$

$R^2 = 0.771$; SER = 0.014 ; dw = 2.06 ; LM(4) = 4.74 (0.31) ; *JB = 14.8 (0.0)*; Arch(4) = 8.55 (0.07) ; 1981:1-1997:4

The logistic trend implies that export volume grows structurally (i.e. independent from price competition) slower than world demand by 1.2 % in 1997 as compared to by 1.7% per year in 1990.

Export deflator

$$\Delta xtd_t = -0.174(0.076) \ [\ xtd_{t-1} -$$
$$\{0.245 \ (0.060) \ yvad_{t-1} + (1-0.245) \ cxbef_{t-1}) \ \} + 0.124 \ (0.325) \ duc_{t-1} \ ]$$
$$+ 0.372 \ (0.090) \ \Delta cxbef_t + 0.339 \ (0.081) \ \Delta cxbef_{t-1}$$
$$- 0.082 \ (0.257) +0.04 \ (0.01) \ (d854-d854(-1)) +0.04 \ (0.01) \ (d864-d864(-1))$$

$R^2 = 0.705$; Ser = 0.0113 ; dw = 1.80 ; *Lm(4) = 10.25 (0.04)* ; JB = 0.41 (0.81) ; Arch(4) = 2.77 (0.59) ; 1980:3-1997:4.

Import volume

The composition of imports by category of final demand is obtained from input-output tables:

$$mtr^* = -0.65 (cmbef-yvad)$$
$$+ 1 . \{0.22 \ ior + 0.44 \ xtr + 0.34 \ pcr\} + 0.393 \ duc$$

Δmtr_t = -0.20 *(0.08)* [mtr_{t-1} - mtr^*_{t-1}]

$\qquad$ + 1 {0.22 Δior_t + 0.34 Δpcr_t + 0.44 Δxtr_t}

$\qquad$ - 0.20 *(0.10)* ($cmbef_t$-$yvad_t$)

$\qquad$ -0.033 *(0.01)* ($d881_t$-$d881_{t-1}$) - 0.034 *(0.01)* ($d884_t$-$d884_{t-1}$)

R^2 = 0.647; Ser = 0.015; dw = 2.37; Lm(4) = 6.66 (0.15); JB = 1.71 (0.42); Arch(4) = 3.09 (0.54); 1981:1-1997:4.

Import deflator

The import deflator is mainly explained by the competitors' price but domestic prices enter for more than 30 p.c. reflecting a non-negligible degree of pricing to the market by international exporters to the Belgian market. The price of energy and the degree of capacity utilisation have been included as additional explanatory variables:

Δmtd_t = $\qquad$ -0.20 *(0.088)* [mtd_{t-1} -

$\qquad$ {0.27 *(0.10)* $yvad_{t-1}$ +0.13 *(0.05)* pei_{t-1}

$\qquad$ + (1-0.27-0.13) $cmbef_{t-1}$ +0.24 *(0.34)* duc_{t-1} }]

$\quad$ + 0.09 *(0.01)* Δpei_t +0.44 *(0.09)* $\Delta cmbef_t$

$\quad$ -0.06 *(0.01)* (d861-d861(-1))+0.03 *(0.01)* (d864-d864(-1)) -0.29 *(0.33)*

R^2 = 0.763; Ser = 0.0124; dw = 2.17; Lm(4) = 5.87 (0.20); JB = 2.41 (0.29); Arch(4) = 3..97 (0.41); 1980:3-1997:4.

<u>GOVERNMENT</u>

Wages and pensions

In addition to being indexed to a "health" CPI, government wages and pensions partly follow real wages in the private sector. A one percent increase in the private sector real wage tends to be followed by approximately half a percent increase in public sector real wages . This demonstration effect takes more than four years to be complete:

$\Delta log(GCD234_t)$ = 0.22 *(0.08)* -0.06 *(0.02)* (log($GCD234_{t-1}$/$PCD2_{t-1}$)-

$$0.44 \ (0.14) \ \log(WRH_{t-1}/(1+GR\_SOCO_{t-1})/PCD2_{t-1}))$$
$$+0.33 \ (0.09) \ \Delta\log(GCD234_{t-1}/PCD2_{t-1})$$
$$+0.24 \ (0.08) \ \Delta\log(GCD234_{t-3}/PCD2_{t-3})$$
$$+0.54 \ (0.11) \ \Delta\log(PCD2_t) + (1-0.54) \ \Delta\log(PCD2_{t-1})$$
$$-0.02 \ (d833_t-d833_{t-2})+ 0.01 \ d911$$

$R^2 = 0.58$; Ser = 0.0048; dw = 1.75; Lm(4) = 8.95 (0.06); JB = 1.25 (0.53); Arch(4) = 1.63 (0.80);1981:2-1996:4.

Other public expenditures deflators

Government consumption of goods and services and public investment are exogenous in real terms. Their deflators are linked to the private value-added deflator and to the price of energy;

$$\Delta_4 \log(GCD1) = 0.03-0.66 \ (0.22) \ (\log(0.25 \ \textstyle\sum_{l=1 \ to \ 4} \ GCD1_{t-l}))$$
$$-(0.92 \ (\log(0.25 \ \textstyle\sum_{l=1 \ to \ 4} \ YVAD_{t-l})) + (1-0.92) \ (\log(0.25 \ \textstyle\sum_{l=1 \ to \ 4} \ PEI_{t-l}))))$$
$$+0.61 \ (0.09) \ \Delta_4 \log(GCD1_{t-1})+ 0.05 \ (0.02) \ \Delta_4 \log(PEI)$$
$$+ 0.05(d834-d834_{t-1}+0.07*d921$$

$R^2 = 0.58$; Ser = 0.0253; dw = 1.83; Lm(4) = 2.45 (0.65); JB = 0.51 (0.77); *Arch(4) = 10.54 (0.03)*; 1981:2-1996:4.

Transfers

Most transfers to households are exogenous in real terms but are indexed to a "health" CPI. Unemployment benefits are the only business cycle sensitive component.

$$TRN\_EXO = (1+GRTRN) \ . \ TRN\_EXO_{t-4}/PCD2_{t-4} \ . \ PCD2$$
$$TRN\_UNN = TRNUNR \ . \ PCD2 \ . \ UNN$$

Moreover they are adjusted to ensure that once the debt to GDP ratio has reached its steady-state value it remains around that level thereby preventing it from following an unstable path:

$$TRN = (TRN\_EXO+TRN\_UNN) + \tau \ [\ (GDN/YEN)^{target} - (GDN/YEN) \ ]$$

Withholding tax on earned income

Because of the progressive tax schedule, the average tax rate is a function of the income level.

$$PTN2/(WIN+OPNI-PIN-OSN) = 0.25$$
$$+0.12 \log((1000/131.745)\ (WIN_{t-1}+OPNI_{t-1}-PIN_{t-1}-OSN_{t-1})/LNN_{t-1}))$$
$$-0.24 \log(FSCINDTB)$$
$$-0.18 \log(FSCINDTS)$$

$$TW3 = PTN2/(WIN+OPNI-PIN-OSN)$$

Companies tax

$$OTN = 3492 +0.08\ GR\_ISOC\ GON + 0.12\ (\ 1/(1+e^{-0,1*T92end}))\ GR\_ISOC\ GON$$

Capital income tax

$$PTN1/OPN1 = 0.09\ GR\_TBTI + 0.72\ PTN1_{t-1}/OPN1_{t-1}$$

Social security contributions

Implicit tax rates on employers', employees' and self-employeds' incomes are related to official rates:

$$TW1 = 0.04+0.66\ GR\_SOCO+0.09d911$$
$$OSN = TW1\ .\ WRH/(1+TW1)\ .\ HOURS\ .\ (LPN-INDEP)$$

$$TW2 = 0+0.72\ GR\_TRAHH + 0.36\ TW2_{t-1} +0.02d911$$
$$PIN1 = TW2\ .\ WRH/(1+TW1)\ .\ HOURS\ .\ (LPN-INDEP)$$

$$PIN2 = -219.76+1.26\ .\ GR\_TR2R2\ .\ 0.25\ .\ (\textstyle\sum_{l=9\ to\ 12} OPNI_{t-l}\ PCD2/PCD2_{t-12}\ .\ (1+DF89\ .$$
$$GR\_TR2R2)$$

Indirect taxes

TIN = -22136 + 1.08

    (PCN . GR_EXCISE/(1+GR_EXCISE)/(1+GR_VATCONS)

    +PCN . (GR_VATCONS)/(1+GR_VATCONS)

    +IHN . GR_VATLOG/(1+GR_VATLOG))

Interest payments

       The implicit interest rate on public debt partially follows market interest rates for each segment:

$RATEGDN = 0.77\ RATEGDN_{t-1} + (1-0.77)\ (GDNIST\ STI_{t-1} + GDNILT\ LTI_{t-1}$

    $+(1-GDNIST-GDNILT)\ LTI\_US_{t-1})/400$

$INN = RATEGDN\ .\ GDN_{t-1}$

<u>**EXPECTED INFLATION and INTEREST RATES**</u>

$INFQE = 0.39\ (0.6\ \Delta pcd_{t-1} + 0.2\ \Delta pcd_{t-2} + 0.2\ \Delta pcd_{t-3}) + 0.34\ (\Delta_4\ pcd_{t+4})/4 + 0.27\ PIEND_{t-1}$

$R^2 = 0.73$; dw = 1.99; J-stat = 0.17; 1982:2-1997:4.

For backward-looking simulations an additional equation is needed to substitute for Δ_4 pcd_{t+4}. This latter is obtained by regressing Δ_4 pcd_{t+4} on its instruments which results in :

$\Delta_4 pcd_{t+4} = 0.71 \bullet (\ 0.39\ \Delta_4\ (wrh_t - (yvar_t - lnhh_t)) + 0.15\ \Delta_4\ (wrh_{t-1} - (yvar_{t-1} - lnhh_{t-1}))$

    $+ 0.11\ \Delta_4\ (wrh_{t-2} - (yvar_{t-2} - lnhh_{t-2})) + 0.35\ \Delta_4\ (wrh_{t-3} - (yvar_{t-3} - lnhh_{t-3})))$

    $+ (1-0.71) \bullet (\ 0.78\ \Delta_4\ mtd_t + 0.22\ \Delta_4\ mtd_{t-2}\ ) + 0.0067$

$STI = STI\_real + 400*INFQE$

$LTI = (1-\psi)\sum_{l=0\ to\ \infty} \psi^l\ E_t(STI_{t+l})$

$RMT = ctse + LTI$

$RCC = cste + LTI$

IDENTITIES

YEN = PCN+GCN+ITN+SCN+XTN-MTN+ZEN

PCN = PCR . PCD

GCR = GCR1+GCR2+GCR3+GCR4

GCN = GCN1+GCN234

GCN1 = GCR1 . GCD1

GCN234 = (GCR2+GCR3+GCR4) . GCD234

GCN2 = GCR2 . GCD234

IHN = IHR . IHD

ION = IOR . IOD

GIN = GIR . GID

XTN = XTR . XTD

MTN = MTR . MTD

YER = PCR+GCR1+GCR2+GCR3+GCR4+IHR+IOR+GIR+SCR+XTR-MTR+ZER

GCD = GCN/(GCR1+GCR2+GCR3+GCR4)

YED = YEN/YER

YVAN = YVAR . YVAD

LNN = LPN+LGN

INDEP = RATI_INDEP . LPN

$\log(LFN) = \log(LFN(-1)) + n$

URX/100 = (LFN-LNN)/LFN

WIN = WRH . HOURS . (LPN-INDEP)+GCN234

$\log(OPNI) = \log(WRH/(1+TW1) . HOURS . INDEP)$

OPN1 = OPN1_RATI . YEN

RETKRH = (RATI_RETKRH . YEN) / $HHOS_{t-1}$

OPN = OPNI+OPN1

GON = YVAD . YVAR-WRH . HOURS . (LPN-INDEP)

YL = WIN+TRN+OPNI-(TDN-PTN1)-OSN

CAN = XTN-MTN+NFN+TWN

NFN = RATE_NFN . NFA_{t-1}

RATE_NFN = $RATE\_NFN_{t-1}$ -0.05 . ($RATE\_NFN_{t-1}$-(ζ+n+PIEND -
   $(XTN_{t-1}-MTN_{t-1}+TWN_{t-1})/NFA_{t-2}$))

NFA = NFA(-1)+CAN

TWN = RATE_TWN . YEN

HW = $YL/PCD+(1-0.000633835)*HW_{t+1}/(1+(0.2+0.01*LTI\_real)/4))$

CC0 = IOD . LTI_real

KRE = (1-DEPKRE) . KRE_{t-1}+IOR

$\Delta log(DUC)= \Delta log(YVAR/KRE_{t-1})$

GYN = PTN+PIN+OTN+OSN+TIN+OGN-TRN+TCN-INN

GLN = GYN-GCN+CGN-GIN

GDN = GDN_{t-1}-GLN

PTN = PTN1+PTN2

PIN = PIN1+PIN2

TDN = PTN+PIN

ODN = OTN+OSN

APPENDIX B: LIST OF SYMBOLS

<u>ENDOGENOUS</u>

CAN	:	current account balance
CC0	:	cost of capital
CCRES	:	households' consumer credits
DUC	:	degree of capacity utilisation
FW	:	market value of households' financial wealth (housing included)
GCD	:	public consumption deflator
GCD1	:	public purchases of goods and services deflator
GCD234	:	public wages and pensions deflator
GCN	:	public consumption, value
GCN1	:	public purchases of goods and services, value
GCN234	:	public wages and pensions, value
GDN	:	public debt
GID	:	public investment deflator
GIN	:	public investment, value
GLN	:	public deficit
GLPRIM	:	primary public deficit
GON	:	gross operating surplus of companies
GYN	:	government disposable income
HHOQ	:	real housing stock
HHOS	:	market value of the housing stock
HOURS	:	effective hours worked per employee
HW	:	households' real human wealth
IHD	:	housing investment deflator
IHN	:	housing investment, value
IHR	:	housing investment, volume
IHXN	:	price of new houses
IHXO	:	price of houses on secondary market
INDEP	:	independent workers
INFQE	:	expected inflation
INN	:	interest payments on public debt
IOD	:	companies investment deflator
ION	:	companies investment, value
IOR	:	companies investment, volume

ITN	:	gross fixed capital formation, value
KRE	:	companies real net capital stock
LFN	:	labour force
LMT	:	households' mortgage debt
LNHH	:	LPN * hours
LNN	:	total employment
LPN	:	private employment
LTI	:	long-term interest rate
M0	:	money stock
MTD	:	imports deflator
MTN	:	imports, value
MTR	:	imports, volume
NFA	:	net foreign assets position
NFN	:	net factor income
ODN	:	OTN+OSN
OPN	:	OPNI+OPN1
OPN1	:	capital income, households
OPNI	:	labour income, independent workers
OSN	:	employers' social security contributions
OTN	:	companies direct taxes
PCD	:	private consumption deflator
PCD2	:	health index
PCN	:	private consumption, value
PCR	:	private consumption, volume
PIN	:	PIN1+PIN2
PIN1	:	employees' social security contributions
PIN2	:	independents' social security contributions
POPULA	:	population
PSN	:	personal sector saving
PTN	:	PTN1+PTN2
PTN1	:	capital income tax
PTN2	:	withholding tax on earned income
PYN	:	personal disposable income
Q	:	index of market value of companies' capital stock
RAGG	:	rate of return on households' financial wealth
RATEGDN	:	implicit interest rate on public debt

RCC	:	rate on investment credit
RETKRH	:	rate of return on housing
STI	:	short-term interest rate
TDN	:	PTN+PIN
TIN	:	indirect taxes
TRN	:	transfers to households
TRN_EXO	:	transfers to households, exogenous component
TRN_UNN	:	transfers to households, unemployment benefits
TW1	:	implicit rate of employers' social contributions
TW2	:	implicit rate of employees' social contributions
TW3	:	implicit rate of tax on earned income
UNN	:	unemployment, number
URX	:	unemployment, rate
W	:	households' real human wealth
WIN	:	compensation of employees
WRH	:	hourly wage cost
XTD	:	exports deflator
XTN	:	exports, value
XTR	:	exports, volume
YED	:	GDP deflator
YEN	:	GDP, value
YER	:	GDP, volume
YL	:	households' disposable labour income
YVAD	:	private GDP deflator
YVAN	:	private GDP, value
YVAR	:	private GDP , volume

EXOGENOUS

AWMPCD	:	foreign competitors price index (eurozone) expressed in BEF
CGN	:	government capital transfers
CMBEF	:	prices in BEF of competitors for Belgian imports
CXBEF	:	competitors' export prices in BEF
DEPKRE	:	rate of depreciation of the capital stock
DF89	:	dummy variable which equals 1 from 1989Q1 and 0 before
DURCON	:	conventional working time
ETPL	:	share of full-time working

FSCINDTB	:	fiscal index, tax brackets (endogenous when indexation of brackets is effective)
FSCINDTS	:	fiscal index tax schedule (reflects fiscal reforms)
GCR1	:	government purchases of goods and services, volume
GCR2+GCR2+GCR3:		public wages and pensions, volume
GDNILT	:	allocation of public debt, long-term BEF
GDNIST	:	allocation of public debt, short-term BEF
GIR	:	public investment, volume
GRTRN	:	exogenous rate of real growth of transfers to households
GR_EXCISE	:	rate of excise duties
GR_ISOC	:	direct tax rate, companies
GR_SOCO	:	rate of employers' social security contribution
GR_TBTI	:	rate of withholding tax on capital income
GR_TR2R2	:	rate of independents' social security contribution
GR_TRAHH	:	rate of traditional employees' social security contribution
GR_VATCONS	:	VAT rate on consumption (average)
GR_VATLOG	:	VAT rate on housing investment
LFN.GR	:	rate of growth of the labour force (also called 'n' in the text)
LGN	:	public employment
LTI_US	:	Euro-dollar long term interest rate
MPN	:	mathematical pension reserves
OGN	:	other government net finance
PEI	:	import price of energy
PIEND	:	perceived inflation target
POP.GROWTH	:	rate of growth of population
RATI_INDEP	:	share of independent workers in private employment
RATI_YVAR	:	correction to go from GDP expenditure to private GDP value-added
REGIS	:	one minus the rate of housing investment that corresponds to registration rights
SCN	:	changes in inventories, value
SCR	:	changes in inventories, volume
STI_real	:	real short term interest rate
T	:	time trend
TCN	:	transfers international co-operation
TRNUNR	:	unemployment benefits in real terms

TWN	:	transfers from ROW
WDR	:	indicator of world demand
ZEN	:	statistical adjustment GDP expenditure, nominal
ZER	:	statistical adjustment GDP expenditure, real
ZVR	:	statistical adjustment GDP value-added, real

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