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GROWTH IN AN OPEN ECONOMY: SOME RECENT DEVELOPMENTS

Stephen J. Turnovsky(*)

The views expressed in this paper are those of the author and do not necessarily reflect the views of the National Bank of Belgium.

(*) University of Washington, Seattle WA 98195.

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Editorial

On May 11-12, 2000 the National Bank of Belgium hosted a Conference on *"How to promote economic growth in the euro area?"*. A number of papers presented at the conference is made available to a broader audience in the Working Papers series of the Bank. This volume contains the first of these papers. The other five papers were issued as Working Papers 6-10.

Abstract

This paper discusses some of the recent developments in growth theory, doing so from the perspective of a small open economy. After setting out a basic generic model, we show how it may yield two of the key models that have played a prominent role in the recent literature, the endogenous growth model and the non-scale growth model. We focus initially on the former, emphasizing how the simplest such model leads to an equilibrium in which the economy is always on its balanced growth path. One aspect of the model is the importance of fiscal policy as a determinant of the equilibrium growth rate, an aspect that is discussed in detail. We also show how the endogeneity or otherwise of the labor supply is crucial in determining the equilibrium growth rate and its responsiveness to macroeconomic policy.

But transitional dynamics are an important aspect of the growth process and indeed much research has been directed to determining the speed with which the economy converges to its balanced growth path. We discuss alternative ways that such transitional dynamics may be introduced. These include (i) restricted access to the world capital market; (ii) the introduction of government capital , and (iii) the two-sector production model, pioneered by Lucas. In the original analysis, the two capital goods relate to physical and human capital and in the international context these naturally can be identified with traded and nontraded capital, respectively.

Criticism of the endogenous growth model has led to the development of the non-scale growth model. This too is characterized by transitional dynamics, which are more flexible than those of the corresponding endogenous growth model. This model is much closer to the neoclassical model; in particular, the long-run growth rate is independent of macroeconomic policy. However, since such models are typically associated with slow

convergence speeds, policy can influence the accumulation of capital for extended periods of time, leading to significant long-run level effects.

The discussion seeks to emphasize the adaptability of the models to a wide range of issues. A final extension addresses the impact of volatililty on growth. This has been extensively analyzed empirically and a stochastic extension of the endogenous growth model provides a convenient framework within which to interpret this research.

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1. INTRODUCTION

Economic growth is arguably the issue of primary concern to economic policy makers in both developed and developing economies. Economic growth statistics are among the most widely publicized measures of economic performance and are always discussed with interest. As a consequence growth theory has long occupied a central role in economics.

1.1 Some Background

Long-run growth was first introduced by Solow (1956) and Swan (1956) into the traditional neoclassical macroeconomic model by introducing a growing population coupled with a more efficient labor force. The direct consequence of this approach is that the long-run growth rate in these models is ultimately tied to demographic factors, such as the growth rate of population, the structure of the labor force, and its productivity growth (technological change), all of which were typically taken to be exogenously determined. Hence, the only policies that could contribute to long-run growth were those that would increase the growth of population, and manpower training programs aimed at increasing the efficiency of the labor force. Conventional macroeconomic policy had no influence on long-run growth performance.

Since then, growth theory has evolved into a voluminous literature, in two distinct generations of models. The Solow-Swan model was the inspiration for a first generation of growth models during the 1960s, which, being associated with exogenous sources of long-run growth, are now sometimes referred to as *exogenous growth models*. Research interest in these models tapered off around 1970, as economists turned their attention to other issues, perceived as being of more immediate significance, such as inflation, unemployment, and oil shocks, and the design of macroeconomic policies to deal with them. Beginning with the seminal work of Romer (1986), there has been a resurgence of interest in economic growth theory giving rise to a second generation of growth models. This revival of activity has been motivated by several issues, which include: (i) an attempt to explain aspects of the data, not discussed by the neoclassical model; (ii) a more satisfactory explanation of international differences in economic growth rates; (iii) a more central role for the accumulation of knowledge; and (iv) a larger role for the instruments of macroeconomic policy in explaining the growth process; see Romer (1994). These new models seek to explain the long-run growth rate as an endogenous equilibrium outcome of the behavior of rational optimizing agents, reflecting the structural characteristics of the economy, such as technology and preferences, as well as macroeconomic policy. For this reason they have become known as *endogenous growth models*.

The new growth theory is far-ranging. It has been analyzed in both closed economies, as well as in open economies. In fact, one of the characteristics of the new growth theory is that it has more of an international orientation; see e.g. Grossman and Helpman (1991). This may reflect the increased importance of the international aspects in macroeconomics in general. In comparison with the first generation of growth models, the newer literature places a greater emphasis on empirical issues and the reconciliation of the theory with the empirical evidence. In this respect a widely debated issue concerns the so-called convergence hypothesis. The question here is whether or not countries have a tendency to converge to a common per capita level of income.

But new growth theory is also associated with important theoretical advances as well, and one can identify two main strands of theoretical literature, emphasizing different sources of economic growth. One class of models, closest to the neoclassical growth model, stresses the *accumulation of private capital* as the fundamental source of economic growth. This differs in a fundamental way from the neoclassical growth model in that it does not require exogenous elements, such as a growing population to generate an equilibrium of ongoing growth. Rather, the equilibrium growth is internally generated, though in order to achieve that, certain restrictions relating to homogeneity must be imposed on the economic framework. Some of these restrictions are of a knife-edge character and have been the source of criticism; see e.g. Solow (1994).

In the simplest such model, in which the only factor of production is capital, the constant returns to scale condition implies that the production function must be linear in physical capital, being of the functional form $Y = AK$. For obvious reasons, this technology has become known as the "AK model". As a matter of historical record, explanation of growth as an endogenous process in a one-sector model is not new. In fact it dates back to Harrod (1939). The equilibrium growth rate characterizing the AK model is essentially of the Harrod type, the only difference being that consumption (or savings) behavior is derived as part of an intertemporal optimization, rather than being posited directly. These one-sector models assume (often only implicitly) a broad interpretation for capital, taking it to include both human, as well as nonhuman, capital; see Rebelo (1991). A direct extension to this basic model are two-sector investment based growth models, originally due to Lucas (1988), that disaggregate private capital into human and nonhuman capital; see also Mulligan and Sala-i-Martin (1993) and Bond, Wang, and Yip (1996).

A second class of models, emphasizes the endogenous development of knowledge, or research and development, as the engine of growth. The basic contribution here is that of Romer (1990), which develops a two-sector model of a closed economy, where new knowledge produced in one sector is used as an input in the production of final output. The knowledge sector has been extended in various directions by a number of authors; see e.g. Aghion and Howitt (1992) and more recently, Eicher (1996). A related class of models deals with innovation and the diffusion of knowledge across countries, and a comprehensive discussion is provided by Barro and Sala-i-Martin (1995, Chapter 8).

One is beginning to see a merger evolving between the old and the new growth theory. The new growth models are often characterized by having scale effects, meaning that variations in the size or scale of the economy, as measured by say population, affect the size of the long-run growth rate. For example, the Romer (1990) model of research and development implies that a doubling of the population devoted to research will double the growth rate. Whether the AK model is associated with scale effects depends upon whether there are production externalities that are linked to the size of the economy; see Barro and Sala-i-Martin (1995). By contrast, the neoclassical Solow model has the property that the equilibrium growth rate is independent of the scale (size) of the economy; it is therefore not subject to such scale effects.

Empirical evidence does not support the existence of scale effects. For example, Jones (1995a) finds that variations in the level of research employment have exerted no influence on the long-run growth rates of the OECD economies. Backus, Kehoe, and Kehoe (1992) find no conclusive empirical evidence of any relationship between US GDP growth and measures of scale. These empirical observations are beginning to stimulate interest in the development of non-scale models. Such models are hybrids in the sense that they share some of the characteristics of the neoclassical model, yet their equilibrium is derived from intertemporal optimization as in the new growth models[1]. Jones (1995b) has proposed a specific model, in which the steady-state growth rate is determined by the growth rate of population, in conjunction with certain production elasticities, in his case pertaining to the knowledge producing sector.

[1] Jones (1995a) referred to such models as “semi-endogenous” growth models.

1.2 Scope of this Paper

It is beyond the scope of this paper to present an exhaustive discussion of growth theory. That is a specialized topic in itself and for that the reader should refer to Grossman and Helpman (1991), Barro and Sala-i-Martin (1995), and Aghion and Howitt (1998) who provide comprehensive treatments of the subject from different perspectives. Rather, the purpose of this paper is to exposit the investment-based growth models, but from an international perspective. In following this approach, we try to cover the main developments, but from a viewpoint that hopefully is of greater relevance for a small open economy such as Belgium.

We begin our discussion in Section 2 by expositing a canonical model of a small open economy, which is sufficiently general to encompass alternative models. Sections 3 and 4 then develop two alternative versions of the AK growth model. Such models have been extensively used to analyze the effects of fiscal policy on growth performance; see e.g. Barro (1990), King and Rebelo (1990), Jones and Manuelli (1990), Rebelo (1991), Jones, Manuelli, and Rossi (1993), Ireland (1994) and Turnovsky (1996a). Most of these endogenous growth models have been developed for a closed economy, although some applications to an open economy are beginning to develop; see Razin and Yuen (1992, 1996), Rebelo (1992), van der Ploeg (1996), Mino (1996), Turnovsky (1996b, 1996d, 1997c), van der Ploeg (1996), Baldwin and Forslid (1999, 2000).

Section 3 begins with the simplest Romer (1986) model with fixed labor supply and then modifies this model to the case where labor is supplied elastically. It emphasizes how going from one assumption to the other fundamentally changes the determination of the equilibrium growth rate and the impact of fiscal policy. Section 4 discusses the Barro (1990) model where government expenditure is productive and analyzes optimal fiscal policy in that setting.

These initial models all abstract from transitional dynamics, so that the economy is always on its balanced growth path. This is an obvious limitation and Section 5 discusses in some detail two ways that transitional dynamics may be introduced. The first is through restricted access to world financial markets, in the form of increasing borrowing costs, likely to be relevant for a small developing economy. The second is through the introduction of government capital, so that in contrast to the Barro model, government expenditure impacts production as a stock, rather than as a flow. But transitional dynamics can also be introduced in other ways, and these are discussed in the next two sections.

Section 6, discusses the case where the production technology is augmented to two sectors, a traded and a nontraded sector, showing the nature of the dynamics this introduces. The two sector model, where the two sectors consist of physical (nonhuman) and human capital, respectively, was one of the original models of endogenous growth pioneered by Lucas (1988). Other authors to analyze the two-sector model include: Mulligan and Sala-i-Martin (1993), Devereux and Love (1994), and Bond, Wang and Yip (1996). This aspect is particularly relevant for international economies.

As we have already noted, the endogenous growth model has been subject to criticism along two lines. First, it is often associated with "scale effects" meaning that long-run growth rates are linked to the size of the economy, a characteristic that is not supported by the empirical evidence. Second, it holds only if strict "knife-edge" conditions on the technology hold. In response to this, we have seen the development of non-scale growth models, which have the property that long-run growth rates are independent of the scale of the economy. This model is also associated with transitional dynamics and is discussed in Section 7.

An important issue that has received prominent attention in the recent literature on growth theory concerns the notion of convergence. This issue has two aspects: (i) whether or not economies converge to a common growth rate, and (ii) the rate at which the economy converges along its transitional growth path. This is discussed in Section 8. The speed of convergence is important, because even if policy is unable to affect long-run growth rates, it can nevertheless have significant long-run level effects if the rates of convergence are sufficiently slow, as much empirical evidence suggests. The final issue addressed is the relationship between volatility and growth, discussed in Section 9. This issue has been the subject of empirical investigation, and the growth model developed in this paper provides a natural analytical framework for assessing this work.

Throughout the paper, our main objective is to exposit the structures of the various models in their basic form rather than analyzing any in detail. The models provide powerful analytical tools that can be adapted to various needs and circumstances. One key issue that distinguishes the endogenous growth model from the non-scale model is the impact of policy on the long-run equilibrium growth rate. Before embarking further, we should acknowledge that the empirical evidence pertaining to this issue is mixed. If one takes the evidence on non-scale growth models seriously, and accepts that the long-run growth rate is determined as suggested by Jones (1995b), the scope for fiscal policy is limited, although less so than in the Solow model. Indeed, empirical evidence by Easterly and Rebelo (1993) and Stokey and Rebelo (1995) suggests that the effects of tax rates on long-run growth rates are insignificant, or weak at best. Stokey and Rebelo argue that their findings provide evidence against those models, such as AK models, that predict large growth effects from taxation. In order for the predictions of these models to be consistent with their evidence, these growth effects would have to be largely offset by changes in other determinants of the long-run growth rate. But other studies such, as Grier and Tullock (1989), Barro (1991), and Barro and Lee (1994) obtain negative relationships between growth and government consumption expenditure, while Barro and Lee also find that government expenditure on education has a positive effect on growth. Taken together, we do not view the empirical evidence as necessarily contradicting the ability of fiscal policy to influence the growth rate. It may well be the case that a higher income tax has a significant negative effect on the growth rate, but that this is roughly offset by a significant positive growth effect of the productive government expenditure it may be financing, thus having a small overall net effect. Indeed, the welfare-maximizing rate of taxation in the simple Barro (1990) model of productive government expenditure coincides with the growth-maximizing tax rate, so that if the tax rate is in fact close to being optimal there should be little effect on the growth rate, precisely as the empirical evidence seems to suggest. But to understand this relationship, it is important to develop a model in which the various components of fiscal policy are introduced explicitly, and their separate and possibly conflicting effects on the growth rate analyzed. It is in this vein that we view the AK model as providing an instructive framework for analyzing fiscal policy on growth.

2. A CANONICAL MODEL OF A SMALL OPEN ECONOMY

We begin by describing the generic structure of a small open economy which consumes and produces a single traded commodity. There are N identical individuals, each of whom has an infinite planning horizon and possesses perfect foresight. Each agent is endowed with a unit of time that can be allocated either to leisure, l, or to labor, (1 - l). Labor is fully employed so that total labor supply, equal to population, N, grows exponentially at the steady rate $\dot{N} = nN$. Individual domestic output, Y_i, of the traded commodity is determined by the individual's private capital stock, K_i, his labor supply, (1 - l), and the aggregate capital stock $K = NK_i$ [2]. In order to accommodate growth under more general assumptions with respect to returns to scale, we assume that the output of the individual producer is determined by the Cobb-Douglas production function[3]:

$$Y_i = \alpha(1-l)^{1-\sigma} K_i^{\sigma} K^{\eta} \qquad 0 < \sigma < 1, \ \eta \gtrless 0 \tag{1a}$$

This formulation is akin to the earliest endogenous growth model of Romer (1986). The spillover received by an individual from the aggregate stock of capital can be motivated in various ways. One is to interpret K as knowledge capital, as Romer suggested. Another, is to assume N specific inputs (subscripted by i) with aggregate K representing an intra-industry spillover of knowledge[4].

Each private factor of production has positive but diminishing marginal physical productivity. To assure the existence of a competitive equilibrium the production function exhibits constant returns to scale in the two private factors [Romer (1986)]. In contrast to the standard neoclassical growth model, we do not insist that the production function exhibits constant returns to scale, and indeed total returns to scale are $1+\eta$, and are increasing or decreasing, according to whether the spillover from aggregate capital is positive or negative.

As we will show in subsequent sections, the production function is sufficiently general to encompass a variety of models. For example, we will show that the model is

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2 Since all agents are identical, all aggregate quantities are simply multiples of the individual quantities, $X = NX_i$. Note that since all agents allocate the same share of time to work, there is no need to introduce the agent's subscript to l.

3 When production functions exhibit non-constant returns to scale in all factors, the existence of a balanced growth equilibrium requires the production function to be Cobb-Douglas, as assumed in (1a); see Eicher and Turnovsky (1999a).

4 negative exponent can be interpreted as reflecting congestion, along the lines of Barro and Sala-i-Martin (1992a).

consistent with long-run stable growth, provided returns to scale are appropriately constrained. This contrasts with models of endogenous growth and externalities in which exogenous population growth can be shown to lead to explosive growth rates; see Romer (1990). We should also point out that the standard AK model emerges when $\sigma + \eta = 1, n = 0$, and the neoclassical model corresponds to $\eta = 0$.

Aggregate consumption in the economy is denoted by C, so that the per capita consumption of the individual agent at time t is $C / N = C_i$, yielding the agent utility over an infinite time horizon represented by the intertemporal isoelastic utility function:

$$\Omega \equiv \int_0^{\infty} (1/\gamma)\left(C_i l^{\theta}\right)^{\gamma} e^{-\rho t} dt; \quad -\infty < \gamma < 1; \theta > 0, 1 > \gamma(1+\theta), 1 > \gamma\theta \tag{1b}$$

where $l/(1-\gamma)$ equals the intertemporal elasticity of substitution, and θ measures the substitutability between consumption and leisure in utility[5]. The remaining constraints on the coefficients in (1b) are required to ensure that the utility function is concave in the quantities C and l.

Agents accumulate physical capital, with expenditure on a given change in the capital stock, I_i, involving adjustment (installation) costs that we incorporate in the quadratic (convex) function

$$\Phi(I_i, K_i) \equiv I_i + hI_i^2 / 2K_i = I_i(1 + hI_i / 2K_i) \tag{1c}$$

This equation is an application of the familiar Hayashi (1982) cost of adjustment framework, where we assume that the adjustment costs are proportional to the *rate* of investment per unit of installed capital (rather than its level). The linear homogeneity of this function is necessary if a steady-state equilibrium having ongoing growth is to be sustained[6].

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5 This form of utility function is consistent with the existence of a balanced growth path; see Ladron et al (1997). The specification in (1b) is the case of pure leisure; they also consider the case where utility derived from leisure depends upon its interaction with human capital.

6 Many applications of the cost of adjustment in the Ramsey model assume that adjustment costs depend upon the absolute rate of investment, rather than its rate relative to the size of the capital stock. They also often assume only that it is convex; the assumption of a quadratic function is made for convenience, simplifying the solution for the equilibrium growth rates in the endogenous growth model

Convex adjustment costs are a standard feature of models of capital accumulation in small open economies with tradable capital facing a perfect world capital market, being necessary for such models to give rise to nondegenerate dynamics; see Turnovsky (1997a). They are, however, less common in endogenous growth models of closed economies, which typically treat the accumulation of capital as being determined residually; see e.g. Rebelo (1991), Barro (1990)[7].

Adjustment costs turn out to have at least two important roles in this model, particularly in the basic AK version of the model to be discussed in Section 3. First, they may preclude the existence of a steady-state equilibrium growth path. Second they introduce an important flexibility into the equilibrating process. In equilibrium, the after-tax rates of return on the two assets available to the economy, traded bonds and capital, must be equal. Given the linear technology, the marginal physical product of capital is also constant, so that the equality between these two after-tax rates of return in general constrains the feasible choice of tax rates. By contrast, the presence of adjustment costs introduces a variable shadow value of capital (the Tobin q), which equilibrates the rates of return on these two assets, for any arbitrarily specified tax rates.

For simplicity we assume that the capital stock does not depreciate, so that the net rate of capital accumulation is given by:

$$\dot{K}_i = I_i - nK_i \tag{1d}$$

In addition, agents have unrestricted access to a world capital market, being able to accumulate foreign bonds, B_i, which pay an exogenously determined fixed rate of return, r. We shall assume that income from current production is taxed at the rate τ_y, income from bonds is taxed at the rate τ_b, while in addition, consumption is taxed at the rate τ_c. We shall illustrate the contrasting implications of different models by analyzing the purely distortionary aspects of taxation and assume that revenues from all taxes are rebated to the agent as lump sum transfers T_i. Thus the individual agent's instantaneous budget constraint is described by:

$$\dot{B}_i = (1-\tau_y)Y_i + [r(1-\tau_b)-n]B_i - (1+\tau_c)C_i - I_i[1+(h/2)(I_i/K_i)] + T_i \tag{1e}$$

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7 There are some exceptions; see Turnovsky (1996c) in a closed economy. Baldwin and Forslid (1999, 2000) emphasize the q-theoretic approach in an open economy.

The agent's decisions are to choose his rates of consumption, C_i, leisure, l, investment, I_i, and asset accumulation, B_i, K_i, to maximize the intertemporal utility function, (1a), subject to the accumulation equations, (1d) and (1e). The discounted Hamiltonian for this optimization is:

$$H \equiv e^{-\rho t}\frac{1}{\gamma}\left(C_i l^{\theta}\right)^{\gamma} + \lambda e^{-\rho t}\left[(1-\tau_y)Y_i - \Phi_i - (1+\tau_c)C_i + r(1-\tau_b)B - T_i - \dot{B}_i\right]$$
$$+ q' e^{-\rho t}\left[I - nK_i - \dot{K}_i\right]$$

where λ is the shadow value of wealth in the form of internationally traded bonds and q' is the shadow value of the agent's capital stock. Exposition of the model is simplified by using the shadow value of wealth as numeraire. Consequently, $q \equiv q'/\lambda$ can be interpreted as being the market price of capital in terms of the (unitary) price of foreign bonds.

The optimality conditions with respect to C_i, l, and I_i are respectively

$$C_i^{\gamma-1} l^{\theta\gamma} = \lambda(1+\tau_c) \tag{2a}$$

$$\theta C_i^{\gamma} l^{\theta\gamma-1} = \frac{\lambda(1-\tau_y)(1-\sigma)Y_i}{(1-l)} \tag{2b}$$

$$1 + h\left(I_i / K_i\right) = q \tag{2c}$$

Equation (2a) equates the marginal utility of consumption to the tax-adjusted shadow value of wealth, while (2b) equates the marginal utility of leisure to its opportunity cost, the after-tax marginal physical product of labor (real wage), valued at the shadow value of wealth. The third equation equates the marginal cost of an additional unit of investment, which is inclusive of the marginal installation cost hI_i / K_i, to the market value of capital. Equation (2c) may be solved to yield the following expression for the rate of capital accumulation:

$$\dot{K}_i / K_i = I_i / K_i - n = (q-1)/h - n \equiv \phi_i \tag{3}$$

With all agents being identical, equation (3) implies that the growth rate of the aggregate capital stock, is $\phi = \phi_i + n$, so that

$$\dot{K}/K = \dot{K}_i/K_i + n = (q-1)/h \equiv \phi \qquad (3')$$

This describes a "Tobin q" theory of investment, with $\dot{K} \gtrless 0$ according to whether $q \gtrless 1$.

Optimizing with respect to B_i and K_i implies the arbitrage relationships

$$\rho - \left(\dot{\lambda}/\lambda\right) = r(1-\tau_b) - n \qquad (4a)$$

$$\left((1-\tau_y)\sigma Y_i / qK_i\right) + \left(\dot{q}/q\right) + (q-1)^2 / 2hq = r(1-\tau_b) \qquad (4b)$$

Equation (4a) is the standard Keynes-Ramsey consumption rule, equating the marginal return on consumption to the growth-adjusted after-tax rate of return on holding a foreign bond. Likewise (4b) equates the after-tax rate of return on domestic capital to the after-tax rate of return on the traded bond. The former comprises three components. The first is the marginal after-tax output per unit of installed capital, (valued at the price q), while the second is the rate of capital gain. The third element reflects the fact that an additional benefit of a higher capital stock is to reduce the installation costs (which depend upon I_i / K_i) associated with new investment. Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must be imposed[8]:

$$\lim_{t\to\infty} \lambda B_i e^{-\rho t} = 0; \quad \lim_{t\to\infty} \lambda q K_i e^{-\rho t} = 0 \qquad (4c)$$

The government in this canonical economy plays a limited role. It levies income taxes on output and foreign interest income, it taxes consumption, and then rebates all tax revenues. In aggregate, these decisions are subject to the balanced budget condition

$$\tau_y Y + \tau_b rB + \tau_c C = T \qquad (5)$$

[8] The transversality condition on debt is equivalent to the national intertemporal budget constraint.

Aggregating (1e) over the N individuals, and imposing (5) and (1d) leads to:

$$\dot{B} = Y + rB - C - I[1 + (h/2)(I/K)] \quad (6)$$

which describes the country's current account.

The model can thus be summarized by the 5 optimality conditions (2a) – (2c), (4a), and (4b), together with the current account relationship, (6). Many models assume that labor is supplied inelastically, in which case that the optimality condition for labor, (2b), ceases to be operative.

3. THE ENDOGENOUS GROWTH MODEL

The investment-based endogenous growth model has been the subject of intensive research since 1986. Most such models assume that labor is supplied inelastically and, as we shall demonstrate, the endogeneity or otherwise of labor is an important determinant of the equilibrium growth rate.

The key feature of the endogenous growth model is that it is capable of generating ongoing growth in the absence of population growth; i.e. n = 0. For this to occur, the production function, (1a), must have constant returns to scale in the accumulating factors, individual and aggregate capital, that is,

$$\sigma + \eta = 1 \tag{7}$$

Substituting this into (1a), this implies individual and aggregate production functions of the form

$$Y_i = \alpha\left[(1-l)K\right]^{\eta} K_i^{1-\eta}; \quad Y = \alpha\left[(1-l)N\right]^{\eta} K \tag{8}$$

The individual production function is thus constant returns to scale in private capital, K_i, and in labor, measured in terms of "efficiency units" (1-l)K. Summing over agents, the aggregate production function is thus linear in the endogenously accumulating capital stock. Note that as long as $\eta \neq 0$ so that there is an aggregate externality, the average (and marginal) productivity of capital depends upon the size of the population. Increasing the population, holding other technological characteristics constant, increases the productivity of capital and the equilibrium growth rate. The economy is thus said to have a "scale effect"; see Jones (1995a). Such scale effects run counter to the empirical evidence and have been a source of criticism of the AK growth model; see Backus, Kehoe and Kehoe (1992). These scale effects can be eliminated from the AK model if either (i) there are no externalities ($\eta = 0$), or (ii) if the individual production function (1a) is modified to

$$Y_i = \alpha(1-l)^{1-\sigma} K_i^{\sigma} \left(K/N\right)^{\eta}$$

so that the externality depends upon the average, rather than the aggregate capital stock; see Mulligan and Sala-i-Martin (1993). Henceforth throughout this section, we shall normalize the size of the population at N = 1 and thereby eliminate the issue of scale effects.

3.1 Inelastic Labor Supply

We begin with the case where labor is supplied inelastically, i.e. $l = \bar{l}$. With population normalized, the individual and aggregate production functions are of the pure AK form:

$$Y_i = AK_i; \;\; Y = AK \tag{9}$$

where $A \equiv \alpha(1-\bar{l})^{1-\sigma}$ is a fixed constant. With the labor supply fixed, both the marginal and average productivity of capital are constant. The specification of the technology, consistent with ongoing growth, is a very strong knife-edge condition, one for which the endogenous growth model has been criticized; see Solow (1994)[9].

To determine the macroeconomic equilibrium, we first take the time differential of (2a), and then combine the resulting equation with (4a), which, with fixed employment (and normalized population), implies

$$\frac{\dot{C}}{C} = \frac{r(1-\tau_b)-\rho}{1-\gamma} \equiv \psi \tag{10}$$

An immediate consequence of (10) is that the equilibrium growth rate of domestic consumption is proportional to the difference between the after-tax return on foreign bonds and the (domestic) rate of time preference. From a policy perspective, it also implies that the consumption growth rate varies inversely with the tax on foreign interest income, but is independent of all other tax rates. Solving this equation implies that the level of consumption at time t is

$$C(t) = C(0)e^{\psi t} \tag{11}$$

9 Note that the technology (9) is identical to that of the original Harrod-Domar model, to which the AK model is a modern counterpart. It was Harrod, himself, who originally referred to the "knife-edge" characteristics of his model.

where the initial level of consumption C(0) is yet to be determined.

The critical determinant of the growth rate of capital is the relative price of installed capital, q, the path of which is determined by the arbitrage condition (4b). To analyze this further, we rewrite (4b) as the following nonlinear differential equation with constant coefficients:

$$\dot{q} = 2h[r(1-\tau_b)q - A(1-\tau_y)] - (q-1)^2 \tag{12}$$

In order for the capital stock domiciled in the economy ultimately to follow a path of steady growth (or decline), the stationary solution to this equation attained when $\dot{q} = 0$, must have (at least) one real solution. Setting $\dot{q} = 0$ in (12), implies that the steady-state value of q, $\tilde{q}$ say, must be a solution to the quadratic equation:

$$A(1-\tau_y) + \frac{(q-1)^2}{2h} = rq(1-\tau_b) \tag{13}$$

A necessary and sufficient condition for the capital stock ultimately to converge to a steady growth path is that this equation have real roots, and by examining this equation it can be seen that this involves a tradeoff between the size of the adjustment cost, h, and the productivity of capital, A. If, for a given h, A is too large, the returns to capital dominate the returns to bonds, irrespective of the price of capital, so that no long-run balanced equilibrium exists along which the returns on the two assets are equalized.

Suppose that (13) has real roots which we denote by q_1 (smaller) and q_2 (larger), respectively. It is easily shown that the equilibrium point that corresponds to the smaller equilibrium value, q_1, is unstable, while the equilibrium that corresponds to the larger value, q_2, although locally stable, violates the transversality condition (4c). The behavior of equilibrium q can thus be summarized by:

Proposition 1: The only solution for q which is consistent with the transversality condition is that q always be at the (unstable) steady-state solution q_1, given by the smaller root to (13). *Consequently there are no transitional dynamics in the market price of capital q.* In response to any shock, q immediately jumps to its new equilibrium value. Correspondingly, domestically domiciled capital is always on its steady growth path, growing at the rate $\phi = \left(q_1 - 1\right)/h$.

The domestic government is assumed to maintain a continuously balanced budget, in accordance with (5), while the domestic current account is given by (6). Substituting the expressions for I and K(t) from (3'), and C(t) from (11) into (6), and invoking the transversality conditions, we can determine (i) C(0) consistent with the nation being internationally solvent , and (ii) the resulting time path for traded bonds; see Turnovsky (1996b). This equation, together with (3') and (11), the solution for q, and the initial condition on C(0), comprise a closed form solution describing the evolution of the small open economy starting from given initial stocks of traded bonds B_0 and capital stock K_0. The following general characteristics of this equilibrium can be observed.

(i) Consumption and "tax-adjusted" wealth, on the one hand, and physical capital and output, on the other, are always on their steady-state growth paths, growing at the rates ψ and ϕ respectively[10]. The former is driven by the difference between the after-tax rate of return on foreign bonds and the domestic rate of time preference. The latter by q, which is determined by the technological conditions in the domestic economy, as represented by the (fixed) marginal physical product of capital A, and adjustment costs h, relative to the return on foreign assets. For the simple linear production function, the rate of growth of capital also determines the equilibrium growth of domestic output $\dot{Y}/Y$.

An important feature of this equilibrium is that it can sustain differential growth rates of consumption and domestic output. This is a consequence of the economy being small and open. It contrasts with the closed economy in which, constrained by the growth of its own resources, all real variables, including consumption and output, would ultimately have to grow at the same rate[11].

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[10] It turns out that consumption is proportional to the quantity $B + qK + (A\tau_y - qr\tau_b)/(r - \phi)$, which we refer to as "tax adjusted" wealth. In the absence of taxes, it reduces to the usual measure of wealth, $B + qK$.

[11] We assume that the country is sufficiently small so that it can maintain a growth rate that is unrelated to that in the rest of the world. Ultimately, this requirement imposes a constraint on the growth rate of the domestic economy. If it grows faster than does the rest of the world, at some point it will cease to be small. While we do not attempt to resolve this issue here, we should note that the question of convergence of international growth rates is an area of intensive research activity, which we shall briefly discuss in Section 8 below.

(ii) Holdings of traded bonds are subject to transitional dynamics, in the sense that their growth rate $\dot{B}/B$ varies through time. Asymptotically the growth rate converges to max[ψ,ϕ] and which it will be depends critically upon the size of the consumer rate of time preference relative to the rates of return on foreign investment opportunities. In the case that domestic agents are relatively patient, they choose to consume only a small fraction of their wealth. The economy runs a current account surplus, thus generating a positively growing stock of foreign assets. The income from these assets then enables the small economy to sustain a long-run consumption growth rate in excess of the growth rate of productive capacity. If agents are relatively impatient, the opposite applies. The economy accumulates an ever increasing foreign debt, the servicing of which prevents it from being able to maintain a consumption growth rate equal to that of domestic production.

(iii) With all taxes being fully rebated and labor supply fixed, the consumption tax is completely neutral. It has no effect on any aspect of the economic performance and acts like a pure lump-sum tax.

3.2 Taxes, Growth, and Welfare

With the neutrality of consumption taxes, we can focus on the two forms of capital taxation. Differentiating the solution for q, (see (8)), together with the definition of the growth rate of capital ϕ, and consumption ψ, leads to:

Proposition 2: An increase in the tax on bond income increases the growth rate of capital and reduces the growth rate of consumption; an increase in the tax on capital reduces the growth rate of capital, but leaves the growth rate of consumption unaffected.

Intuitively, an increase in the tax on bond income lowers the rate of return on bonds, thereby inducing investors to increase the proportion of capital in their portfolios, raising the price of capital and inducing growth in capital. In addition, this tax induces agents to switch from saving to consumption, increasing the ratio of consumption to tax adjusted wealth. This slows down the rate of growth of consumption. An increase in the tax on capital income generates the opposite portfolio response, lowering the growth rate of capital.

A key issue concerns the effects of tax changes, on the level of welfare of the representative agent, when consumption follows its optimal path. This is given by the expression:

$$\Omega = \int_0^{\infty} \frac{1}{\gamma}\left[C(0)e^{\psi t}\right]^{\gamma} e^{-\rho t} dt = \frac{\left[C(0)\right]^{\gamma}}{\gamma(\rho - \gamma\psi)} \qquad (14)$$

Thus, the overall intertemporal welfare effects of any policy change depend upon their effects on (i) the initial consumption level, and (ii) the growth rate of consumption. From (14) we may derive:

Proposition 3: Starting from zero taxes, an increase in either form of income tax leaves the overall level of welfare unchanged. However, the two taxes do have fundamentally different effects on the time profile of consumer welfare.

Consider first a change in the tax on capital τ_y. Starting from zero taxes, it leaves the initial tax-adjusted price of capital, tax-adjusted wealth, and therefore the initial consumption level and initial welfare, all unchanged. Moreover, since the capital income tax has no effect on the growth rate of consumption, the time path of utility and total overall discounted welfare are unchanged as well.

Starting from a zero tax situation, a tax on bond income also has no initial impact on tax-adjusted wealth. However, it raises the fraction of tax-adjusted wealth that is consumed, thereby increasing initial consumption and improving initial welfare. But since it also reduces the consumption growth rate, these short-run gains come at the expense of longer-run losses. Indeed, these two effects can be shown to be exactly offsetting, so that starting from zero taxes, the imposition of a tax on bond income (with full rebating) has no effects on overall intertemporal welfare. All it does is to redistribute the time path of consumer welfare.

3.3 Elastic Labor Supply

The endogenous growth model we have been discussing includes two interdependent criticial knife-edge restrictions: (i) inelastic labor supply, and (ii) fixed productivity of capital. The structure of the equilibrium changes fundamentally when the labor supply is endogenized. This introduces two key changes. The first is that the production function is modified to (8), so that the productivity of capital now depends positively upon the fraction of time devoted to labor. Second, the fixed endowment of a unit of time leads to the requirement that the steady-state allocation of time between labor and leisure must be constant.

This latter condition provides a link between the long-run rate of growth of consumption and the rate of growth of output. This can be seen by dividing the optimality condition (2a) by (2b). On the left hand side we see that with the allocation of time remaining finite, the marginal rate of substitution between consumption and leisure grows with consumption. On the right hand side we see that, given the fixed labor supply, the wage rate grows with output. For these condition to remain compatible over time, the equilibrium consumption-output ratio must therefore remain bounded, being given by:

$$\frac{C_i}{Y_i} = \frac{C}{Y} = \left(\frac{l}{1-l}\right)\frac{1-\sigma}{\theta} \tag{15}$$

To obtained the macroeconomic equilibrium we take the time derivatives of: (i) the optimality condition for consumption, (2a), (ii) the equilibrium consumption-output ratio, (15), and, (iii) the production function, (8). Combining the resulting equations with (8), (3'), and (4b), the macroeconomic equilibrium can be expressed by the pair of differential equations in q and l:

$$\dot{q} = r(1-\tau_b)q - \frac{(q-1)^2}{2h} - (1-\tau_y)\alpha(1-l)^{1-\sigma} \tag{16a}$$

$$\dot{l} = \frac{1}{F(l)}\left[r(1-\tau_b) - \rho - \frac{(1-\gamma)(q-1)}{h}\right] \tag{16b}$$

where $F(l) \equiv \frac{(1-\gamma)\sigma}{1-l} + \frac{1-\gamma(1+\theta)}{l} > 0$.

The steady state to (16) is obtained by setting $\dot{q} = \dot{l} = 0$ and is therefore characterized by the relative price of capital, q, and the fraction of time devoted to leisure, l, both being constant. Linearizing (16) around its steady state, we can easily show that the two eigenvalues to the linearized approximation are both positive; see Turnovsky (1999). Hence the only bounded equilibrium is one in which both q and l adjust instantaneously to ensure that the economy is always on its balanced growth path (denoted by ~) namely[12]:

$$\tilde{\psi} = \frac{\tilde{q}-1}{h} = \frac{r(1-\tau_b)-\rho}{1-\gamma} \tag{17a}$$

$$\frac{(1-\tau_y)\alpha(1-\tilde{l})^{1-\sigma}}{\tilde{q}} + \frac{(\tilde{q}-1)^2}{2h\tilde{q}} = r(1-\tau_b) \tag{17b}$$

where the transversality condition now implies:

$$\tilde{\psi} = \frac{\tilde{q}-1}{h} < r(1-\tau_b)$$

Equation (17a) implies that the equilibrium is one in which domestic output, capital, and consumption all grow at a common rate determined by the difference between the world rate of interest and the domestic rate of time preference, all multiplied by the intertemporal elasticity of substitution. The form of the expression is analogous to the equilibrium growth rate in the simplest AK model; see Barro (1990). The only difference is that for the small open economy the (fixed) marginal physical product of capital is replaced by the (given) foreign interest rate. Given this growth rate, (17a) determines the equilibrium price of capital, $\tilde{q}$, which will ensure that domestic capital grows at this equilibrium rate. Having obtained $\tilde{q}$, (17b) then determines the fraction of time devoted to leisure (employment) such that the marginal physical product of capital ensures that the rate of return on domestic capital equals the (given) world rate of interest. Hence in this small open economy with elastically supplied labor, the growth rate of output and capital is independent of production characteristics such as the productivity parameter, α, and the

[12] This local instability of the dynamic path depends in part upon our assumptions of a Cobb-Douglas production function and constant elasticity utility function, and justifies our focus on that equilibrium in the present analysis. For more general production functions one cannot dismiss the possibility that the dynamics has a stable eigenvalue, giving rise to potential problems of indeterminate equilibria. In a model with both physical and nonhuman capital, Benhabib and Perli (1994), Ladrón-de-Guevara, Ortigueira, and Santos (1997) show how the steady-state equilibrium may become indeterminate. Other authors have emphasized the existence of externalities as sources of indeterminacies of equilibrium; see Benhabib and Farmer (1994).

marginal cost of adjustment, h. Changes in these parameters are reflected in the labor-leisure choice $\tilde{l}$ [13].

The viability of the equilibrium requires that the nation's intertemporal budget constraint, which in this growth context is

$$B_0 + \frac{K_0}{r-\psi}\left[\left[1-\left(\frac{\tilde{l}}{1-\tilde{l}}\right)\frac{(1-\sigma)}{\theta}\right]\frac{\tilde{Y}}{\tilde{K}} - \left(\frac{\tilde{q}^2-1}{2h}\right)\right] = 0 \qquad (18)$$

be met. The initial value of its foreign bonds plus the capitalized value of the current account surplus along the balanced growth path must sum to zero. Having determined the equilibrium values of $\tilde{l}, \tilde{q}$, and $\tilde{Y/K}$, the intertemporal constraint (18) determines the combination of the initial capital stock, K_0, and the initial stock of foreign bonds, B_0, necessary for the equilibrium to be intertemporally viable. Substituting (18) into the current account relationship, we find that the equilibrium stock of traded bonds accumulate at the common equilibrium growth $\tilde{\psi}$.

If the inherited stocks of these assets violate (18) we assume that the appropriate adjustment is attained through initial lump-sum taxation (if necessary), of the form $dT_0 + dB_0 + \tilde{q}dK_0 = 0$, whereby the private agent is forced to readjust his portfolio to attain the intertemporally viable ratio consistent with (18). Using the government's balanced budget condition (5) one can determine the required level of lump sum taxes at each point in time[14].

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[13] In order for the equilibrium to be viable, the implied fraction of time devoted to leisure must satisfy $0 < l < 1$. This will be so if and only if: $0 < r(1-\tau_b) + \left(\frac{h}{1-\gamma}\right)(r(1-\tau_b)-\rho)\left[\frac{r(1-\tau_b)(1-2\gamma)+\rho}{2(1-\gamma)}\right] < (1-\tau_y)\sigma\alpha$, a condition that is plausibly met; see Turnovsky (1999).

[14] Along the balanced growth path, this is of the form: $T(t) = (aK_0 + bB_0)e^{\tilde{\psi}t}$, where a, b are constant, easily derived from the balanced growth equilibrium.

3.4 Comparison with Fixed Employment AK Model

Endogenizing labor fundamentally changes the macrodynamic equilibrium from its determination in the pure AK model, described in Section 3.1, where the labor supply is fixed. The key to this is the marginal rate of substitution relationship, (15), which implies that, because in equilibrium the allocation of time must be constant, the long-run consumption-output ratio must also be constant, forcing equilibrium consumption and output to grow at the same rate. The divergence in the equilibrium growth rates between consumption and output, which with fixed labor supply could prevail, is eliminated through the adjustment in the time allocation between work and leisure.

This contrasting mechanism is reflected in the corresponding dynamics of the external sector. As noted, with fixed employment, these differential growth rates are sustained by the transitional dynamics in the accumulation of traded bonds from their arbitrarily given initial stock, B_0, with the initial consumption being determined to ensure that international solvency is met. In the present case, however, the initial consumption level is determined by the optimality condition (15). Now, however, the initial stocks of assets B_0, K_0 must be chosen (through some initial intervention) to ensure that the economy remains internationally solvent.

This seemingly modest change in economic structure has significant consequences. It implies that factors that under fixed employment are reflected in the equilibrium growth rate of output are now reflected in the allocation of work time. Thus, for example, whereas in the fixed employment AK model the growth rate of output depends upon the domestic production parameters, α and h, it is now independent of these parameters, which instead influence l. Particularly important differences arise with respect to fiscal policy. From (17a) and (17b) we can summarize their effects as follows:

> **Proposition 4:** An increase in the domestic income tax increases the time devoted to work, that is reduces leisure. It has no effect on either the growth rate of consumption or output. An increase in the tax on bond income leads to a reduction in work and an increase in leisure. It also reduces the growth rates of both consumption and output.

The intuition is as follows. With the equilibrium growth rate and the equilibrium price of capital, $\tilde{q}$, fixed, a higher domestic income tax reduces the after-tax return to capital. In order to maintain equilibrium among rates of return, the productivity of capital must be increased. This is achieved by an increase in the fraction of time devoted to labor, that is, by a decline in leisure. An increase in the tax on foreign bond income generates the opposite portfolio response. The lower return on foreign bonds requires a lower equilibrium return on domestic capital, which is accomplished by a reduction in the productivity of capital brought about by an increase in leisure.

Both these responses contrast with the fixed-employment open economy AK model, where each affects the equilibrium growth rate of output as summarized in Proposition 2. On the other hand, the consumption tax has no effect either on the growth rate or on employment. Its only effect is on the consumption-output ratio, which is reduced. In this respect, the tax acts very much like a lump-sum tax, as in the fixed employment AK model of the open economy [15].

The fact that the growth rate is independent of most income taxes (except τ_b) offers an interesting perspective to the following issue. As we observed at the outset, one of implications of the basic endogenous growth model, a feature that distinguishes it from the traditional neoclassical model, is that its equilibrium growth rate varies inversely with distortionary income taxes. The fact that empirical evidence by Easterly and Rebelo (1993), Stokey and Rebelo (1995) and Jones (1995a, 1995b) does not support this implication, has been used as evidence against these endogenous growth models. Our results suggest some caution might be required in reaching this conclusion. For small economies facing a perfect world capital market, the equilibrium growth rate is in fact independent of most tax rates. Instead, such economies respond to changes in tax rates through variations in their equilibrium labor-leisure choice.

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[15] As a further comparison, the effects of both the domestic income tax and the consumption tax contrast sharply with the analogous closed economy with endogenous labor. In such an economy both lead to a reduction in the growth rate together with an increase in leisure; see Turnovsky (2000). With the equilibrium growth rate fixed from the consumption side, this tradeoff ceases to exist in the small open economy.

4. PRODUCTIVE GOVERNMENT EXPENDITURE

4.1 The Barro-Model

Thus far we have focused on the taxation side of the government budget. But most tax revenues are used to finance government expenditures, which provide some benefits to the economy. We shall focus on government expenditure that enhances the productive capacity of the economy, identifying such expenditures as being on some form of infrastructure. This model was first applied to a closed economy by Barro (1990), and like Barro, we shall make the simplifying assumption that the benefits are derived from the *flow* of productive government expenditures[16]. In Section 5.2 below, we shall briefly discuss the more plausible case where it is the accumulated *stock* of government expenditure that is relevant.

We abstract from labor (setting $l = 0$) and assume that the production function of the representative firm is now specified by

$$Y_i = \alpha G_s^{\eta} K_i^{1-\eta} \equiv \alpha \left(G_s / K_i \right)^{\eta} K_i \qquad 0 < \eta < 1 \qquad (19)$$

where G_s denotes the flow of productive services enjoyed by the individual firm. As in Section 3, we assume that the population growth $n = 0$. Thus productive expenditure has the property of positive, but diminishing, marginal physical product, while enhancing the productivity of private capital.

We shall assume that the services derived from aggregate expenditure, G, are

$$G_s = G \left(\frac{K_i}{K} \right)^{1-\varepsilon} \qquad (20)$$

where K denotes the aggregate capital stock. As Barro and Sala-i-Martin (1992a) have argued most public services are characterized by some degree of congestion; there are few pure public goods. It is straightforward to parameterize the degree of congestion. This is important since the degree of congestion is to some extent the outcome of a policy

[16] In Turnovsky (1996a) we discuss the parallel case where government expenditure is on a utility-enhancing consumption good.

decision, and once determined, the degree of congestion turns out to be a critical determinant of optimal tax policy.

Equation (20) is one convenient formulation of congestion that builds on the public goods literature; see Edwards (1990). It implies that in order for the level of public services, G_s, available to the individual agent to remain constant over time, given its individual capital stock, K_i, the growth rate of G must be related to that of K in accordance with $\dot{G}/G = (1-\varepsilon)\dot{K}/K$. Congestion increases if aggregate usage increases relative to individual usage, and a good example of this type of congestion is the service provided by highway usage. Unless an individual drives his car (uses his capital), he derives no service from a publicly provided highway, and in general, the service he derives depends upon his own usage relative to that of others in the economy, as total usage contributes to congestion.

The parameter ε can be interpreted as describing the degree of relative congestion associated with the public good and the following special cases merit comment. If $\varepsilon = 1$, the level of services derived by the individual from the government expenditure is fixed at G, independent of both the firm's own usage of capital and aggregate usage. The good G is a non-rival, non-excludable public good that is available equally to each individual; there is no congestion. Since few, if any such public good exist, it is probably best viewed as a benchmark. At the other extreme, if $\varepsilon = 0$, then only if G increases in direct proportion to the aggregate capital stock, K, does the level of the public service available to the individual firm remain fixed. We shall refer to this case as being one of *proportional* (relative) congestion. In that case, the public good is like a private good, in that since $K = NK_i$, the individual receives his proportionate share of services.

In order to sustain an equilibrium of ongoing growth, government expenditure cannot be fixed at some exogenous level, but rather, must be tied to the scale of the economy. This can be achieved most conveniently by assuming that the government sets its level of expenditure as a share of aggregate output, $Y = NY_i$:

$$G = gY \tag{21}$$

In an environment of growth this is a reasonable assumption. Government expenditure thus increases with the size of the economy, with an expansionary government expenditure being denoted by an increase in g.

Summing (19) over the N identical agents and substituting (20) and (21), we obtain

$$G = \left(\alpha g N^{\eta\varepsilon}\right)^{1/(1-\eta)} K; \;\; Y = \left(\alpha g^{\eta} N^{\eta\varepsilon}\right)^{1/(1-\eta)} K \tag{22}$$

Aggregate output thus has the fixed AK technology, where the productivity of capital depends (positively) upon the productive government input. Notice that provided $\eta\varepsilon > 0$, the productivity of capital depends upon the size (scale) of the economy, as parameterized by the fixed population, N. This is because the size of the externality generated by government expenditure increases with the size of the economy, playing an analogous role to aggregate capital in the Romer (1986) model. As in that model, this scale effect disappears if $\varepsilon = 0$, so that there is proportional congestion and each agent receives his own individual share of government services, G/N.

We now re-solve the representative individual's optimization problem. In so doing, he is assumed to take aggregate government spending, G, and the aggregate stock of capital, K, as given, insofar as these impact on the productivity of his capital stock. Performing the optimization, the optimality conditions (2a), (2c), and (4a) remain unchanged. The optimality condition with respect to capital is now modified to:

$$\frac{(1-\tau_y)(1-\eta\varepsilon)Y_i}{qK_i} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r(1-\tau_b) \tag{4b'}$$

The difference is that the private marginal physical product of capital is now proportional to $(1-\eta\varepsilon)$, depending both upon the degree of congestion and the productivity of government expenditure. The less congestion (the larger ε) the smaller the benefits of government expenditure are tied to the usage of private capital, thus lowering the return.

The other modification are to the government budget constraint, (5) and the current account relationship (6), which are modified to:

$$\tau_y Y + \tau_b rB + \tau_c C = G \tag{5'}$$

and

$$\dot{B} = Y + rB - C - I[1 + (h/2)(I/K)] - G \tag{6'}$$

4.2 Optimal Fiscal Policy

It is clear from Section 4.1 that growth and economic performance are heavily influenced by fiscal policy in this model. This naturally leads to the important question of the optimal tax structure. To address this issue it is convenient to consider, as a benchmark, the first-best optimum of the central planner, who controls resources directly, against which the decentralized economy can be assessed. The central planner is assumed to internalize the equilibrium relationship $NK_i = K$, as well as the expenditure rule (21). The optimality conditions are now modified to

$$C_i^{\gamma-1} = \lambda \tag{2a'}$$

$$1 + h(I_i/K_i) = q \tag{2c'}$$

$$\rho - \frac{\dot{\lambda}}{\lambda} = r \tag{4a''}$$

$$\frac{(1-g)Y_i}{qK_i} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = r \tag{4b''}$$

The key difference is that the social return to capital nets out the fraction of output appropriated by the government.

It is straightforward to show that the decentralized economy will mimic the first-best equilibrium of the centrally planned economy if and only if

$$\tau_b = 0 \tag{23a}$$

$$(1-g) = (1-\tau_y)(1-\varepsilon\eta) \tag{23b}$$

The first condition follows from the fact that since there is no distortion to correct in the international bond market the optimal tax on foreign bond income should be zero. By contrast, government expenditure, by being tied to the stock of capital in the economy, induces spillovers into the domestic capital market, generating distortions that require a tax on capital income in order to ensure that the net private return on capital equals its social return.

To understand (23b) better, it is useful to observe that the welfare-maximizing share of government expenditure, is (Barro, 1990)[17]:

$$\hat{g} = \eta \tag{24}$$

Substituting (24) into (23b) and simplifying, the optimal income tax can be expressed in the form

$$\hat{\tau}_y = \frac{g - \eta\varepsilon}{1 - \eta\varepsilon} = \frac{g - \hat{g}}{1 - \eta\varepsilon} + \frac{\hat{g}(1-\varepsilon)}{(1 - \eta\varepsilon)}. \tag{23b'}$$

In order to finance its expenditures, (5'), the government must, in conjunction with $\hat{\tau}_y$, set a corresponding consumption tax $\hat{\tau}_c$:

$$\hat{\tau}_c = \frac{\eta\varepsilon(1-g)}{(1-\eta\varepsilon)(C/Y)} \tag{23c}$$

Equation (23b') emphasizes that the optimal tax on capital income corrects for two distortions. The first is due to the deviation in government expenditure from the optimum, the second is caused by congestion. Comparing (23b') and (23c) we see that there is a tradeoff between the income and the consumption tax in achieving these objectives, and that this depends primarily upon the degree of congestion. In the case where $\varepsilon = 1$, so that there is no congestion, capital income should be taxed only to the extent that the share of government expenditure deviates from the social optimum. The tradeoff between the two taxes is seen most directly if *g* is set optimally in accordance with (24). In this case, if there is no congestion, government expenditure should be fully financed by a consumption tax alone; capital income should remain untaxed. As congestion increases (ε

[17] This is obtained by maximizing utility with respect to *g*.

declines), the optimal consumption tax should be reduced and the income tax increased until with proportional congestion, government expenditure should be financed entirely by an income tax.

It is useful to compare the present optimal tax on capital with the well-known Chamley (1986) proposition which requires that asymptotically the optimal tax on capital should converge to zero. The Chamley analysis did not consider any externalities from government expenditure. Setting $\eta = 0$, we still find that the optimal tax on capital is equal to the share of output claimed by the government ($\hat{\tau}_y = g$). The difference is that by specifying government expenditure as a fraction of output, its level is not exogenous, but instead is proportional to the size of the growing capital stock. The decision to accumulate capital stock by the private sector leads to an increase in the supply of public goods in the future. If the private sector treats government spending as independent of its investment decision (when in fact it is not), a tax on capital is necessary to internalize the externality and thereby correct the distortion. Thus, in general, the Chamley rule of not taxing capital in the long run will be nonoptimal, although it will emerge in the special case where $g = \eta\varepsilon$, in which case there is no spillover from government expenditure to the capital market.

5. TRANSITIONAL DYNAMICS AND GROWTH

The models discussed thus far all have the characteristic that consumption and output (capital) are on their respective balanced growth paths; there are no transitional dynamics. Instead, the economy adjusts infinitely fast to any exogenous shock, thus contradicting the empirical evidence pertaining to convergence, which we shall discuss briefly in Section 8, below. The point of this literature is that the economy adjusts relatively slowly, with the rate of 2% per annum being a benchmark estimate[18]. This implies that the economy is mostly off its balanced growth path, on some transitional path, which only gradually converges to steady state. It is therefore important to modify the model to allow for such transitional dynamics, and this can be achieved in several ways, all of which assign a central role to a second state variable to the dynamics.

In this section, we consider two important modifications to the basic model, which accomplish this objective. These include: (i) limited access to the world financial market; and (ii) the introduction of public capital[19].

As we will show in Section 6, extending the model to two sectors, having traded and nontraded capital may introduce transitional dynamics, depending upon the relative sectoral capital intensities. But one further way transitional dynamics can be introduced is through a modification of the technology to allow it to have non-scale growth, along the lines initially proposed by Jones (1995a, 1995b). This model will be discussed in more detail in Section 7.

5.1 Upward Sloping Supply Curve of Debt

The assumption that the economy is free to borrow or lend as much as it wants at the given world interest rate in a perfect world capital market is a strong one. While it is convenient and may be a reasonable approximation for developed economies having access to highly integrated capital markets it is less plausible for developing economies. The equilibrium structure changes dramatically when the economy faces restricted access

[18] This benchmark value was originally established by Mankiw, Romer, Weil (1992), Barro and Sala-i-Martin (1992b) and others.

[19] This can also be achieved in other ways by (i) introducing Uzawa (1968) preferences and (ii) the Blanchard (1985)-Weil (1989) overlapping generations model.

to the world capital market. Not only does this generate transitional dynamics, but it also imposes some of the characteristics of a closed economy, thereby linking the equilibrium growth rates of consumption and output.

This can be most conveniently addressed in the case of a debtor nation by postulating that the rate of interest at which it may borrow is an increasing function of its debt. This type of constraint was originally proposed by Bardhan (1967) and has been introduced by many authors since then; see Obstfeld (1982), Bhandari, Haque, and Turnovsky (1990), Fisher (1995). While these specifications are essentially ad hoc, more formal justification of this type of relationship, in terms of default risk, has been provided by Eaton and Gersovitz (1989).

One issue that arises is whether the specification of debt cost is expressed in terms of the absolute level of debt, as originally proposed by Bardhan, or relative to some earnings capacity to service the debt, as argued by Sachs (1984) and others. The latter formulation is appropriate if an equilibrium of ongoing growth is to be sustained, and indeed has been adopted in previous models of growing open economies; see e.g. van der Ploeg (1996).

To illustrate the impact of endogenizing the borrowing rate, we focus on a debtor economy and assume that the cost of borrowing from abroad is:

$$r(Z/K) = r^* + \upsilon(Z/K); \upsilon' > 0 \tag{25}$$

where $Z \equiv -B$ denotes the aggregate stock of debt. For simplicity, we return to the specification of Section 3.1, where we consider the impact of distortionary taxes (including now a tax on debt), the revenues of which are rebated. It is important to emphasize that in performing his optimization, the representative agent takes the interest rate as given. This is because the interest rate facing the debtor nation, is an increasing function of the economy's *aggregate* debt, which the representative agent, in making his decisions, assumes he is unable to influence.

With this in mind, the optimality conditions are now:

$$C^{\gamma-1} = \lambda(1+\tau_c) \qquad (2a")$$

$$1 + h(I_i/K_i) = q \qquad (2c')$$

together with the arbitrage conditions

$$\rho - \frac{\dot{\lambda}}{\lambda} = (1+\tau_z)r\left(\frac{Z}{K}\right) \qquad (4a")$$

$$(1-\tau_y)\frac{A}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} = (1+\tau_z)r\left(\frac{Z}{K}\right) \qquad (4b")$$

and the current account relationship

$$\dot{Z} = C + I\left[1 + \frac{h}{2}\frac{I}{K}\right] - AK + r\left(\frac{Z}{K}\right)Z \qquad (6')$$

These equations are analogous to (2a), (2c), (4a), (4b), and (6') in the basic model, but now the interest rate is endogenously determined as a function of the nation's debt-capital ratio Z/K. This changes the dynamics fundamentally.

To derive the macrodynamic equilibrium, we need to express it in terms of stationary variables. For this purpose, it is convenient to express it in terms of $c \equiv C/K, z \equiv Z/K$, and q. Taking time derivatives of these quantities and combining with (2c') and (4a"), the equilibrium dynamics may be summarized by:

$$\dot{c} = c\left(\frac{1}{1-\gamma}\left((1+\tau_z)r(z) - \rho\right) - \frac{q-1}{h}\right) \qquad (26a)$$

$$\dot{z} = c + \left(\frac{q^2-1}{2h}\right) - \left(\frac{q-1}{h}\right)z - A + r(z)z \qquad (26b)$$

$$\dot{q} = (1+\tau_z)r(z)q - (1-\tau_y)A - \frac{(q-1)^2}{2h} \qquad (26c)$$

Linearizing these equations it is easily verified that the equilibrium is characterized by saddlepath dynamics[20]. Starting from an initial debt-capital ratio, z_0 the economy follows a unique one-dimensional stable transitional adjustment path, taking it to the steady state.

The steady state growth path will be obtained when $\dot{c} = \dot{z} = \dot{q} = 0$, so that the corresponding steady state values of c,z,q, denoted by tildes, are determined by:

$$\frac{1}{1-\gamma}\left((1+\tau_z)r(\tilde{z})-\rho\right)=\frac{\tilde{q}-1}{h} \tag{27a}$$

$$\tilde{c}+\left(\frac{\tilde{q}^2-1}{2h}\right)-\left(\frac{\tilde{q}-1}{h}\right)\tilde{z}-A+r(\tilde{z})\tilde{z}=0 \tag{27b}$$

$$(1-\tau_y)\frac{A}{\tilde{q}}+\frac{(\tilde{q}-1)^2}{2h\tilde{q}}=(1+\tau_z)r\left(\tilde{z}\right) \tag{27c}$$

The key difference to observe from the basic model is that the constancy of the long-run consumption-capital ratio forces consumption and output to grow at the same long-run equilibrium rate. In effect, the economy incurs an equilibrium level of debt to capital such that these two growth rates are brought into equality. This is in sharp contrast to the equilibrating mechanism in the elastic labor supply model of Section 3.3; see (17a), (17b). In that case, it was the time devoted to leisure that adjusted to ensure that the productivity of capital and its equilibrium growth rate of capital was consistent with the exogenously determined growth rate of consumption.

This model responds very differently to policy shocks from the basic model developed in Section 3.1, in that any such disturbance will generate transitional dynamics, before ultimately converging to the equilibrium described by (27). Furthermore, the effects on long-run growth may also be very different. This can be seen by combining (27a) and (27b), to obtain[21]

$$(1-\tau y)\frac{A}{\tilde{q}}+\frac{(\tilde{q}-1)^2}{2h\tilde{q}}=\rho+(1-\gamma)\left(\frac{\tilde{q}-1}{h}\right)$$

[20] That is there are one stable and two unstable eigenvalues, with one "sluggish" variable, z, and two "jump" variables q and c.

[21] This equation coincides with what obtains in the closed economy with costs of adjustment in investment; see Turnovsky (1996c).

From this equation we show that an increase in the tax on capital will reduce the (common) equilibrium growth rate in the economy, while an increase in the tax on bonds will leave the equilibrium growth rate unaffected. Instead, an increase in τ_z will discourage the accumulation of higher debt such that its after-tax cost just equals the (unchanged) equilibrium rate of return on capital. This response is very different from that described in Section 3.1, where with a perfect world bond market a higher tax on bonds would raise the equilibrium growth rate of capital, but reduce the growth rate of consumption and wealth.

Finally, the upward-sloping supply curve of debt introduces an externality that assigns a role to a tax on debt. In contrast to the representative agent, a central planner, maximizing the welfare of the representative agent, takes account of the fact that as he accumulates debt and capital that he influences the marginal borrowing cost. The key optimality conditions corresponding to (4a") and (4b") for the centrally planned economy are modified to

$$\rho - \frac{\dot{\lambda}}{\lambda} = r(z) + r'(z) \cdot z \tag{28a}$$

$$\frac{A}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2hq} + \frac{r'(Z/K)}{q} \cdot \left(\frac{Z}{K}\right)^2 = r\left(\frac{Z}{K}\right) + r'\left(\frac{Z}{K}\right) \cdot \left(\frac{Z}{K}\right) \tag{28b}$$

Because of the transitional dynamics, a time-varying income tax rate is necessary in order to replicate the entire time path of the first-best centrally planned economy. An example of this is provided by Turnovsky (1997d). Here we simply observe that the steady-state of the decentralized economy will coincide with that of the centrally planned economy if and only if

$$\hat{\tau}_z = \frac{r'(\tilde{z})\tilde{z}}{r(\tilde{z})} > 0 \tag{29a}$$

$$\hat{\tau}_y = -\frac{r'(\tilde{z})\tilde{z}^2}{A} < 0 \tag{29b}$$

Intuitively, agents in the decentralized economy fail to take account of the fact that as they collectively increase their amount of borrowing, they raise the aggregate debt-capital ratio, thus raising the cost of debt. Similarly, as they invest in capital and increase the productivity capacity of the economy and its ability to finance debt capital, they lower the aggregate debt-capital ratio, and reduce the cost of debt. Accordingly, by underestimating the true cost of borrowing and underestimating the true benefits to accumulating capital the agents in the decentralized economy underinvest in capital and overborrow, relative to the first best optimum. To correct for this misallocation, debt should be taxed, while capital should be subsidized.

5.2 Public and Private Capital

Most models analyzing productive government expenditure treat the current *flows* of government expenditure as the sources of contributions to productive capacity. This is true of Aschauer (1988), Barro (1989), and Lee (1995) within the context of the stationary Ramsey model and also of Barro (1990), Turnovsky (1996b) in the context of the AK model, discussed in Section 4. While the flow specification has the virtue of tractability, it is open to the criticism that insofar as productive government expenditures are intended to represent public infrastructure, such as roads and education, it is the accumulated *stock*, rather than the current flow, that is the more appropriate argument in the production function.

Despite this, within the Ramsey framework, relatively few authors have adopted the alternative approach of modeling productive government expenditure as a stock. Arrow and Kurz (1970) were the first authors to model government expenditure as a form of investment. More recently, Baxter and King (1993) study the macroeconomic implications of increases in the stocks of public goods. They derive the transitional dynamic response of output, investment, consumption, employment, and interest rates to such policies by calibrating a real business cycle model. Fisher and Turnovsky (1998) address similar issues from a more analytical perspective.

The literature introducing both private and public capital into growth models is sparse. Futagami, Morita, and Shibata (1993), Glomm and Ravikumar (1994), and Turnovsky (1997b) do so in a closed economy, while an open economy version is developed by Turnovsky (1997c). Private capital in the Glomm-Ravikumar model fully depreciates each period, rather than being subject to at most gradual (or possibly zero) depreciation. This enables the dynamics of the system to be represented by a single state variable alone, so that the system behaves much more like the Barro model in which government expenditure is introduced as a flow. In particular, under constant returns to scale in the reproducible factors, there are no transitional dynamics and the economy is always on a balanced growth path.

With a perfect world capital market, the introduction of public capital leads to an equilibrium in which the transitional dynamics can be specified in terms of (i) the ratio of public to private capital; (ii) the consumption to capital ratio, and (iii) the shadow value of private capital. One result stemming from these models is that the long-run growth-maximizing level of g, $\bar{g}$ say, *exceeds* the long-run welfare maximizing level, $\hat{g}$. This is in contrast to Barro (1990), who, introducing government expenditure as a flow in the production function, finds that the welfare-maximizing and growth-maximizing shares of government expenditure coincide. The difference is accounted for by the fact that when government expenditure influences production as a flow, maximizing the marginal product of government expenditure net of its resource cost maximizes the growth rate of capital. But it also maximizes the social return to public expenditure, thereby maximizing overall intertemporal welfare. By contrast, when government expenditure affects output as a stock, public capital needs to be accumulated to attain the growth maximizing level. This involves foregoing consumption, leading to welfare losses relative to the social optimum. Intertemporal welfare is raised by reducing the growth rate, thereby enabling the agent to enjoy more consumption.

The model with government capital is relevant for analyzing the policies of unilateral capital transfers instituted by the European Union in order to facilitate joining members attain the level of infrastructure that will enable them to attain the growth rate consistent with that of the Union. These transfers have been tied to investment in public capital. Chatterjee, Sakoulis, and Turnovsky (2000) have recently developed an endogenous growth model analyzing this process and have reached a number of striking conclusions with respect to the effects of such policies on both the growth performance and welfare of the recipient economy. For example, whether or not a tied transfer of capital from abroad benefits the growth rate and welfare of the recipient economy depends upon its initial endowment of public capital, as well as the costs of installation.

6. TWO-SECTOR GROWTH MODEL WITH TRADED AND NONTRADED CAPITAL

Two-sector models of economic growth were pioneered by Uzawa (1961), Takayama (1963), and others. In this early literature, the two sector corresponded to the production of the consumption good and the production of the investment good, respectively. The key result in that early literature focused on the uniqueness and stability of equilibrium, which was shown to depend critically upon the capital intensity of the investment good sector relative to the consumption sector.

In a seminal paper, Lucas (1988) introduced the two-sector endogenous growth model. The model included two capital goods, physical capital and human capital. The former is produced together with consumption goods in the output sector, using both human and physical capital. Human capital is produced in the education sector using both physical capital and human capital. The agent's decisions at any instant of time are (i) how much to consume, (ii) how to allocate his physical and human capital across the sectors, and (iii) at what rate to accumulate total physical and human capital over time. Having two capital goods, this model is characterized by transitional dynamics. However, because two-sector endogenous growth models initially proved to be intractable, much of the analysis was restricted to balanced growth paths (e.g. Lucas (1988), Devereux and Love (1994)) or to analyzing the transitional dynamics using numerical simulation methods (see e.g. Mulligan and Sala-i-Martin (1993), Pecorino (1993), Devereux and Love (1994)). One important exception to this is Bond, Wang, and Yip (1996), which using the methods of the standard two-sector trade model provided an effective analysis of the dynamic structure of the two-sector endogenous growth model.

In fact, if one identifies physical capital with traded capital and human capital with nontraded capital, it is very natural to interpret the Lucas model within an international context, as being an endogenous growth version of the dependent economy model. This model has played an important role in international economics since the first formal analyses of nontraded goods were carried out by the Australian school of Salter (1959), Swan (1960), and extended by others[22]. It is also particularly relevant for analyzing issues pertaining to real exchange rates.

[22] An extensive review of this model is provided by Turnovsky (1997a, Chapter 4).

6.1 Outline of the Model

We shall merely sketch the main features of the model. The representative agent now accumulates two types of capital for rental at the competitively determined rental rate. The first is physical capital, K, which is traded, and the second is human capital, H, which is nontraded. For expositional simplicity there is no government.

These two forms of capital are used by the agent to produce a tradable good, Y_T, taken to be the numeraire, by means of a linearly homogeneous production function:

$$Y_T = aK_T^{\alpha} H_T^{(1-\alpha)}; \quad 0 < \alpha < 1 \tag{30a}$$

where, K_T and H_T denote the allocation of the respective capital good to the production of the traded good. The agent also produces a nontraded good, Y_N, using an analogous production function

$$Y_N = bK_N^{\beta} H_N^{(1-\beta)}; \; 0 < \beta < 1 \tag{30b}$$

The fact that both production functions are linearly homogeneous in the two reproducible factors, K and H, is critical for an equilibrium with steady endogenous growth to exist. The relative price of nontraded goods, p, is taken as exogenously given by the agent, and is determined by market clearing conditions in the economy. Both forms of capital are costlessly and instantaneously mobile across the two sectors, with the sectoral allocations being constrained by:

$$K_T + K_N = K \tag{31a}$$

$$H_T + H_N = H \tag{31b}$$

The accumulation of traded capital, $\dot{K}$, involves adjustment costs, as specified in (1c). Nontraded (or human) capital is accumulated costlessly in accordance with

$$\dot{H} = Y_N - C_N$$

where C_N is the agent's consumption of the nontraded good. In addition to accumulating the two types of capital, the agent accumulates net foreign bonds, B, that pay an exogenously given world interest rate, r. Thus the agent's instantaneous budget constraint is specified by:

$$\dot{B} = aK_T^{\alpha}H_T^{1-\alpha} + pbK_N^{\beta}H_N^{1-\beta} + rB - C_T - pC_N - I(1 + hI/2K) - p\dot{H} \tag{32}$$

where C_T denotes consumption of the traded good.

The agent's optimization decision is to choose the rate of consumption (C_T, C_N), capital allocation decisions (K_T,K_N,H_T,H_N), and rates of capital accumulation, I and $\dot{H}$, to maximize the intertemporal isoelastic utility function:

$$\int_0^{\infty} \frac{1}{\gamma}\left(C_T^{\theta}C_N^{1-\theta}\right)^{\gamma} e^{-\rho t} dt \tag{33}$$

subject to the constraints (31), (32), (1d), and the initial stocks of assets.

The following optimality conditions with respect to C_T,C_N,K_T,K_N,H_T,H_N, and I obtain:

$$\theta C_T^{\theta\gamma-1}C_N^{\gamma(1-0)} = \lambda \tag{34a}$$

$$(1-\theta)C_T^{\theta\gamma}C_N^{\gamma(1-\theta)-1} = \lambda p \tag{34b}$$

$$a\alpha K_T^{\alpha-1}H_T^{1-\alpha} = pb\beta K_N^{\beta-1}H_N^{1-\beta} \equiv r_k \tag{34c}$$

$$a(1-\alpha)K_T^{\alpha}H_T^{-\alpha} = pb(1-\beta)K_N^{\beta}H_N^{-\beta} \equiv r_h \tag{34d}$$

$$1 + h(I/K) = q \tag{34e}$$

The first pair are the usual intertemporal envelope conditions relating the marginal utility of the two consumption goods to the shadow value of wealth and are analogous to (2a). Equations (34c) and (34d) equate the marginal returns to traded and nontraded capital across the two sectors. The quantities r_k and r_h define the marginal physical products of traded and nontraded capital, respectively, measured in terms of traded output (the numeraire). Equation (34e) is analogous to (2c) and may be immediately solved to yield the following expression for the rate of accumulation of traded capital:

$$I/K = \dot{K}/K = (q-1)/h \equiv \phi(t) \tag{35}$$

Applying the standard optimality conditions with respect to traded bond b, and the two forms of capital, K and H, leads to the arbitrage conditions:

$$\rho - \dot{\lambda}/\lambda = r \tag{36a}$$

$$(r_k/q) + \left(\dot{q}/q\right) + \left((q-1)^2/2hq\right) = r \tag{36b}$$

$$(r_h/p) + \left(\dot{p}/p\right) = r \tag{36c}$$

These are analogous to (4a), (4b). Finally, in order to ensure that the agent's intertemporal budget constraint is met, the following transversality conditions must be imposed:

$$\lim_{t\to\infty} \lambda B e^{-\rho t} = 0; \quad \lim_{t\to\infty} q' K e^{-\rho t} = 0; \quad \lim_{t\to\infty} \lambda p H e^{-\rho t} = 0 \tag{37}$$

6.2 Determination of Macroeconomic Equilibrium

We define aggregate consumption C, expressed in terms of the traded good as numeraire, by

$$C \equiv C_T + pC_N$$

This definition, together with the optimality conditions (34a), (34b), implies that consumptions of the two goods are

$$C_T = \theta C; \qquad pC_N = (1-\theta)C \tag{38}$$

from which aggregate consumption grows at the rate

$$\dot{C}/C = \left(\left[r-\rho-\gamma(1-\theta)\left(\dot{p}/p\right)\right]/(1-\gamma)\right) \equiv \psi(t) \qquad (39)$$

where at this point the rate of inflation of the relative price, $\dot{p}/p$, is yet to be determined.

The derivation of the macroeconomic equilibrium involves three stages. The first determines the static allocation of existing resources in terms of (i) the relative price of nontraded to traded goods, p, and (ii) the gradually evolving aggregate stocks K and H. The second stage determines the dynamics of the relative prices, p and q, which, characteristic of two-factor two-sector economies having the Heckscher-Ohlin technology, decouple, and evolve independently of quantities. Having determined prices, the third stage then solves for the equilibrium rates of accumulation of the aggregate stocks of assets, K, H, and b.

The key stage is the second stage, the price dynamics . This is described by the pair of equations:

$$\dot{p} = rp - a(1-\alpha)\delta^{\alpha}p^{\alpha/(\alpha-\beta)} \qquad (40a)$$

$$\dot{q} = rq - (q-1)^2/2h - a\alpha\delta^{\alpha-1}p^{((\alpha-1)/(\alpha-\beta))} \qquad (40b)$$

where $\delta \equiv \left[(\beta/\alpha)^{\beta}((1-\beta)/(1-\alpha))^{1-\beta}(b/a)\right]^{1/(\alpha-\beta)}$. These equations are recursive; the relative price of the two goods evolves autonomously in accordance with (40a), and in turn determines the evolution of the market price of installed traded capital, the price of which is determined by the solution to the pair of differential equations (41a), (41b). The steady-state solution for the relative price of $\tilde{p}$ is

$$\tilde{p} = \left[r/\left(a(1-\alpha)\delta^{\alpha}\right)\right]^{((\alpha-\beta)/\beta)} \qquad (41a)$$

and the corresponding steady-state value of q, $\tilde{q}$, is the solution to the quadratic equation

$$a(1-\alpha)\delta^{\alpha}\tilde{p}^{(\alpha/(\alpha-\beta))} + (\tilde{q}-1)^2/2h = r\tilde{q}\,. \qquad (41b)$$

which is analogous to (13) in the basic one sector model. The nature of the price dynamics depends crucially upon the sectoral capital intensities and there are two cases to be considered.

If $\beta > \alpha$ so that the nontraded sector is relatively intensive in traded capital, the only solutions for p and q, which are consistent with the transversality condition on traded capital are $p = \tilde{p}$ given by (41a), and $q = q_1$, the (unstable) steady-state solution given by the negative root to (41b). In this case there are *no transitional dynamics in either the relative price of nontraded to traded goods or the market price of capital*. In response to any shock, these prices immediately jump to their respective new steady state values. These in turn imply that the growth rate of consumption and traded capital are on their respective balanced growth paths; see (39) and (35) respectively.

By inserting the solutions for $C(t)$ and $K(t)$ into the market clearing condition for nontraded capital, $\dot{H} = Y_N - C_N$, we obtain the solution for H starting from the initial stock of nontraded capital H_0. Invoking the transverality condition we derive the initial consumption and furthermore show that the evolution of nontraded capital $H(t)$ involves a transitional adjustment path, with the growth rate of nontraded capital converging to that of traded capital. From these solutions for W, K, and H, we can derive the long-run implications for traded bonds, which therefore are also subject to transitional dynamics.

If $\alpha > \beta$ so that the traded sector is relatively intensive in traded capital, the only solutions for p and q which are consistent with the transversality condition on traded capital are that p and q lie on the stable saddlepath LM, ultimately converging to $p = \tilde{p}$ given by (41a), and $q = q_1$, the (unstable) steady-state solution given by the negative root to (41b). In this case, a shock to the economy *will generate transitional adjustment paths in both p and q*, which in turn generate transitional dynamics in the growth rates of all quantities, C,K, H, and B. An interesting aspect of the dynamics is that the initial values of prices p(0) and q(0) are determined by the stable saddlepath solution to (40a) and (40b) in conjunction with the international solvency condition for the economy. Details of these arguments are provided by Turnovsky (1996d).

7. NON-SCALE GROWTH MODEL

As noted, the endogenous growth model has been criticized on both empirical and theoretical grounds. We therefore now turn to an alternative model, the non-scale model, which has been proposed in part in response to these criticisms. The increased flexibility of the production function is associated with a higher order dynamics in comparison to the corresponding AK growth model. Thus, in cases where the AK model is always on its balanced growth path, the corresponding non-scale model will follow a first-order adjustment path.

7.1 Inelastic Labor Supply

Our objective is to analyze the dynamics of the aggregate economy about a balanced growth path. Along such an equilibrium path, aggregate output and the aggregate capital stock are assumed to grow at the same constant rate, so that the aggregate output-capital ratio remains unchanged. Summing the individual production functions (1a) over the N agents, the aggregate production function with inelastic labor supply is:

$$Y = \alpha(1-\bar{l})^{1-\sigma}K^{\eta+\sigma}N^{1-\sigma} \equiv AK^{\sigma_K}N^{\sigma_N} \tag{42}$$

where $A \equiv \alpha(1-\bar{l})^{1-\sigma}$, $\sigma_N \equiv 1-\sigma$ = share of labor in aggregate output, $\sigma_K \equiv \eta+\sigma$ = share of capital in aggregate output. Thus $\sigma_K + \sigma_n = 1+\eta$ measures total returns to scale of the social aggregate production function. Taking percentage changes of (42) and imposing the long-run condition of a constant Y/K ratio, the long-run equilibrium growth of capital and output, g, is

$$g \equiv (\sigma_n/(1-\sigma_K))n > 0 \tag{43}$$

Equation (43) exhibits the key feature of the non-scale growth model, namely, that the long-run equilibrium rate is proportional to the population growth rate by a factor that reflects the productivity of labor and capital in the aggregate production function[23]. In particular, it is independent of any macro policy instrument. We shall show below that as long as the dynamics of the system are stable, $\sigma_K < 1$, in which case the long-run equilibrium growth is $g > 0$, as indicated. Under constant returns to scale, $g = n$, the rate of population growth, as in the standard neoclassical growth model, to which the present one-sector model reduces. Otherwise g exceeds n or is less than n, that is there is positive or negative per capita growth, according to whether returns to scale are increasing or decreasing, $\eta \gtrless 0$.

The implication that long-run growth cannot be sustained in the absence of population growth has itself been the source of criticism. Accordingly, a second, alternative class of non-scale growth models, that eliminates the effect of country size, but still permits growth in the absence of population growth, has more recently emerged. These models permit at least a limited role for government policy to influence the long-run growth rate, through taxes and subsidies to research and development[24]. The model we present falls into the first category and we shall interpret what we shall view as its success in replicating the economy as providing some support for the original form of non-scale growth model.

To analyze the transitional dynamics of the economy about its long-run stationary growth path, it is convenient to express the system in terms of the relative price of installed capital, q, and the following stationary variables:

$$c \equiv C / N^{(\sigma_N/(1-\sigma_K))}; \qquad k \equiv K / N^{(\sigma_N/(1-\sigma_k))}; \qquad b \equiv B / N^{(\sigma_N/(1-\sigma_K))}$$

Under standard conditions of constant social returns to scale $[\sigma_N = \sigma_K = 1]$ and these reduce to standard per capita quantities; i.e. $c = C/N = C_i$ etc. Otherwise they represent "scale-adjusted" per capita quantities.

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23 The non-scale model, originally proposed by Jones (1995a, 1995b) was originally for a two-sector economy in which the two sectors are final output and new knowledge, which is endogenously determined; see also Segerstrom (1998) and Young (1998). Eicher and Turnovsky (1999a) provide a detailed characterization of the determinants of long-run equilibrium growth in such a two-sector non-scale growth model. The one-sector model we are discussing does not incorporate this important aspect.

Consumption Dynamics

To determine the growth rate of consumption we take the time derivative of (2a) and combine with (4a) to find that the individual's consumption grows at the constant rate:

$$\dot{C}_i / C_i = \left(\left(r\left(1-\tau_b\right)-\rho-n\right)/\left(1-\gamma\right)\right) \equiv \psi_i \tag{44}$$

With all individuals being identical, the growth rate of aggregate consumption is $\psi = \psi_i + n$, so that

$$\dot{C}/ C = \left(\left(r\left(1-\tau_b\right)-\rho-\gamma n\right)/\left(1-\gamma\right)\right) \equiv \psi \tag{45}$$

Differentiating c and using (45), the growth rate of the scale-adjusted per capita consumption is:

$$\dot{C}/ C = \left(\left(r\left(1-\tau_b\right)-\rho-\gamma n\right)/\left(1-\gamma\right)\right)-\left(\sigma_N /\left(1-\sigma_K\right)\right)n \equiv \psi - g \tag{46a}$$

Equations (44), (45) and (46a) all share the property that with a perfect world capital market, the corresponding consumption growth rates are constant and independent of the production characteristics of the domestic economy. In addition, these equilibrium growth rates vary inversely with the tax on foreign bond income, but are independent of all other tax rates. These aspects of the dynamics of consumption remain unchanged from the AK model discussed in Section 4.1

Capital and the Price of Capital

The dynamics of capital accumulation are, however, distinctly different from those of the AK model with fixed labor supply, in which, like consumption, capital always lies on its balanced growth path. In the present model we find that the scale-adjusted

[24] Examples of the second form of non-scale model include Young (1998), Peretto (1998), Aghion and Howitt (1998), Dinopoulos and Thompson (1998), and Howitt (1999). Jones (1999) provides a convenient characterization of these two classes of non-scale models

capital-labor ratio, k, and the relative price of capital, q, converge to a long-run steady growth path along a transitional locus. To derive this path we differentiate k with respect to time and combine with (3'), to obtain:

$$\dot{k}/k = \left[\left((q-1)/h\right) - \left(\sigma_N / (1-\sigma_k)\right)n\right] = \phi - g \tag{46b}$$

To derive the law of motion for the relative price of the capital good, we substitute the production function, (1a), the aggregation condition, $K = NK_i$, and k into the arbitrage condition (4b). The latter can then be expressed as

$$\dot{q} = r(1-\tau_b)q - (q-1)^2/2h - (1-\tau_y)A\sigma k^{\sigma_K - 1} \tag{46c}$$

Thus (46b) and (46c) comprise a pair of equations in q and k, that evolve independently of consumption.

In order for the domestic capital stock ultimately to follow a path of steady growth, the stationary solution to (46b), (46c), attained when $\dot{q} = \dot{k} = 0$, must have (at least) one *real* solution. Setting $\dot{q} = \dot{k} = 0$ we see that the steady-state values of q and k, $\tilde{q}$ and $\tilde{k}$, are determined recursively as follows. First, the steady-state price of installed capital is:

$$\tilde{q} = 1 + h\left(\sigma_N / (1-\sigma_K)\right)n = 1 + hg \tag{47a}$$

Having determined $\tilde{q}$ from this equation, the equilibrium scale-adjusted capital-labor ratio, $\tilde{k}$, is then determined from the steady-state arbitrage condition:

$$(1-\tau_y)A\sigma\tilde{k}^{\sigma_K - 1} + (\tilde{q}-1)^2/2h = r(1-\tau_b)\tilde{q} \tag{47b}$$

To be viable, the long-run equilibrium must satisfy the transversality conditions. Substituting (4a), (3') into (4c) and evaluating this requires that:

$$r(1-\tau_b) > g > 0 \tag{48}$$

That is, the after-tax interest rate on foreign bonds must exceed the growth rate of domestic aggregate output. Observe that the condition (48) ensures that (47a) and (47b) imply a unique steady state equilibrium having: (i) a positive equilibrium growth rate of capital (output), $\tilde{\phi} = g$, and, (ii) a positive scale-adjusted capital-labor ratio, $\tilde{k}$ [25].

The equilibrium of the production side is thus fundamentally different in the non-scale economy from that of the simple AK technology. First, it is characterized by transitional dynamics, (46b), (46c), the nature of which will be discussed below. Second, in contrast to the AK technology where the existence of a balanced equilibrium growth rate depends upon the size of the adjustment costs relative to the productivity of capital, the transversality condition (4a) *always* ensures the existence of a unique equilibrium growth rate of capital. Moreover, as is evident from (43), the steady-state growth rate of aggregate capital (and output) shares the standard characteristic of non-scale growth models, namely that it is: (i) strictly positive and (ii) depends upon the returns to scale in the production function. Specifically, the growth rate is greater than or less than that of labor, according to whether there are increasing or decreasing returns to scale in aggregate production. Furthermore, it is independent of the taxes levied upon interest or capital income. This implication contrasts with the AK model of Section 4.1 in which the growth of output increases with the former tax rate and decreases with the latter. In the non-scale model, the response to taxes occurs through the adjustment in k.

One further contrast is the response of the relative price of capital, q, to the cost of adjustment h. From (13), we see that in the AK model an increase in h lowers the return to capital arising from its favorable impact on installation costs; see (4b). For the return to capital to remain equal to the fixed return on foreign bonds, a q must *decline*. In the non-scale economy, with the equilibrium growth rate determined by production elasticities, an increase in h requires a *higher* q, in order for the growth rate of capital to equal the equilibrium growth rate of output; see (35a).

[25] This may be shown as follows:

$$(1-\tau_y)\alpha\sigma\tilde{k}^{\sigma_K - 1} = r(1-\tau_b)\tilde{q} - (\tilde{q}-1)^2/2h$$

Using (10a) and (11) the right hand of this equation exceeds $g(1+hg/2) > 0$, thus implying $\tilde{k}^{\sigma_K - 1} > 0$ and hence $\tilde{k} > 0$.

Linearizing (46b) and (46c) around (47a) and (47b), it is immediately apparent that $\sigma_K < 1$ is a necessary and sufficient condition for saddlepoint stability, in which case (43) ensures that the equilibrium growth rate of output, g is positive. The stability condition asserts $\eta < \sigma_N = 1 - \sigma$, so that the share of external spillover generated by private capital accumulation, and hence the overall social increasing returns to scale, cannot exceed the exogenously growing factor's share (labor) in production. As usual, we assume that the capital stock accumulates slowly, so that k evolves gradually from its initial value, k_0, while the shadow value of capital may adjust instantaneously to new information.

As in the AK model with inelastic labor supply, an important aspect of this equilibrium is that differential growth rates of consumption and domestic output can be sustained. This is a consequence of the economy being small in the world bond market and as in that model is associated with transitional dynamics in foreign asset accumulation[26].

7.2 Elastic Labor Supply

A key condition in the above version of the non-scale model is the inelastic labor supply. Once this is imposed, all other parameters are unrestricted. However, when this assumption is relaxed and labor is endogenously supplied, a much stronger condition is required for a consistent equilibrium to obtain. This is because the optimality condition (15) must now be taken into account. With the fraction of time allocated to work constant in steady state, this relationship implies that the steady-state consumption-output ratio must be constant, thereby imposing the equality of the long-run growth rates of consumption and output (and capital). Thus equating (45) to (43) we must have:

$$\psi \equiv \frac{r(1-\tau_b) - \rho - \gamma n}{1-\gamma} = \left(\frac{\sigma_N}{1-\sigma_K}\right) n \equiv g \qquad (49)$$

[26] Eicher and Turnovsky (1999b) consider the case where the economy faces an upward-sloping supply curve of debt, as in Section 5.1. This leads to a fourth order dynamic system, having a two-dimensional stable locus. This has interesting implications for the dynamics of debt in the face of financial liberalization. Specifically, they show that a single policy of financial liberalization leads to an initial capital inflow, followed by an eventual capital outflow, as part of the internal dynamics.

That is, the return on foreign bonds, given the taste parameters, must be such that the implied growth rate of consumption is driven to that of capital, which is determined by the population growth rate in conjunction with the productive elasticities, in accordance with the non-scale growth model.

Condition (49) is the growth analogue to the well-known knife-edge condition in the stationary Ramsey model, $r = \rho$, necessary for an interior equilibrium to exist, and to which it reduces in the absence of growth $(n = 0)$. Note, further, that if there are constant returns to scale, (49) simplifies to

$$r(1-\tau_b) = \rho + n \tag{49'}$$

which is the familiar long-run viability condition for the standard Ramsey growth model. But now, with the more general productive structure, (49) involves both the productive elasticities, σ_K, σ_N as well as the intertemporal elasticity measure, γ.

Macrodynamic Equilibrium

Being a generalization of previous models, the macrodynamic equilibrium of the present model includes elements of the earlier discussion and here we merely sketch its structure. Following the procedures of Sections 2 and 7.1, the macrodynamic equilibrium includes the set of equations:

$$\dot{k} = \left(\frac{q-1}{h} - g\right)k \tag{50a}$$

$$\dot{q} = r(1-\tau_b)q - \frac{(q-1)^2}{2h} - (1-\tau_y)\alpha(1-l)^{\sigma_N}\sigma k^{\sigma_K - 1} \tag{50b}$$

$$\dot{l} = \frac{(1-\gamma)\sigma_K}{F(l)}\left(g - \frac{q-1}{h}\right) \tag{50c}$$

where in deriving (50c) we have used the condition (49). Note that (50a) and (50c) imply that scale-adjusted capital, k, and leisure, l, move in inverse proportion, and to a linear approximation, the distances from their respective steady states (denoted by tildes) are related by:

$$l(t) - \tilde{l} = -\frac{(1-\gamma)\sigma_K}{F(\tilde{l})\tilde{k}}\left(k(t) - \tilde{k}\right) \tag{51}$$

Intuitively, as capital increases, the return to labor rises and the desirability of leisure declines. Note from (51) that employment is now subject to transitional dynamics that mirrors the path of capital.

Equation (51) introduces a linear dependence into the three dynamic equations, thus implying that the stationary equations corresponding to (50a), (50b) and (50c) do not suffice to determine the steady state. Setting $\dot{k} = \dot{l} = 0$, we see that both (50a) and (50c) imply that the steady-state price of installed capital, $\tilde{q}$, remains as determined by (47a). Given this value of $\tilde{q}$, the remaining steady-state relationship [obtained by setting $\dot{q} = 0$ in (50b)] determines only the equilibrium marginal physical product of capital, which except in polar cases depends upon *both* $\tilde{l}$ and $\tilde{k}$. If employment is fixed (as in Section 7.1), then this determines $\tilde{k}$; if $\sigma_K = 1$, so that we have an AK technology (as in Section 3.3), then this determines $\tilde{l}$.

But in the present case, where both $\tilde{l}$ and $\tilde{k}$ are endogenously determined, further consideration is required to pin each down. The additional relationship is the current account. This requires that $\tilde{l}$ be appropriately chosen to ensure that (15) generates a consumption path that is consistent with the nation's intertemporal budget constraint. The argument is basically similar to that of the stationary Ramsey model, as set out in Turnovsky (1997a) and has the characteristic that the long-run equilibrium values of $\tilde{l}$ and $\tilde{k}$ are dependent upon initial conditions.

8. CONVERGENCE

The issue of convergence has been widely debated in the new growth theory. There are several aspects to this discussion. The first concerns whether or not countries have a tendency to converge to a common per capita level of income. The second is the speed of convergence. A related issue is whether the speed of convergence is constant across sectors and over time.

The notion of convergence is somewhat loose and more precise notions have been proposed, depending upon the structural similarity or lack, thereof, of the countries being compared; see Galor (1996). Evidence by Mankiw, Romer, and Weil (1992) suggesting that poorer countries having low initial income levels tend to grow faster than richer countries has been viewed as evidence in favor of conditional convergence, meaning that the convergence is dependent upon the structural parameters of the economies being identical, with the exception of initial conditions. On the other hand, Quah (1996) provides evidence against conditional convergence, but in support of a weaker form of convergence, known as club convergence, meaning that countries having the same structural characteristics and similar initial conditions as well, converge to similar levels of per capita income; i.e. poor countries and rich countries converge to low and high income levels, respectively.

Neoclassical and endogenous growth models offer strikingly different predictions with respect to both the determinants of long-run growth rates and speeds of convergence along transitional growth paths. According to the Solow-Swan model, countries with similar technologies should converge to similar long-run levels of per capita income. Thus to the extent that countries start from different initial income levels, one would expect poorer countries to catch up to richer countries. By contrast, endogenous growth models imply that long-run national growth rates are in general sensitive to national characteristics, such as tastes, technology, and tax structure. Controlling for technological change, Mankiw, Romer, and Weil (1992) and Barro and Sala-i-Martin (1995) present empirical evidence to suggest that countries do converge to identical growth rates, thus supporting the neoclassical growth model's explanation of long-run growth rates over that of the endogenous growth model.

Influential empirical work by Barro (1991), Barro and Sala-i-Martin (1992b, 1995), Barro and Lee (1994), Sala-i-Martin (1994), Mankiw, Romer and Weil (1992) established 2-3% as a benchmark estimate of the convergence rate. Both the one-sector neoclassical growth model and the two-sector Lucas endogenous growth model generate a one-dimensional locus of transitional dynamics. The speed of convergence along this locus is measured by the magnitude of the unique stable eigenvalue, which implies that (at least to a linear approximation) all variables converge at the same rate. However, in both cases the implied common convergence rate grossly exceeds the empirical benchmark of 2%, being approximately 7% in the neoclassical model and 10% in the Lucas model[27].

However, the neoclassical approach to convergence overemphasizes capital accumulation at the expense of technological change. Bernard and Jones (1996a, 1996b) have shown that while growth rates of output among OECD countries converge, the growth rates of manufacturing technologies exhibit markedly different time profiles. These distinct growth and convergence patterns of technology and output also challenge the first generation of endogenous growth models. Because the dynamics in these models are typically described by a one-dimensional locus, the first generation of endogenous growth models constrain output and technology to accumulate in proportion, much like in the neoclassical model.

Non-scale models generally imply much slower rates of convergence. In a preliminary examination of this issue, Jones (1995a) obtained excessively slow rates of convergence, mainly because he assumed that the sectoral allocation of each factor remains constant during the transition. Dinopoulos and Thompson (1998) analyze the transition dynamics of the alternative non-scale model numerically to find that the rate of convergence is approximately equal to the rate of population growth, but they do not highlight separate transition paths of output, capital and technology. Eicher and Turnovsky (1999c) show how the empirical findings regarding the differential rates of convergence of output and technology can be reconciled by introducing a two-sector *non-scale* model of capital accumulation that incorporates endogenous technological change and population growth. Their key contribution is to show how the stable transition path in the two-sector non-scale growth model is characterized by a two-dimensional stable saddlepath, which permits the growth rates and the convergence speeds *to vary both across time and variables*. The presence of a two-dimensional stable manifold introduces important

27 Ortigueira and Santos (1997) show how the introduction of adjustment costs in the endogenous growth model reduces the rate of convergence to about 2%.

flexibility to the convergence characteristics, by allowing capital, output, and technology to converge at different time-varying rates toward possibly different long-run equilibrium growth rates. These properties are consistent with Bernard and Jones (1996a, 1996b) who show that different sectors exhibit distinctly different convergence time profiles, suggesting that the process of convergence is more complex than indicated by changes in any single aggregate measure. Furthermore, they show that reasonable asymptotic convergence speeds are achieved with a wide variety of parameter values.

Recent studies have questioned the accuracy of the original benchmark estimates of 2-3%, suggesting that these estimates ignore a number of econometric issues, as a result of which they are downwardly biased. Once one controls for factors such as omitted variables (country specific effects), the endogeneity of the dependent variables, and measurement errors the estimates of the convergence rates both increase above 2% and are much more sensitive to the time period, the set of countries and their stages of development. The new consensus that seems to be emerging is that convergence rates potentially differ significantly across countries and over time, leading to a much wider range of estimates of the convergence rate. For example, using a panel data approach taking account of fixed effects, Islam (1995) estimates the rate of convergence to be 4.7% for nonoil countries and 9.7% for OECD countries. Caselli, Esquivel, and Lefort (1996), use a GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables and obtain a convergence rate of around 10%. Evans (1997) using an alternative method to generate consistent estimates of convergence finds them to be around 6%. Finally, Temple (1998) finds that allowing for measurement error produces estimates of the convergence rate for OECD countries of between 1.5% and 3.6% and for nonoil countries between 0.3% and 6.7%. This finding is also consistent with the time-varying convergence rates generated by the non-scale model.

9. VOLATILITY AND GROWTH

The impact of volatility on economic growth and performance has been widely discussed in recent years. Various aspects of this issue have received both theoretical and empirical attention. Empirical research has focused primarily on cross-country volatility, thus naturally setting the issue within an international context. Three primary sources of volatility have been studied, and the general conclusion is that volatility tends to be harmful to growth. Ramey and Ramey (1995) present evidence to suggest that mean output growth rates are adversely affected by their volatility. Aizenman and Marion (1993, 1999) confirm this result for a set of developing countries, for which they find that volatility also affects private investment negatively. These findings, however, are in contrast to earlier studies by Kormendi and Meguire (1985) and Grier and Tullock (1989) who, using cross-country comparisons, find that higher standard deviations of output growth are associated with higher mean growth rates. Other recent empirical evidence by Gavin and Hausmann (1995) find that when other measures of volatility are included, the volatility of GDP has an insignificant positive impact on output.

A second body of empirical literature examines the relationship between the volatility of policy instruments and the growth rate. Kormendi and Meguire (1985) and Aizenman and Marion (1993) find that output growth is adversely affected by the volatility of monetary policy, while the latter find the same applies for fiscal policy as well. A third and more extensive literature examines the relationship between external sources of volatility and growth. This evidence generally supports the proposition that volatility in the terms of trade and in the real exchange rate adversely affect the growth rate; see Mendoza (1997), Gavin and Hausmann (1995). But other evidence is less conclusive; see Lutz (1994).

The basic growth model we have developed in this paper provides a convenient theoretical framework for analyzing this issue. Stochastic versions have been analyzed by Devereux and Smith (1991), Turnovsky (1993, 1997a), Grinols and Turnovsky (1994), and Obstfeld (1994). These models yield a macroeconomic equilibrium in which the growth rate is related to the various sources of exogenous risk impinging on the economy, and their interaction with policy variables. These models generate an aggregate risk-growth tradeoff, a key determinant of which is the magnitude of the degree of relative risk aversion.

9.1 *A Canonical Model of a Stochastic Small Open Economy*

We consider a small open economy that is specialized in the production of a single good. The economy is inhabited by a representative agent who consumes both the domestically produced good, and a second good that he imports from abroad. The agent supplies L units of labor inelastically, producing output in accordance with the stochastic Cobb-Douglas production function, analogous to (8):

$$dY_i = \alpha(LK)^{\sigma} K_i^{1-\sigma}(dt + dy) \equiv F(dt + dy) \tag{52}$$

where K_i denotes the individual firm's stock of capital and K is the average economy-wide stock of capital, so that (LK) measures the supply of labor in efficiency units. The variable dy is a temporally independent, normally distributed, stochastic process with mean zero and variance $\sigma_y^2 dt$ over the instant dt. Aggregate (average) output is thus represented by:

$$dY = \alpha L^{\sigma} K(dt + dy) \equiv fK(dt + dy) \tag{52'}$$

where $\alpha L^{\sigma} \equiv f$ is constant, thereby yielding endogenous growth.

We assume that the wage rate, a, over the period $(t,\ t+dt)$ is determined at the start of the period and is set equal to the expected marginal physical product of labor over that period, namely

$$a = E\left(\frac{\partial F}{\partial L}\right)_{K_i = K} = \sigma\alpha L^{\sigma-1} K = \frac{\sigma f K}{L}$$

The total rate of return to labor, dA, over the period is thus specified non-stochastically by:

$$dA = a\,dt = E\left(\frac{\partial F}{\partial L}\right)_{K_i = K} dt = \sigma\alpha L^{\sigma-1} K dt = \frac{\sigma f K}{L} dt \tag{53a}$$

The private rate of return on capital, dR_K over the period (t, t+dt), is thus determined residually by

$$dR_K = \frac{d\bar{Y} - LdA}{\bar{K}} \equiv r_K dt + du_K \tag{53b}$$

where

$$r_K = (1-\sigma)f\ ; \qquad du_K \equiv fdy$$

According to this specification, the wage rate, *a*, is fixed over the period (t, t+dt), with all short-run fluctuations in output being reflected in the stochastic return to capital. While this allocation of risk may seem extreme, casual empirical evidence suggests that the returns to capital are far more volatile than are wages[28]. Equations (53a) and (53b) imply, further, that the mean rate of return to capital is constant through time, while the wage rate grows with the equilibrium capital stock.

The agent has access to a world capital market, being able to borrow (or lend) internationally a traded bond, the relative price of which in terms of the traded good, P, is taken as given and is assumed to be generated by the exogenous Brownian motion process:

$$\frac{dP}{p} = \pi dt + dp \tag{54}$$

where π is the instantaneous expected rate of change in the relative price, and dp is a temporally independent, normally distributed, random variable with mean zero and variance $\sigma^2{}_p dt$. The borrowing rate charged on foreign debt, Z, expressed in terms of domestic output, is thus assumed to be of the stochastic form:

$$dR_F = r_F dt + du_F \tag{53c}$$

$$r_F \equiv i^* + \pi\ ;\quad du_F \equiv dp$$

where i* denotes the exogenously given world interest rate.

28 In the US, for example, the relative variability of real stock returns over the period 1955-95 have been around 32% p.a., while the relative variability of wages has been comparable to that of output, 2%.

The representative consumer's real wealth, W, expressed in terms of the domestic good as numeraire is given by:

$$W = K - PZ \tag{55}$$

In addition, over the instant dt he is assumed to purchase output of the traded commodities at the nonstochastic rates $C(t)dt$. For simplicity, we abstract from the government.

The agent's objective is to select his rates of consumption, together with his portfolio of assets, to maximize the expected value of discounted utility[29]

$$\Omega \equiv E\int_0^\infty \frac{1}{\gamma} C^\gamma e^{-\rho t} dt \qquad -\infty < \gamma < 1 \qquad 0 \le \theta \le 1 \tag{56a}$$

subject to the wealth constraint (55) and the stochastic wealth accumulation equation, expressed in terms of the domestic good as:

$$dW = W\left[n_K dR_K - n_F dR_F\right] + aLdt - (C_D + PC_M)dt \tag{56b}$$

where $n_K \equiv K / W$ = share of portfolio held in the form of capital; $n_F \equiv PZ / W$ = share of portfolio held in the form of debt[30].

Performing the optimization, the first order optimality conditions are

$$\frac{C}{W} = \frac{1}{1-\gamma}\left\{\rho - \gamma(n_K r_K - n_F r_F) + (1-\gamma)\left(\frac{aL - \tau}{W}\right) - \frac{1}{2}\gamma(\gamma - 1)\sigma_w^2\right\} \tag{57a}$$

$$\left[r_K - r_F\right]dt = (1-\gamma)\,\text{cov}\left[dw, du_K - du_F\right] \tag{57b}$$

[29] If $n_f > 0$ the country is a borrower; if $n_f < 0$ it is a lender.

[30] For the constant elasticity utility function, the coefficient of relative risk aversion, *r*, and the intertemporal elasticity of substitution, s, are related by $r = 1/s = 1 - \gamma$, so that the exponent γ reflects both these measures. An interesting extension is to generalize the constant elasticity utility function to the recursive utility function, in which case *r* and *s* can be set independently. In the absence of labor income this extension is straightforward, and Obstfeld (1994) provides an example in an international setting. However, it becomes more cumbersome with the introduction of labor income.

where

$$dw = n_K f dy - n_F dp \tag{57c}$$

denotes the stochastic shock to aggregate wealth.

Equation (57a) is the solution for the aggregate consumption-wealth ratio[31]. Equation (57b) expresses the differential between the real rate of return on domestic capital and the cost of borrowing in terms of their relative risk differentials, as measured by the covariance of their returns with the return on the overall portfolio. Solving (57b) in conjunction with the normalized wealth constraint, one can determine the agent's portfolio shares n_K, n_F.

To derive the macroeconomic equilibrium we shall assume that the two stochastic variables, domestic output, dy and the terms of trade shocks, dp, are mutually uncorrelated and from (57c), σ_w^2, σ_{pw} can be computed. Recalling also $du_K = fdy$ and $du_F = dp$, the macroeconomic equilibrium can then be summarized as follows:

$$n_K = \frac{(1-\sigma)f - \left(i^* + \pi\right)}{(1-\gamma)\left[f^2\sigma_y^2 + \sigma_p^2\right]} + \frac{\sigma_p^2}{\left[f^2\sigma_y^2 + \sigma_p^2\right]} \tag{58a}$$

$$n_F = \frac{(1-\sigma)f - \left(i^* + \pi\right)}{(1-\gamma)\left[f^2\sigma_y^2 + \sigma_p^2\right]} - \frac{f^2\sigma_y^2}{\left[f^2\sigma_y^2 + \sigma_p^2\right]} \tag{58b}$$

$$\psi = \frac{1}{1-\gamma}\left\{\left[n_K(1-\sigma)f - n_F(i^* + \pi)\right] - \rho - \frac{1}{2}\gamma(1-\gamma)\sigma_w^2\right. \tag{58c}$$

$$\sigma_w^2 = (n_K f)^2\left[\sigma_y^2 + \sigma_z^2\right] + n_F^2\sigma_p^2 \tag{58d}$$

$$\frac{C}{W} = n_K f - n_F\left[i^* + \pi\right] - \psi \tag{58e}$$

The macroeconomic equilibrium is a stochastic growth path along which all real quantities grow at the common stochastic rate,

$$\frac{dW}{W} = \psi dt + dw \tag{59}$$

This equilibrium has a simple recursive structure. First, equations (58a) and (58b) jointly determine the portfolio shares, $\hat{n}_K$, $\hat{n}_F$, such that the risk-adjusted rate of return to capital equals the risk-adjusted cost of debt. These expressions highlight the two determinants of the optimal portfolio shares. The first is the speculative component, which depends upon the expected differential between the return to capital and borrowing costs, while the second reflects the hedging behavior on the part of the investor and depends upon the relative variances associated with the returns on these two assets. Having obtained the optimal portfolio shares, (58d) determines the equilibrium variance, $\hat{\sigma}_w^2$, along the balanced growth path, with (58c) and (58e) then sequentially determining the equilibrium mean growth rate, $\hat{\psi}$, and consumption-wealth ratio $C\hat{/}W$.[32]

Of particular significance is the welfare of the representative agent as the economy evolves along its stochastic equilibrium growth path. This can be shown to be given by

$$\Omega \equiv E\int_0^\infty \frac{1}{\gamma}C^\gamma e^{-\rho t}dt = \frac{W_0^\gamma\,(C\hat{/}W)^\gamma}{\gamma\left[C\hat{/}W - n_K f(\sigma-g)\right]} \tag{58g}$$

Given the transversality condition, and assuming $C\hat{/}W > 0$, (58g) implies $\gamma\Omega > 0$. Taking the differential of this expression yields

$$d\Omega = (\gamma\Omega)\frac{d(C/W)}{C/W} - \Omega\frac{[d(C/W) - f(\sigma-g)dn_K]}{[C/W - f(\sigma-g)n_K]}$$

Two key elements are seen to influence the agent's welfare. First, a higher C/W ratio raises the current stream of consumption benefits and this is welfare improving. But at the same time it reduces the growth rate, the welfare effect of which depends upon whether $\gamma \gtrless 0$. This latter effect also applies (in reverse) to an increase in n_K.

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[31] In the case of the logarithmic utility function $\gamma = 0$ and in the absence of labor income, (57a) reduces to the familiar relationship $C/W = \rho$.

Equations (58) thus provide the basis for assessing the effect of alternative sources of risk on growth and economic performance. In the one sector closed economy, (58c) implies that the tradeoff between growth and its variability depends upon γ. If $\gamma < 0$ as empirical evidence suggests, then higher variability of output and growth will be associated with a higher mean growth rate. Empirical evidence examining the growth-volatility relationship, although mixed, tends to favor a negative tradeoff; see Ramey and Ramey (1995), Gavin and Hausmann (1995). For an open economy, the risk-growth tradeoff is more complex, involving the international portfolio adjustment as an important component; see Turnovsky (1993).

[32] The equilibrium must satisfy the transversality condition, which for the constant elasticity utility function is given by $\lim_{t \to \infty} E\left[W^{\gamma} e^{-\rho t}\right] = 0$.

10. CONCLUDING COMMENTS

In this paper we have discussed some of the recent developments in growth theory. Many of these developments from the perspective of a small open economy. After setting out a basic generic model, we have shown how it may yield two of the key models that have played a prominent role in the recent literature on economic growth theory, the endogenous growth model and the non-scale growth model. We focused initially on the former, emphasizing how the simplest such model leads to an equilibrium in which the economy is always on its balanced growth path. We have also shown how the endogeneity or otherwise of the labor supply is crucial in determining the equilibrium growth rate and the responsiveness of the equilibrium growth rate to macroeconomic policy.

But transitional dynamics are an important aspect of the growth process and indeed, much of the recent discussion of convergence concerns the speed with which the economy approaches its balanced growth path. We have discussed alternative ways that such transitional dynamics may be introduced. The first is through restricted access to the world capital market in the form of an upward sloping supply curve of debt, an aspect particularly relevant to a developing economy. The introduction of government capital is also an important extension of the basic model that causes the economy to approach its balanced growth path only gradually. Transitional dynamics also characterize the two-sector production model, pioneered by Lucas (1988). In his analysis the two capital goods relate to physical and human capital. In the international context, these naturally can be identified with traded and nontraded capital, respectively.

As we have noted, the endogenous growth model has been the source of criticism, leading to the development of the non-scale model. This too is characterized by transitional dynamics, which are more flexible than those of the corresponding endogenous growth model. The fact that the long-run growth rate is independent of policy in such models does not mean that policy is unimportant. On the contrary, since such models are typically associated with slow convergence speeds (of the order of 2-3%) policy can influence the accumulation of capital for extended periods of time, leading to significant long-run level effects.

As we have tried to emphasize, the models we have been discussing are easily adaptable to a wide range of issues. The last extension we discuss has been to address the impact of volatility on growth. This has been extensively analyzed empirically and a stochastic version of the endogenous growth model provides a convenient framework to interpret this research. Other extensions are also possible. For example, although our analysis has been restricted to real aspects, one can also introduce monetary aspects, including inflation and monetary policy. Some work along these lines has been carried out by Palivos and Yip (1995) for a closed economy and can be adapted, as appropriate to a small international economy.

As noted at the outset, our discussion has been necessarily limited and restricted. We have focused entirely on models where the growth occurs through capital accumulation, emphasizing particularly the role of the technological conditions. We have not addressed issues pertaining to innovation and technology transfer and the related issue of the indigenous development of technology for which the knowledge-based models of Romer (1990), Grossman and Helpman (1991), Eicher (1996), Aghion and Howitt (1998) are particularly relevant.

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