Imperfect information, macroeconomic dynamics and the yield curve: an encompassing macro-finance model

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Imperfect Information, Macroeconomic Dynamics and the Yield Curve: An Encompassing Macro-Finance Model

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Abstract

In this paper we estimate an encompassing Macro-Finance model allowing for time variation in the equilibrium real rate, mispricing and learning dynamics. The encompassing model specification incorporates (i) a small-scale (semi-) structural New-Keynesian model, (ii) flexible price of risk specifications, (iii) liquidity premiums in the form of (constant) deviations from (Gaussian) no-arbitrage and (iv) learning dynamics. This model is estimated on US data using MCMC techniques. We find that the encompassing model outperforms significantly standard Macro-Finance models in terms of marginal likelihood and BIC. Three findings stand out. First, unlike standard Macro-Finance models, a substantial fraction of the variation in long-term yields is attributed to changes in the perceived equilibrium real rate. Second, statistically and economically significant learning effects, especially for inflation expectations, are found. Finally, historical decompositions show that the model can replicate the US yield curve dynamics over the period 1960-2007.

JEL code: E43, E44 and E52

Keywords: Imperfect information, New-Keynesian macroeconomic dynamics, equilibrium real rate, affine yield curve models.

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1 Introduction

The macroeconomic interpretation of the yield curve plays an essential role in analysts' assessment of the current state and outlook of the economy. In particular, (changes in) the yield curve factors such as level, slope and curvature are commonly associated with macroeconomic events and/or developments. For instance, inverted yield curves are typically interpreted as signaling an imminent slow-down or recession through the slope factor (e.g. Estrella and Mishkin, 1996 or Estrella and Trubin, 2005), while so-called inflation scares are inferred from: "A significant long-rate rise in the absence of an aggressive funds rate tightening" (see Goodfriend, 1993 p. 8).

Central in this macroeconomic interpretation of the yield curve is the long-standing idea that a substantial part of the yield curve dynamics can be linked to macroeconomic factors. Recently, this link between macroeconomic factors and the yield curve dynamics has been theoretically formalized and empirically verified by structural Macro-Finance models.[1] This class of models has in common three important modeling features. First, the macroeconomic dynamics are described by means of a (semi-) structural New-Keynesian (NK) macroeconomic model, allowing for a structural identification of the factors and shocks. Standard in these NK models is the assumption of full information rational expectations. Second, the monetary policy rate serves as the main interface between the macroeconomic and the financial model components. In particular, the policy rate set in function of macroeconomic conditions (e.g. in the form of a Taylor-rule) determines to a large extent the short-run money market rates. Finally, no-arbitrage conditions provide the link between the short and the long end of the yield curve. The no-arbitrage conditions used in the Macro-Finance models replace and extend the standard 'expectation hypothesis' by (i) endogenizing the prices of risk and (ii) imposing the 'expectations hypothesis' under the risk-neutral measure only. With the short end of the yield curve being largely determined by the policy rate (and thus by the macroeconomic state), no-arbitrage conditions connect the entire yield curve to the macroeconomic factors.

The main aim of this paper is to extend the Macro-Finance model by introducing an encompassing model. The encompassing model embeds the current set of Macro-Finance models as special cases and, in addition, allows for learning dynamics, liquidity effects and time variation in the equilibrium real interest rate. Time variation in the equilibrium real interest rate is incorporated in the encompassing model by introducing a stochastic natural real rate in the New-Keynesian model. This extension of the structural model is motivated by recent empirical evidence, suggesting substantial volatility and persistence in the equilibrium real rate dynamics (e.g. Laubach and Williams (2003), Clarck and Kozicki (2004), Bjornland et al. (2007), Trehan and Wu (2007)). Liquidity effects are introduced in the encompassing model by allowing for non-zero mispricing terms relative to the Gaussian no-arbitrage model. By construction, such mispricing terms generate arbitrage possibilities which are difficult to justify within a pure Macro-Finance framework. We interpret these mispricing terms as liquidity or preferred habitat effects. Learning dynamics are integrated in the model by relaxing the assumption of full-information rational expectations,

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[1] Versions within this class of models have been studied by e.g. Ang and Piazzesi (2003), Bekaert, Cho and Moreno (2005), Brandt and Yaron (2003), Dewachter and Lyrio (2006), Diebold, Rudebusch and Aruoba (2006), Evans and Marshall (2001), Rudebusch and Wu (2004) and Doh (2006).

as maintained in the benchmark models. We allow for information asymmetries between the central bank and the private agents concerning the long-run trends of the economy. In particular, the private sector is assumed not to observe/believe the inflation target nor the equilibrium real interest rate. Instead agents infer (learn) the latter two variables from observable macroeconomic dynamics. Following Kozicki and Tinsley (2005a), the learning model contains both an endogenous and an exogenous component. The endogenous component is based on constant gain learning. Agents adaptively adjust long-run expectations in function of the endogenous forecast errors. Adaptive, constant gain learning introduces persistent deviations between the actual and perceived law of motions (e.g. Orphanides and Williams (2005) or Kozicki and Tinsley (2005a)).[2] The encompassing model thus potentially explains the observed difficulty of anchoring inflation expectations around the inflation target of the central bank, as acknowledged for instance by Volcker: "With all its built-in momentum and self-sustained expectations, [the inflationary process] has come to have a life on its own".[3] Exogenous belief shocks complement the endogenous learning dynamics. Such belief shocks represent exogenous non-macroeconomic shocks, directly impacting on agents' beliefs. Examples of belief shocks include the so-called 'inflation scares' or confidence and risk appetite shocks.

The contribution of these extensions is assessed by comparing the performance of the encompassing model to that of alternative types of the Macro-Finance model. The encompassing model is compared to two types of benchmark models. The first type of benchmark model assumes the validity of the (extended) expectations hypothesis.[4] Small-scale versions of this type have been estimated by Bekaert et al. (2005), Doh (2006), Wu (2006), while recently De Graeve et al. (2007) have studied a medium-scale version. In general, this type of model fits the data well, despite the maintained assumption of constant risk premiums. The second type of benchmark model is the generally affine version of the structural model. This type of model, introduced in their structural form by Rudebusch and Wu (2004) or Hördahl, Tristani and Vestin (2006), trades off the advantage of flexibility in modeling risk premiums against the loss of full consistency between the (linearized) NK model and the yield curve.[5] The generally affine models are the most flexible and the best performing benchmark Macro-Finance models.

Next to comparing model performances, we interpret the estimation results of the encompassing models in the light of the ongoing debates in the Macro-Finance literature. In particular, we focus on two issues: (i) the decomposition of long-term yield dynamics and (ii) the expectations hypothesis puzzle. First, related to the decomposition of long-term yields, most benchmark Macro-Finance models maintain the assumption of a constant equilibrium real interest rate. Consequently, standard Macro-Finance models attribute, by construction, most of the variation in the long-term yields to the variation in long-run inflation expectations. Explaining the variation in long-term yields only in terms of the

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[2] Recent empirical studies on US data, e.g. Kozicki and Tinsley (2005a) or Dewachter and Lyrio (2008), report substantial differences between the estimated time series for the actual and perceived inflation targets.

[3] As quoted in Orphanides and Williams (2005).

[4] Note that this type of model typically comes in either a fully consistent or a semi-structural version. In the structural version of the model, consistency of the macroeconomic and the yield curve part of the model is imposed. This requires a unique pricing kernel underlying both the IS curve as well as the yield curve. A less restrictive form of this type of model, i.e. the semi-structural form, does not impose consistency across the modeling parts of the Macro-Finance model.

[5] Bekaert et al. (2005) argue that, within the context of a (linearized) NK model, consistency implies constant prices of risk and term premiums.

variation in inflation expectations is, however, not fully satisfactory for two reasons. First, implied inflation expectations are excessively volatile relative to most measures (surveys) of inflation expectations. Also, within a full-information rational expectations context, they imply unrealistically high volatility in the inflation target of the central bank. The encompassing model introduces, next to the long-run inflation expectations, the equilibrium real rate as an additional factor. Consequently, variation in long-term yields is decomposed in either variations in the inflation expectations or in the equilibrium real rate. The estimation results indicate that the equilibrium real rate dynamics explain an important part of the variation of long-term yields. Also, inflation expectations of the encompassing model are better aligned to the survey measures of inflation expectations. Second, we interpret the estimation results of the encompassing model in the context of the ongoing debate concerning the expectations hypothesis puzzle. Empirical tests have consistently rejected the joint null hypothesis of rational expectations and the (extended) expectations hypothesis (see Schiller et al. 1983) for the yield curve. In general, these rejections have been interpreted in terms of the rejection of the expectations hypothesis. Recently, however, Kozicki and Tinsley (2005b) have pointed out that learning dynamics, of the type introduced in this paper, generate sufficiently strong deviations from rational expectations to 'explain' the expectations hypothesis puzzle. The encompassing model, incorporating both learning dynamics and time-varying prices of risk, can be used to identify the relative importance of each argument. The estimation results indicate that both time variation in the prices of risk and learning dynamics are relevant and important features of the data.

All versions of the model are estimated and compared using Bayesian techniques.[6] Although computationally more intensive than FIML, the Bayesian approach has the advantage of integrating informative priors. Theory-consistent priors facilitate estimation and identification of the structural shocks by avoiding 'unreasonable' regions of the parameter space and/or numerical near singularities. The posterior distribution of the parameters is obtained using MCMC methods based on three information sources: macroeconomic variables, the term structure of interest rates and surveys of inflation expectations. The inclusion of survey data in the measurement equation and thus likelihood function is an additional contribution of this paper and is motivated by the need for the identification of the perceived macroeconomic dynamics.[7] Model versions are compared using the marginal likelihoods of the respective models and the BIC criterion.

The remainder of the paper is organized as follows. In Section 2, the general framework of the encompassing Macro-Finance model is explained. In particular, the small-scale semi-structural NK model is described, the learning dynamics are introduced and the implied yield curve and inflation expectations are derived. Section 3 contains the econometric methodology used in the paper, while Section 4 discusses the estimation results. In particular, Section 4 compares the alternative versions of the Macro-Finance model, analyzes the posterior density of the parameters of the encompassing model and, finally, discusses the macroeconomic decomposition of the yield curve. Section 5 concludes by summarizing the main

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[6] The Bayesian approach is not common in the Macro-Finance literature which relies almost entirely on the FIML. Doh (2006, 2007) also uses Bayesian methods.

[7] Surveys of private expectations have been used in other contexts. For instance, Kim and Orphanides (2005) use surveys of interest rate expectations in a latent factor model of the yield curve.

findings of the paper.

2 The model

This section introduces the class of macro-finance models. This class of models is built around (i) a macroeconomic framework, describing the dynamics of observable macroeconomic variables under the historical probability measure and (ii) a financial part, modeling the term structure of interest rates via discounting under the risk neutral probability measure. This section first explains the macroeconomic dynamics. Subsequently, standard arbitrage-free pricing techniques are used to derive the affine representation of the yield curve.

2.1 Macroeconomic dynamics

2.1.1 Structural dynamics

The macroeconomic model is the hybrid New-Keynesian benchmark model incorporating the Phillips curve (AS equation), the IS equation and a monetary policy rule. The benchmark model is extended by introducing two unobserved macroeconomic variables, the inflation target of the central bank, π_t^*, and the output-neutral real interest rate, ρ_t.[8]

The Phillips curve relates current inflation π_t to real marginal costs s_t. By assuming proportionality between real marginal costs on the one hand and the output gap, y_t, and a cost-push shock, $v_{\pi,t}$, on the other, we obtain the standard aggregate supply curve, relating inflation to the output gap[9]:

$$\pi_t = c_{\pi,t} + \mu_{1,\pi} E_t \pi_{t+1} + \mu_{2,\pi} \pi_{t-1} + \kappa y_t + v_{\pi,t}, \tag{1}$$

with

$$v_{\pi,t} = \varphi_\pi v_{\pi,t-1} + \sigma_\pi \varepsilon_{\pi,t}.$$

Microfoundations for this hybrid aggregate supply function are well-established (e.g. Bekaert et al., 2005) and build on the following two assumptions. First, Calvo-pricing is assumed such that in each period only a fraction of firms reoptimizes prices. Second, following Galí and Gertler (1999), we assume that the fraction of firms not reoptimizing prices uses a 'rule-of-thumb' indexation. Denoting this 'rule-of-thumb' indexation scheme by $z_{\pi,t}$, the formal indexation rule becomes:

$$z_{\pi,t} = \pi_t^* + \delta_\pi (\pi_{t-1} - \pi_t^*). \tag{2}$$

This rule stipulates that indexation occurs as a convex combination between the inflation target, π_t^*, and the observed inflation rate, π_{t-1}. The indexation parameter, $0 \leq \delta_\pi \leq 1$, determines the relative

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[8] The introduction of a time-varying equilibrium real rate is motivated by the recent estimates of the equilibrium real rate by Laubach and Williams (2003), Clarck and Kozicki (2004) and Bjornland et al. (2007). These authors report significant time variation in the equilibrium real rate in the US. Also, as discussed in Trehan and Wu (2007), accounting for the time variation in equilibrium real rates is important in the analysis of monetary policy. In this paper we use a short-cut by introducing the equilibrium real rate as a purely exogenous process. This exogenous process captures persistent shocks in productivity, preferences, fiscal policy or financial premiums.

[9] The output gap is defined as the difference between total GDP and potential GDP. Positive gaps thus refer to output *above* potential, while negative values refer to output *below* potential.

weight assigned to respectively the inflation target and the past inflation rate. The higher δ_π, the higher the weight on past realized inflation and the lower the weight of the inflation target in the inflation indexation rule. The above assumptions allow for a structural decomposition of the parameters of the aggregate supply curve in function of the deep parameters, δ_π, the discount factor, β, and the central bank inflation target, π_t^* :[10]

$$\begin{gathered} c_{\pi,t} = (1-\mu_{1,\pi}-\mu_{2,\pi})\pi_t^*, \\ \mu_{1,\pi} = \frac{1}{1+\delta_\pi},\ \mu_{2,\pi} = \frac{\delta_\pi}{1+\delta_\pi}. \end{gathered} \tag{3}$$

A Fuhrer-type of IS equation is used. By introducing a utility function incorporating (external) habit formation, we allow for endogenous inertia in the output gap dynamics. More specifically, we assume that agents maximize expected utility:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, F_s),\ \text{with}\ \ U(C_s,F_s) = \frac{F_s C_s^{1-\sigma}-1}{1-\sigma}, \tag{4}$$

with C_s consumption and F_s a combined factor consisting of preference shocks, G_s, and habit, H_s: $F_s = G_s H_s$. We furthermore assume a standard habit formation function, specifying the habit as a function of past consumption: $H_s = C_{s-1}^{\eta}$, $\eta = h(\sigma-1)$ and $0 \leq h \leq 1$. Solving the maximization under the standard budget and resource constraints delivers the hybrid IS equation:

$$y_t = \mu_y E_t y_{t+1} + (1-\mu_y) y_{t-1} - \phi(i_t - E_t \pi_{t+1} - \rho_t) + v_{y,t}, \tag{5}$$

where ρ_t represents the output neutral real interest rate and $v_{y,t} = \phi \ln G_t$ follows a first order autoregressive process:

$$v_{y,t} = \varphi_y v_{y,t-1} + \sigma_y \varepsilon_{y,t}. \tag{6}$$

Equation (5) formally introduces the output-neutral real rate, ρ_t. This rate is implicitly defined as the long-run equilibrium level of the real interest rate. In particular, as follows from equation (5), ex ante real rates $(i_t - E_t\pi_{t+1})$ above the output-neutral real rate, ρ_t, lead to a decline in output while ex ante real rates below the output neutral real rate are in general expansionary. The parameters of the hybrid IS curve, μ_y and ϕ can be interpreted in terms of the structural parameters, i.e. relative risk aversion σ and habit persistence h :

$$\mu_y = \frac{\sigma}{\sigma + h(\sigma-1)},\ \phi = \frac{1}{\sigma+h(\sigma-1)}. \tag{7}$$

The macroeconomic model is closed by modeling the (risk-free) monetary policy interest rate, i_t, in terms of an extended Taylor rule. Extended Taylor rules formalize monetary policy (up to a policy shock) as a convex combination of the previous policy rate, i_{t-1}, and the target interest rate i_t^T:

$$i_t = (1-\gamma_i) i_t^T + \gamma_i i_{t-1} + v_{i,t}, \tag{8}$$

with

$$v_{i,t} = \varphi_i v_{i,t-1} + \sigma_i \varepsilon_{i,t}. \tag{9}$$

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[10] In the remainder of the paper we assume a vertical Phillips curve in the long run. Within the class of empirical New-Keynesian models this assumption is usually imposed by restricting the discount factor to 1, $\beta \approx 1$. Note also that given that $\mu_{1,\pi} = (1-\mu_{2,\pi})$ we refer to $\mu_{1,\pi}$ as μ_π.

Following the Taylor-rule literature, the target interest rate is modeled in function of both the inflation and the output gaps:

$$i_t^T = \rho_t + E_t\pi_{t+1} + \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y y_t. \tag{10}$$

Our specification of the target interest rate rule ensures that for $\gamma_\pi > 0$ and/or $\gamma_y > 0$ central banks follow an activist policy. For $\gamma_\pi > 0$ and $\gamma_y > 0$, the targeted ex ante real interest rate $(i_t^T - E_t\pi_{t+1})$ increases above (decreases below) the neutral real interest rate level, ρ_t, in function of positive (negative) inflation and/or the output gaps:

$$i_t^T - E_t\pi_{t+1} - \rho_t = \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y y_t. \tag{11}$$

The specification of the interest rate target implies a second (equivalent) interpretation of the neutral real rate, ρ_t, as the real interest rate target of the central bank. As shown in equation (10), the central bank aims at a steady state real rate target ($\pi_t = \pi_t^*$, $y_t = 0$) equal to ρ_t. In the remainder of the paper we use both interpretations of ρ_t, i.e. as the output-neutral rate or as the real rate target of the central bank, interchangeably.

Finally, the dynamics of the neutral real interest rate, ρ_t, and the inflation target, π_t^*, are modeled as random walks:

$$\begin{aligned} \pi_t^* &= \pi_{t-1}^* + \sigma_{\pi^*}\varepsilon_{\pi^*,t} \\ \rho_t &= \rho_{t-1} + \sigma_\rho\varepsilon_{\rho,t}. \end{aligned} \tag{12}$$

This random walk specification is particularly important as it introduces stochastic endpoints for inflation and the risk-free interest rate in the model.[11] Under the random walk dynamics, π_t^* can be interpreted as the inflation target of the central bank and ρ_t as the real rate target. Equivalently, π_t^* and ρ_t are the stochastic endpoints since, by construction, inflation (π_t) and the real rate ($i_t - \pi_t$) converge (in expectation) towards π_t^* and ρ_t, respectively ($\lim_{s\to\infty} E_t\pi_{t+s} = \pi_t^*$ and $\lim_{s\to\infty} E_t i_{t+s} = \pi_t^* + \rho_t$).

2.1.2 Subjective expectations

We follow and extend the approach taken in Kozicki and Tinsley (2005a) or Doh (2007) by explicitly differentiating between the beliefs of private agents and those held by the central bank. Differences in beliefs arise either as a consequence of imperfectly informed private agents or, alternatively, as a consequence of imperfect credibility of the central bank policy and/or announcements (see Kozicki and Tinsley, 2005a). We restrict the imperfect information to the (unobserved) long-run tendencies (stochastic trends/endpoints) of the economy, i.e. the inflation target and the output-neutral real interest rate. Formally, imperfect information is introduced by differentiating between stochastic trends as perceived

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[11] As shown by Kozicki and Tinsley (2001), stochastic endpoints are crucial in modeling the link between macroeconomic variables and the yield curve. While most Macro-Finance models limit the number of stochastic endpoints to one, i.e. the inflation target, we allow for two endpoints. In the data section, it is shown that long-run inflation expectations, identified by survey data, cannot account for the time variation in long-term yields. One interpretation of this finding is that the unexplained variation in yields is due to a persistent real rate factor. This interpretation is followed in this paper and entails the introduction of a second stochastic endpoint.

by private agents, π_t^{*P} and ρ_t^P, and actual stochastic trends (as inferred by the central bank), π_t^* and ρ_t. Conditional on the perceived stochastic endpoints, ρ_t^P and π_t^{*P} agents form expectations rationally.[12]

Perceived stochastic endpoints evolve in function of both endogenous forecast errors (constant gain learning) and exogenous (belief) shocks. Specifically, following Kozicki and Tinsley (2005a), we posit updating rules for the perceived stochastic endpoints as:

$$\begin{aligned} \pi_t^{*P} &= \pi_{t-1}^{*P} + \omega_\pi \sigma_{\pi^*} \varepsilon_{\pi^*,t} + (1-\omega_\pi)\left[\sigma_{\pi^{*b}} \eta_{\pi,t} + g_\pi \left(\pi_t - E_{t-1}^P \pi_t\right)\right], \\ \rho_t^P &= \rho_{t-1}^P + \omega_\rho \sigma_\rho \varepsilon_{\rho,t} + (1-\omega_\rho)\left[\sigma_{\rho^b} \eta_{\rho,t} + g_\rho \left(i_t - \pi_t - E_{t-1}^P (i_t - \pi_t)\right)\right], \end{aligned} \tag{13}$$

with initial conditions π_0^{*P} and ρ_0^P. The expectation E^P denotes the subjective expectations operator. The updating rule implies two alternative sources of information: (i) observed shocks (public signals) to the (actual) stochastic trends ε_{π^*} and ε_ρ on the one hand and (ii) subjective inferences (private signals) for the (changes in the) stochastic trends, i.e. $\sigma_{\pi^{*b}}\eta_{\pi,t} + g_\pi(\pi_t - E_{t-1}^P\pi_t)$ and $\sigma_{\rho^b}\eta_{\rho,t} + g_\rho(i_t - \pi_t - E_{t-1}^P(i_t - \pi_t))$ on the other. The private signals are composed of two shocks: (i) exogenous belief shocks (η_π, η_ρ) and (ii) endogenous and adaptive revisions (with gains g_π and g_ρ) of the perceived endpoints induced by prediction errors for inflation and the ex post real interest rate, ($g_\pi\left(\pi_t - E_{t-1}^P\pi_t\right)$ and $g_\rho\left(i_t - \pi_t - E_{t-1}^P(i_t - \pi_t)\right)$). The parameters ω_π and ω_ρ measure the implicit weight attached to each of these signals. The higher ω_π and ω_ρ, the more weight is attached to the public signals ('true' shocks) in the updating of the stochastic trends. The parameters ω_π and ω_ρ are interpreted as indices measuring the subjectively perceived quality of information contained in the public signals.

The learning model (equation (13)) incorporates a number of standard expectations formation processes as special cases. First, the full-information, rational expectations case (RE) is obtained by assuming full information with respect to the stochastic endpoints. In terms of the learning rule, RE implies (i) that agents perceive the signals, ε_{π^*} and ε_ρ, as fully informative, such that the true shocks are fully incorporated in the perceived target, i.e. $\omega_\pi = 1$ and $\omega_\rho = 1$ and (ii) that agents have perfect information on the initial state: $\pi_0^* = \pi_0^{*P}$ and $\rho_0^P = \rho_0$. In this paper we refer to the full-information, rational expectations case as the learning model with $\omega_\pi = \omega_\rho = 1$, while the alternative, $\omega_\pi < 1$, $\omega_\rho < 1$ is referred to as the imperfect information case. Second, standard constant gain learning rules (where agents update stochastic endpoints adaptively, e.g. Dewachter and Lyrio (2008)) are obtained by setting $\omega_\pi = 0$, $\omega_\rho = 0$ and $\sigma_{\rho^b} = \sigma_{\pi^{*b}} = 0$. Agents thus attach no informative value at all to the public signals and only update in function of prediction errors. Finally, the updating rule introduced by Kozicki and Tinsley (2005a) is recovered by eliminating the actual shocks from the updating equation: $\omega_{\pi^*} = \omega_\rho = 0$.

2.1.3 Actual and Perceived Law of Motion

The model introduced in equations (1), (5), (8), (12) and (13) can be written and solved in a state space framework. We collect observable macroeconomic variables in the data vector, X_t^o, $X_t^o = [\pi_t,\ y_t,\ i_t,\ v_{\pi,t},\ v_{y,t},\ v_{i,t}]'$, actual stochastic trends in X_t^c, $X_t^c = [\pi_t^*, \rho_t]'$, and perceived trends in X_t^{cP}, $X_t^{cP} = [\pi_t^{*P}, \rho_t^P]'$. Analogous groupings are formed for the shocks $\varepsilon_t = [\varepsilon_{\pi,t},\ \varepsilon_{y,t},\ \varepsilon_{i,t}]'$, $\eta_t = [\eta_{\pi,t}, \eta_{\rho,t}]'$

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[12] We thus assume that agents know the structural dynamics and observe the macroeconomic variables π_t, y_t and i_t as well as the exogenous factors $v_{\pi,t}$, $v_{y,t}$ and $v_{i,t}$.

and $\varepsilon_t^c = [\varepsilon_{\pi^*,t}, \varepsilon_{\rho,t}]'$. The structural dynamics discussed in section 2.1.1 and defined in equations (1), (5), (8) and (12) can be rewritten using the system matrices A, B, C, D and Σ^o and Σ^c:

$$\begin{aligned} AX_t^o &= C + BE_t\left[X_{t+1}^o\right] + DX_{t-1}^o + FX_t^c + \Sigma^o\varepsilon_t, \\ X_t^c &= X_{t-1}^c + \Sigma^c\varepsilon_t^c, \end{aligned} \tag{14}$$

with F defined as: $(A - B - D)H$ and H a matrix containing the cointegrating relations, i.e. the dependence of X_t^o on X_t^c.[13] The full-information, rational expectations solution (conditional on X_t^c) can be written as a reduced form VAR and summarizes the dynamics under rational expectations:

$$\left[\begin{array}{c} X_t^o \\ X_t^c \end{array}\right] = \left[\begin{array}{c} C^{RE} \\ 0 \end{array}\right] + \left[\begin{array}{cc} \Phi^{RE} & (I - \Phi^{RE})H \\ 0 & I \end{array}\right]\left[\begin{array}{c} X_{t-1}^o \\ X_{t-1}^c \end{array}\right] + \left[\begin{array}{cc} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} \\ 0 & \Sigma^c \end{array}\right]\left[\begin{array}{c} \varepsilon_t \\ \varepsilon_t^c \end{array}\right]. \tag{15}$$

The Perceived Law of Motion (PLM) formalizes the macroeconomic dynamics as perceived by private agents. As argued in the previous section, agents have (conditional on the perceived stochastic endpoints) rational expectations. The PLM is thus obtained as the RE solution to the structural equations, replacing actual stochastic endpoints (X_t^c) by their perceived counterparts (X_t^{cP}). It can be shown that the resulting PLM can be written as[14]:

$$\left[\begin{array}{c} X_t^o \\ X_t^{cP} \end{array}\right] = \left[\begin{array}{c} C^{RE} \\ 0 \end{array}\right] + \left[\begin{array}{cc} \Phi^{RE} & (I - \Phi^{RE})H \\ 0 & I \end{array}\right]\left[\begin{array}{c} X_{t-1}^o \\ X_{t-1}^{cP} \end{array}\right] + \left[\begin{array}{cc} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} \\ 0 & \Sigma^{cP} \end{array}\right]\left[\begin{array}{c} \varepsilon_t \\ \varepsilon_t^{cP} \end{array}\right]. \tag{16}$$

The Actual Law of Motion (ALM) is obtained (i) by substituting the expectations $E_t\left[X_{t+1}^o\right]$ by the subjective expectations $E_t^P\left[X_{t+1}^o\right]$, as implied by the PLM, into the structural equations, system (14):

$$\begin{aligned} (A - B\Phi^{RE})X_t^o &= C + B(C^{RE}) + B(I - \Phi^{RE})HX_t^{cP} + DX_{t-1}^o + FX_t^c + \Sigma^o\varepsilon_t, \\ X_t^c &= X_{t-1}^c + \Sigma^c\varepsilon_t^c, \end{aligned} \tag{17}$$

and (ii) by taking into account the dynamics of the subjective beliefs, i.e. the dynamics of the perceived stochastic endpoints:

$$X_t^{cP} = X_{t-1}^{cP} + W\Sigma^c\varepsilon_t^c + (I - W)\left[\Sigma_\eta\eta_t + GV(X_t^o - E_{t-1}^P\left[X_t\right]\right], \tag{18}$$

where equation (18) is the state-space representation of the updating rules specified in equation (13).[15] This ALM (i.e. the system consisting of equations (17) and (18)) can subsequently be summarized with respect to the full state vector $X_t = [X_t^{o\prime}, X_t^{cP\prime}, X_t^c]'$ and the shock vector $\upsilon_t = [\varepsilon_t', \eta_t', \varepsilon_t^{c\prime}]'$as:

$$X_t = C^A + \Phi^A X_{t-1} + \Gamma^A S^A \upsilon_t, \tag{19}$$

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[13] Formally, H is defined such that:

$$\lim_{s\to\infty} E_t\left[X_{t+s}^o\right] = HX_t^c.$$

In this model, the matrix H is given by:

$$H' = \left[\begin{array}{cccccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right]$$

[14] Note that, in line with the learning literature, it is assumed that agents ignore the dynamics of the updating rule (Kreps's anticipated utility assumption). Specifically, in solving the model, agents do not take into consideration the implications of current forecast errors on subsequent inferences of the stochastic endpoints. Instead, we assume agents regard the (perceived) stochastic endpoints as purely *exogenous* martingale processes, with impact matrix Σ^{cP}.

[15] The matrices W, G and Σ_η are diagonal matrices containing respectively the weights ω_π and ω_ρ, the gains g_π and g_ρ and the standard deviations $\sigma_{\pi^{*b}}$ and σ_{ρ^b}. Finally, V is a transformation matrix selecting from $(X_t^o - E_{t-1}^P(X_t^o))$ the inflation and real interest forecast error, respectively. The expectation E_{t-1}^P is the conditional expectation under the PLM.

with

$$C^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I - \Phi^{RE})H & -F \\ -(I - W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} (C + BC^{RE}) \\ -(I - W)GVC^{RE} \\ 0 \end{bmatrix},$$

$$\Phi^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I - \Phi^{RE})H & -F \\ -(I - W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} D & 0 & 0 \\ -(I - W)GV\Phi^{RE} & I - (I - W)GV(I - \Phi^{RE})H & 0 \\ 0 & 0 & I \end{bmatrix},$$

$$\Gamma^A S^A = \begin{bmatrix} (A - B\Phi^{RE}) & -B(I - \Phi^{RE})H & -F \\ -(I - W)GV & I & 0 \\ 0 & 0 & I \end{bmatrix}^{-1} \begin{bmatrix} \Sigma^o & 0 & 0 \\ 0 & (I - W)\Sigma_\eta & W\Sigma^c \\ 0 & 0 & \Sigma^c \end{bmatrix}.$$

Analogously, the Perceived Law of Motion, equation (16) can be stated in the state space form:

$$X_t = C^P + \Phi^P X_{t-1} + \Gamma^P S^P v_t^P, \tag{20}$$

with

$$C^P = \begin{bmatrix} C^{RE} \\ 0 \\ 0 \end{bmatrix}, \Phi^P = \begin{bmatrix} \Phi^{RE} & (I - \Phi^{RE})H & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Gamma^P S^P = \begin{bmatrix} \Sigma_{1,1}^{RE} & \Sigma_{1,2}^{RE} & 0 \\ 0 & \Sigma^{cP} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Note, finally, that the specific assumptions made in section 2.1.1 imply that all the vectors of constants are zero both in the ALM and PLM, i.e. $C^A = C^P = 0$.

Introducing learning dynamics in the model has a significant impact on the macroeconomic dynamics (ALM). We mention three. First, with subjective expectations conditioned on perceived stochastic trends, the 'expectations channel' implies an impact of perceived trends on the actual macroeconomic outcome. Formally, in the ALM, equation (19), with $B \neq 0$ (implying the existence of an expectations channel) perceived trends, X_t^{cP}, affect actual macroeconomic outcomes X_t^o, see equation (17). Second, exogenous belief shocks, i.e. $\eta_{\pi,t}$ and $\eta_{\rho,t}$, also affect macroeconomic outcomes. The macroeconomic impact of these shocks depends on the information value attributed to these shocks. The more informative content is attributed to these shocks, i.e. the higher is $I - W$, the stronger the impact of exogenous belief shocks. Exogenous belief shocks are neutralized only in the full information rational expectations case, $W = I$. Finally, constant gain learning, $(I - W)G > 0$, implies that actual and perceived stochastic trends are linked. There is a feedback from the macroeconomic dynamics, including the actual stochastic endpoints, to the perceived endpoints. Simulations show that for a large range of parameters, cointegration between the actual and perceived stochastic endpoints is observed. The cointegrating relation between actual and perceived endpoints is generated by the endogenous component in the updating rule, i.e. the constant gain learning.[16] Intuitively, if perceived endpoints are lower (higher) than actual endpoints, agents on average underpredict (overpredict) the inflation rate and the real interest rate, resulting in positive (negative) prediction errors. With adaptive learning (implying positive gains) on average positive (negative)

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[16] Formally, the cointegration between perceived and actual endpoints requires adaptive learning, i.e. $G > 0$. In case no endogenous updating occurs, i.e. $G = 0$, cointegration does not hold since the perceived endpoints also depend on the independent belief shocks.

prediction errors feed into the perceived endpoints and generate a trend-wise convergence of perceived to actual stochastic trends. In the estimation, we impose convergence of the perceived trends towards the actual stochastic endpoints.[17]

2.2 The term structure of interest rates

In this section, we derive the closed-form solution for the bond prices as implied by the Perceived Law of Motion, equation (20). Solving for bond prices under the PLM explicitly recognizes the fact that bond prices are determined in financial markets by private agents, using subjective expectations w.r.t. the macroeconomic dynamics. We follow Ang and Piazzesi (2003) and solve for the affine yield curve representation. To this end, we assume a log-normal stochastic discount factor, M_{t+1} :

$$M_{t+1} = \exp(-i_t - \tfrac{1}{2}\Lambda_t' S^P S^{P\prime} \Lambda_t - \Lambda_t' S^P v_{t+1}^P), \tag{21}$$

with prices of risk, Λ_t, linear in the state variable X_t :

$$\Lambda_t = \Lambda_0 + \Lambda_1 X_t. \tag{22}$$

Imposing the no-arbitrage conditions on (zero-coupon) bond prices with time to maturity τ, $p_t(\tau)$, implies that the bond prices satisfy the equilibrium pricing condition:

$$p_t(\tau) = E_t^P \left[M_{t+1} p_{t+1}(\tau - 1) \right]. \tag{23}$$

Given the above conditions, closed form solutions for bond prices exist and belong to the exponentially affine class:

$$p_t(\tau) = \exp(a(\tau) + b(\tau) X_t), \tag{24}$$

where the price loadings are determined by the no-arbitrage restrictions implicit in the equilibrium condition (equation (23)):

$$\begin{aligned} a(\tau) &= a(\tau-1) + b(\tau-1)(C^P - \Gamma^P S^P S^{P\prime} \Lambda_0) + \tfrac{1}{2} b(\tau-1) \Gamma^P S^P S^{P\prime} \Gamma^{P\prime} b'(\tau-1) - \delta_0, \\ b(\tau) &= b(\tau-1)(\Phi^P - \Gamma^P S^P S^{P\prime} \Lambda_1) - \delta_1', \end{aligned} \tag{25}$$

with the nominal risk-free rate identified as: $i_t = \delta_0 + \delta_1' X_t$ and initial conditions $a(0) = 0$ and $b(0) = 0$.

Exponentially affine bond prices imply affine yield curves. Noting that the time t yield on a zero coupon bond with maturity τ, $y_t(\tau)$, is defined as $y_t(\tau) = -\ln(p_t(\tau))/\tau$, the no-arbitrage yield curve becomes linear in the state vector:

$$y_t(\tau) = A(\tau) + B(\tau) X_t, \tag{26}$$

with yield loadings $A(\tau)$ and $B(\tau)$ defined as $A(\tau) = -a(\tau)/\tau$ and $B(\tau) = -b(\tau)/\tau$. Finally, the no-arbitrage yield curve is extended by including (i) a time-invariant liquidity premium $\phi(\tau)$ and (ii) a measurement error component $\eta_{y,t}$. The final yield curve representation is given by

$$y_t(\tau) = A(\tau) + B(\tau) X_t + \varphi(\tau) + \eta_{y,t}(\tau), \tag{27}$$

---

[17] Denoting the expectations generated under the ALM by E_t^A we impose:

$$\lim_{s \to \infty} E_t^A \left[X_{t+s}^{cP} \right] = X_t^c.$$

with $\phi(1/4) = 0$, i.e. no liquidity premium on the one period bond, and with all measurement errors normally distributed (i.i.d.) with mean zero and maturity specific variance: $E\left[\eta_{y,t}(\tau)\right] = 0$ and $E\left[\eta_{y,t}(\tau)^2\right] = \sigma^2_{\eta,y}(\tau)$.

The yield curve representation in equation (26), although satisfying the no-arbitrage conditions, is not necessarily consistent with the macroeconomic part of the model, as noted by Wu (2006) or Bekaert et al. (2005). Full consistency, as imposed by structural Macro-Finance models, implies additional restrictions on the prices of risk, aligning the pricing kernel, as defined in equation (21), to the IS curve (equation (5)). In particular, within the linearized macroeconomic representation, consistency implies the extended expectations hypothesis, with prices of risk constrained to values implied by the IS curve:[18]

$$\Lambda_0 = \Lambda_0^{IS}, \ \Lambda_1 = 0. \tag{28}$$

Note, however, that imposing consistency within the linearized and homoskedastic macroeconomic framework implies constant risk premiums. In particular, the expected excess holding return, ehr_t, is in general affine in the state vector, X_t :

$$\begin{aligned} ehr_t &= E_t^P \ln(p_{t+1}(\tau-1) - p_t(\tau)) \\ &= b(\tau-1)\Gamma^P S^P S^{P\prime}\Lambda_0 + b(\tau-1)\Gamma^P S^P S^{P\prime}\Lambda_1 X_t && (29) \\ & && (30) \\ &\quad -\frac{1}{2} b(\tau-1)\Gamma^P S^P S^{P\prime}\Gamma^{P\prime} b(\tau-1)'. \end{aligned}$$

Therefore, imposing consistency, i.e. $\Lambda_1 = 0$, necessarily implies constant risk premiums and the expectations hypothesis. In the empirical sections, we distinguish between 'structural' versions of the model, imposing no-arbitrage and consistency, i.e. equation (28), and 'non-structural' models, only imposing no-arbitrage conditions and allowing for flexible price of risk (and risk premium) specifications, i.e. equation (22).

2.3 Subjective inflation expectations

The PLM can be used to generate subjective expectations for each of the macroeconomic variables. Given the linear structure of the PLM, these expectations are affine functions of the current state vector. Denoting the time-t (subjective) expectation of the macroeconomic state at time $t+\tau$ by $E_t^P\left[X_{t+\tau}\right]$:

$$E_t^P\left[X_{t+\tau}\right] = A_s(\tau) + B_s(\tau)X_t, \tag{31}$$

where the expectation is derived from the PLM, equation (20). The affine loadings $A_s(\tau)$ and $B_s(\tau)$ can be obtained from the system of difference equations:

$$\begin{aligned} A_s(\tau) &= \Phi^P A_s(\tau-1) + C^P, \\ B_s(\tau) &= \Phi^P B_s(\tau), \end{aligned} \tag{32}$$

---

[18] A derivation along the lines of Bekaert et al. (2005) shows that for this model the consistent prices of risk are:

$$[1, \sigma, 0, 0, 0, 0, 0, 0, 0, 0]\Gamma^P - [0, \phi/(1-\rho_g), 0, 0, \frac{\sigma}{\sigma + \sigma(1-h)}, 0, 0]$$

with initial conditions $A_s(0) = 0$ and $B_s(0) = I$.

In the empirical part of the paper, we use the Survey of Professional Forecasters (SPF) surveys of inflation expectations to identify subjective expectations. This survey reports subjective *average* inflation expectations over one and ten year horizons. Average inflation expectations over the horizon τ can be obtained by averaging the period-by-period inflation forecasts over the prediction horizon. Denote the subjective expectations for average inflation over the horizon τ by $S(\tau)$. The PLM-implied estimate for $S(\tau)$ is given by:

$$S(\tau) = A_{s,\bar{\pi}}(\tau) + B_{s,\bar{\pi}}(\tau)X_t + \eta_{s,t}(\tau), \tag{33}$$

with

$$\begin{aligned} A_{s,\bar{\pi}}(\tau) &= \frac{1}{\tau}\sum_{j=0}^{\tau-1} e_\pi A_s(\tau), \\ B_{s,\bar{\pi}}(\tau) &= \frac{1}{\tau}\sum_{j=0}^{\tau-1} e_\pi B_s(\tau), \end{aligned} \tag{34}$$

where e_π denotes the selection vector selecting inflation from the state vector X_t. Note that we allow for (normally distributed) maturity-specific measurement errors with mean zero and standard deviation $\sigma_{\eta,\pi}(\tau)$.

3 Econometric methodology

3.1 Posterior distribution

The econometric analysis focuses on (i) estimating the parameters of the encompassing model and on (ii) evaluating the performance of the encompassing model relative to standard benchmark models used in the Macro-Finance literature. Both estimation and model comparison is done within a Bayesian framework. To this end, posterior densities and marginal likelihoods for the each of the alternative model versions are computed.

Denoting the data set used in the econometric analysis by Z^T and the parameter vector for model version i by θ_i, the posterior density of θ_i, $p(\theta_i \mid Z^T)$, is given by:

$$p(\theta_i \mid Z^T) = \frac{L(Z^T \mid \theta_i)p(\theta_i)}{p(Z^T)}, \tag{35}$$

with $p(\theta_i)$ the prior for model version i, $L(Z^T \mid \theta_i)$ the likelihood function and $p(Z^T)$ the marginal likelihood of Z^T (given version i of the model). The likelihood function of the data is constructed under the Actual Law of Motion, treating the latent factors, π_t^*, ρ_t, π_t^{*P} and ρ_t^P, as unobserved variables to the econometrician. Specifically, the likelihood function is constructed based on the prediction errors identified by the measurement equation, with the state space representation of the ALM dynamics serving as transition equation:

$$X_t = C^A(\theta_i) + \Phi^A(\theta_i)X_{t-1} + \Gamma^A(\theta_i)S^A(\theta_i)v_t, \; v_t \sim N(0, I). \tag{36}$$

The measurement equation combines three types of information: (i) the information contained in the observable macroeconomic variables π_t, y_t and i_t, (ii) yield curve information incorporated in the set of yields $y_t(\tau_i)$, $i = 1, ..., n_y$ and (iii) information extracted from surveys of inflation expectations $S(\tau_i)$, $i = 1, .., n_s$. The measurement equation, implied by equations (27) and (33), is affine in the state vector X_t :

$$Z_t = A_Z(\theta_i) + B_Z(\theta_i)X_t + \mu_Z(\theta_i) + \Sigma_Z(\theta_i)\eta_{Z,t},\ \eta_{Z,t} \sim N(0, I), \tag{37}$$

with $Z_t = [\pi_t,\ y_t,\ i_t,\ y_t(\tau_1), ..., y_t(\tau_{n_y}),\ S_t(\tau_1), ..., S_t(\tau_{n_s})]'$. The vector μ_Z contains the liquidity effects in the yield curve, $\phi(\tau_i)$. The variance-covariance matrix of the measurement errors, $\Omega_Z = \Sigma_Z\Sigma_Z'$ is diagonal and singular. We assume strictly positive standard deviations for the measurement errors of all yields and survey data of inflation expectations. However, we impose that the macroeconomic variables contained in Z_t are observed without measurement error, leading to the singularity of Ω_Z.

The likelihood function is obtained by integrating out the unobserved latent factors using the Kalman filter. The log-likelihood, $l(Z^T \mid \theta_i)$, implied by the Kalman filter is:

$$l(Z^T \mid \theta_i) = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\ln(\left|V_{Z,t|t-1}\right|) + (Z_t - Z_{t|t-1})'\left(V_{Z,t|t-1}\right)^{-1}(Z_t - Z_{t|t})\right], \tag{38}$$

with the prediction and updating equations for the mean (for brevity the dependence of system matrices on the parameter vector θ_i is suppressed) :

$$\begin{aligned} Z_{t|t-1} &= A_Z + B_Z X_{t|t-1} + \mu_Z, \\ X_{t|t-1} &= C^A + \Phi^A X_{t-1|t-1}, \\ X_{t|t} &= X_{t|t-1} + K_{t|t-1}(Z_t - Z_{t|t-1}), \end{aligned} \tag{39}$$

and for the variance V_Z :

$$\begin{aligned} V_{Z,t|t-1} &= B_Z P_{t|t-1} B_Z' + \Omega_Z, \\ P_{t|t-1} &= \Phi^A P_{t-1|t-1}\Phi^{A\prime} + \Gamma^A S^A S^{A\prime}\Gamma^{A\prime}, \\ P_{t|t} &= (I - K_{t|t-1}B_Z)P_{t|t-1}, \end{aligned} \tag{40}$$

with Kalman gain:

$$K_{t|t-1} = P_{t|t-1}B_Z'(B_Z P_{t|t-1}B_Z' + \Sigma_Z\Sigma_Z')^{-1}. \tag{41}$$

The posterior density of θ_i, $p(\theta_i \mid Z^T)$ is, in general, not known in closed form. We use MCMC methods, and in particular the Metropolis-Hastings algorithm, to simulate draws from the posterior. We follow the standard two-step procedure. First, a simulated annealing procedure is used to find the mode of the posterior.[19] In a second step, the Metropolis-Hastings procedure is used to trace the posterior

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[19] The following additional conditions are imposed throughout the estimation. We impose determinacy on the solution of the rational expectations models. In case of the learning models, we impose non-explosive behavior on the ALM, by excluding explosive roots. With respect to the Kalman filter, we initialize the Kalman filter by estimating the initial values of the unobserved variables. For the initial variance-covariance matrix of the Kalman filter, we solve for the steady state of the Riccati equation. This steady state exists given the imposed cointegration of the latent factors with the observed macroeconomic factors.

density of θ_i.[20]

Given the likelihood, $L(Z^T \mid \theta_i)$, and the priors of the respective versions, $p(\theta_i)$, the marginal likelihood of the data given the model version i is defined by:

$$p(Z^T) = \int_{\theta_i} L(Z^T \mid \theta_i) p(\theta_i) d\theta_i. \tag{42}$$

The marginal likelihood is used to evaluate the relative performance of the alternative versions of the model. Additionally, we use the BIC criterion as a complementary statistic to compare model performance.

3.2 Model versions

In order to evaluate the encompassing model, a formal comparison to typical Macro-Finance models is performed. In particular, we retain four *types* of Macro-Finance models: the Benchmark models, the Liquidity premium or Mispricing models, the Flexible model and the Encompassing model. Table 1 summarizes the main differences across the model types. Each of these versions is obtained as a special case of the encompassing model.

Insert Table 1

Benchmark models MFS. The benchmark version, labeled the MFS (Macro-Finance Structural), is the fully consistent, full-information rational expectations version of the Macro-Finance model. The model is based on a unique pricing kernel underlying both the NK model and the yield curve, imposes no-arbitrage on the yield curve and endows agents with (full information) rational expectations. As indicated in the Introduction, this model was introduced by Wu(2006), Bekaert et al. (2005) and Doh (2006). As a minor extension relative to the standard representation, we allow for autocorrelation in the exogenous shocks. The parameter vector for the benchmark model is (up to a set of eight standard deviations of measurement errors):

$$\theta_{MFS} = \left[\sigma_\pi, \sigma_y, \sigma_i, \sigma_{\pi^*}, \sigma_\rho, \pi_0^*, \rho_0, \delta_\pi, h, \sigma, \kappa, \gamma_\pi, \gamma_y, \gamma_i, \rho_\pi, \rho_y, \rho_i\right].$$

The parameters π_0^* and ρ_0 are the estimated initial values for the latent variables π_t^* and ρ_t. All other parameters have been defined in Section 2.

Liquidity premium models MFM. Next, the benchmark model is extended by allowing for mispricing in terms of time-invariant, maturity-specific, deviations of the actual yield curve from the one implied by the no-arbitrage conditions. Given the time-invariant nature of the mispricing terms, $\phi(\tau)$, and the fact that they are not systematically related to macroeconomic variables, we refer to these terms as liquidity effects. Formally, we extend the benchmark model with maturity-specific 'liquidity effects', i.e. we extend

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[20] The Metropolis-Hastings algorithm is based on a total of 200000 simulations, with a training sample of 20000. An acceptance ratio of 40% is targeted in the algorithm. Parameters are drawn based on the Gaussian random walk model. Finally, Geweke's test for differences in means and cumulative mean plots are used to assess convergence.

the parameter vector by including the parameters $\phi(\tau)$. Liquidity effects are estimated for yields with maturities $\tau = 1/2$, 1, 3, 5 and 10 years. This extension generates a 'liquidity premium' variant of the fully consistent version, which we label MFM (M refers to mispricing). The parameter vector θ (up to eight measurement error standard deviations) is given by:

$$\theta_{MFM} = \left[\sigma_\pi, \sigma_y, \sigma_i, \sigma_{\pi^*}, \sigma_\rho, \pi_0^*, \rho_0, \delta_\pi, h, \sigma, \kappa, \gamma_\pi, \gamma_y, \gamma_i, \rho_\pi, \rho_y, \rho_i, \phi(1/2), \phi(1), \phi(3), \phi(5), \phi(10)\right].$$

Flexible models MFF. The structural model versions (i.e. the MFS and MFM types discussed above) have been criticized because they imply constant prices of risk. Therefore, given the homoskedastic nature of the NK model, structural models impose the (extended) expectation hypothesis and time-invariant expected excess returns. This modeling feature is clearly at odds with a large literature documenting time-varying risk premiums. In order to accommodate time variation in the risk premiums, Rudebusch and Wu (2004) and Hördahl et al. (2006) introduced an alternative version of the structural model, where the prices of risk contained in Λ_0 and Λ_1 (equation (22)) are treated as free parameters. This version of the model targets maximal flexibility. The label used for this flexible version is MFF. However, within the context of an eight factor model, estimating the maximally flexible model is practically infeasible. With all prices of risk modeled as free parameters, a total of 70 parameters would need to be estimated. Given the complexity of the model, this number of parameters is prohibitively large. To reduce the number of free parameters, we impose some structure on the prices of risk by assuming (i) that the prices of risk and the risk premiums are stationary under the PLM and (ii) that the prices of risk only load on observed macroeconomic variables, π_t, y_t and i_t:[21]

$$\Lambda_0 = \begin{bmatrix} \Lambda_{0,\pi} \\ \Lambda_{0,y} \\ \Lambda_{0,i} \\ \Lambda_{0,\pi^*} \\ \Lambda_{0,\rho} \\ 0 \\ 0 \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} \Lambda_{1,\pi\pi} & \Lambda_{1,\pi y} & \Lambda_{1,\pi i} & 0 & 0 & 0 & -\Lambda_{1,\pi\pi} - \Lambda_{1,\pi i} & -\Lambda_{1,\pi i} & 0 & 0 \\ \Lambda_{1,y\pi} & \Lambda_{1,yy} & \Lambda_{1,yi} & 0 & 0 & 0 & -\Lambda_{1,y\pi} - \Lambda_{1,yi} & -\Lambda_{1,yi} & 0 & 0 \\ \Lambda_{1,i\pi} & \Lambda_{1,iy} & \Lambda_{1,ii} & 0 & 0 & 0 & -\Lambda_{1,i\pi} - \Lambda_{1,ii} & -\Lambda_{1,ii} & 0 & 0 \\ \Lambda_{1,\rho\pi} & \Lambda_{1,\rho y} & \Lambda_{1,\rho i} & 0 & 0 & 0 & -\Lambda_{1,\rho\pi} - \Lambda_{1,\rho i} & -\Lambda_{1,\rho i} & 0 & 0 \\ \Lambda_{1,\pi^*\pi} & \Lambda_{1,\pi^* y} & \Lambda_{1,\pi^* i} & 0 & 0 & 0 & -\Lambda_{1,\pi^*\pi} - \Lambda_{1,\pi^* i} & -\Lambda_{1,\pi^* i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Following the literature, we do not allow for mispricing (liquidity effects). The parameter vector θ_{MFF} then becomes:

$$\theta_{MFF} = \left[\sigma_\pi, \sigma_y, \sigma_i, \sigma_{\pi^*}, \sigma_\rho, \pi_0^*, \rho_0, \delta_\pi, h, \sigma, \kappa, \gamma_\pi, \gamma_y, \gamma_i, \rho_\pi, \rho_y, \rho_\iota, \Lambda_0, \Lambda_1\right].$$

Encompassing models MFE. Finally, the fourth type of model extends the class of Flexible models by additionally allowing for mispricing and learning. This type, labeled MFE, is the encompassing model. The introduction of mispricing and imperfect information (learning) implies that relative to the MFF version of the model, two additional sets of parameters are estimated. In particular, the mispricing terms $\phi(\tau)$ and the parameters modeling learning behavior (ω_π, ω_ρ, g_π, g_ρ, π_0^{*P}, ρ_0^P, $\sigma_{\pi^{*b}}$, σ_{ρ^b}) are added to

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[21] This type of restriction is based on the statistical tests rejecting the unit root hypothesis for term premiums and risk premiums. Within the context of the Macro-Finance models, this observation implies the stationarity of the prices of risk. Note that in the empirical implementation, we restrict the Λ_1 matrix further. Statistical analysis shows that we can set $\Lambda_{1,\pi^*.}$ and $\Lambda_{1,\rho.}$ to zero.

the Flexible model:

$$\begin{aligned}\theta_{MFE} \;=\; & [\sigma_\pi, \sigma_y, \sigma_i, \sigma_{\pi^*}, \sigma_\rho, \pi_0^*, \rho_0, \delta_\pi, h, \sigma, \kappa, \gamma_\pi, \gamma_y, \gamma_i, \rho_\pi, \rho_y, \rho_\iota, \phi(1/2), \phi(1), \phi(3), \phi(5), \phi(10), ... \\ & \Lambda_0, \Lambda_1, \omega_\pi, \omega_\rho, g_\pi, g_\rho, \sigma_{\pi^{*b}}, \sigma_{\rho^b}, \pi_0^{*P}, \rho_0^P].\end{aligned}$$

Finally, note that each of the versions allows for two stochastic endpoints. The models presented in the empirical Macro-Finance literature typically incorporate only one stochastic endpoint. In order to compare model performance to the models estimated in the literature we add two base models: a standard New-Keynesian model (NK0) and a standard Macro-Finance model (MF1). The NK0 model represents the standard New-Keynesian model with *constant* equilibrium real rate and inflation target. The MF1 model retains the *constant* equilibrium real rate assumption but allows for a *time-varying* inflation target. Both the NK0 and MF1 version are set within the class of Liquidity models (MFM) and imply additional parameter restrictions: $\sigma_\rho = \sigma_{\pi^*} = 0$ and $\sigma_\rho = 0$ for the NK0 and MF1 models, respectively.

4 Empirical analysis

4.1 Data

4.1.1 Sources and summary statistics

The data set consists of quarterly observations for the US economy covering the period 1960.Q2 till 2006.Q4, yielding 187 data points. The sample consists of observations on macroeconomic variables, on the term structure of interest rates and subjective inflation expectations. The macroeconomic variables included in the sample represent the inflation rate, the output gap and the policy rate. In particular, inflation is measured by the quarterly inflation rate (in per annum terms) obtained from the GDP deflator. The output gap is measured as the percentage deviation of GDP from potential output, as reported by the CBO. Finally, we use the effective federal funds rate as the policy rate.[22] The term structure of interest rates is represented by observations on six yields, spanning the maturity spectrum of actively traded bonds. We use data on yields with respective maturities of 1/4, 1/2, 1, 3, 5 and 10 years. For the short-term yields, i.e. the 1/4 and 1/2 year maturities, we used observations on the secondary market yields of treasuries.[23] The yields for the 1, 3, 5 and 10 year maturities are sampled combining the data set compiled by Gürkaynak, Sack and Wright (2006) and the McCulloch-Kwon data set.[24] Finally, we use the Survey of Professional Forecasters (SPF) as a source for the 1 and 10 year average (subjective) inflation expectations.[25]

Insert Table 2

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[22] For inflation and for real GDP the data series GDPDEF and GDPC1 were retrieved from the Federal Reserve Economic Data (FRED) data base. We use the 2006 vintage of potential output (spreadsheet 7027_Table2-2.xls available at www.cbo.gov). Data on the effective federal funds rate are obtained from the FRED service.

[23] For the 1/4 and 1/2 year maturities, the FRED series TB3MS and TB6MS are used.

[24] The Gürkaynak, Sack and Wright (2006) data set starts from the 14th of June 1961 for the 1, 3 and 5 year bonds and from the 16th of August 1971 for the 10 year bond. The missing observations are obtained from the McCulloch-Kwon data set, available at: the http://www.econ.ohio-state.edu/jhm/ts/mckwon/mccull.htm.

[25] Note that we are using the 'combined' surveys to obtain the 10 year inflation expectations. The original data are available at the FED of Philadelphia at: http://www.philadelphiafed.org/econ/spf/spfshortlong.html

Table 2 reports descriptive statistics summarizing the main features of the data set. The summary statistics are broadly in line with stylized facts reported in the literature. Inflation averages 3.7% p.a. over the sample period, while the average output gap is close to zero. The yield curve is on average upward sloping with the 10 year yield on average equal to 7% while short-term maturities display a mean around 5.5 % per annum. In line with previous findings, the volatilities of the yields are decreasing in the maturity of the yields, leading to a decreasing term structure of yield volatilities. As far as survey data of inflation expectations are concerned, the average inflation expectations are aligned with the actual inflation with means of respectively 4.0% and 3.9% per annum for the 1 and 10 year inflation expectations.

Based on the correlations coefficients reported in Table 2, one observes a strong and significant relation between the *levels* of the yield curve and macroeconomic variables either in the form of observed variables or inflation expectations. In particular, short-run yields correlate positively with the federal funds rate and the observed inflation rate and negatively with the output gap, while longer maturity yields seem to correlate primarily with long-run inflation expectations.

Insert Table 3

Table 3 reports the summary statistics of the *first differences* of the series in the data set. Two observations stand out. First, the correlation between macroeconomic and yield curve variables is much weaker. This observation holds in particular for the correlation between yields and inflation. The observed decrease in correlations between changes in yields and macroeconomic variables suggests that no single macroeconomic variable can by itself account for a substantial part of the high frequency movements in the yield curve. Second, as for the levels, Table 3 illustrates the relative importance of observed macroeconomic variables across the maturity spectrum of the yield curve. We find that changes in observed macroeconomic variables, such as inflation, output gap and interest rate shocks, correlate positively with changes in the short end of the yield curve. Changes at the longer end of the yield curve correlate primarily with changes in inflation expectations. For example, the correlation between 10-year inflation expectations and the 10-year yield is estimated around 24%.

4.1.2 Long-term yields and inflation expectations

An important finding emerging from the data analysis, as reported in Tables 2 and 3, is that inflation expectations (as measured by the SPF), although related, cannot fully account for the time variation in long-term yields. More precisely, the observed time variation in long-term yields and inflation expectations are not aligned (one-to-one). For instance, comparing the volatility of the 10 year maturity yield to the 10 year average inflation expectations, we observe excessive volatility in yields, i.e. 0.024 vs 0.015 for levels and 0.006 vs 0.002 for first differences. The excess volatility of long-term yields, relative to inflation expectations is also illustrated in Figure 1. This figure compares the dynamics of (shocks in) inflation expectations to (shocks in) long-term yields and underscores the fact that inflation expectations cannot fully account for the variation in long-term yields.

Insert Figure 1

This finding is significant in light of the basic modeling assumptions typically employed in the Macro-Finance literature. In particular, the excess sensitivity of long-term yields relative to long-term inflation expectations contradicts the maintained hypothesis that all variation in long-term yields can be attributed to time variation in long-run inflation expectations. We observe that significant and persistent differences exist between (demeaned) long-term yields and inflation expectations. Such persistent deviations suggest additional factors impacting on the long-term yields. In order to align the variation in the long-term yields to the macroeconomic dynamics, we extend the Macro-Finance model by introducing a second stochastic endpoint, representing the neutral real rate. The excess volatility (relative to long-run inflation expectations) of the long-term yields is then 'explained' by the volatility of the long-run real interest rate.[26]

4.2 Priors

Tables 4 and 5 report the prior distributions and the implied mean and standard deviation of the parameters of the respective models. Table 4 lists the priors for the structural and learning parameters while Table 5 contains the priors for the non-structural parameters related to mispricing (liquidity), measurement errors and (non-structural) prices of risk. In general, we use informative priors for the structural and learning parameters. The priors for the structural parameters conform to the benchmark NK macroeconomic model, while recent empirical results from the learning literature are used to set priors on the learning parameters. For the non-structural parameters we use relatively uninformative priors.

Insert Tables 4 and 5

The prior distributions for the standard deviations of the structural shocks, σ_π, σ_y and σ_i (Table 4) follow the Inverse Gamma distribution with mean 1% and standard deviation 0.25%. The standard deviations for the permanent shocks (i.e. the inflation target and the natural rate shocks), σ_{π^*} and σ_ρ, are uniformly distributed with support between 0 and 1%. The latter priors are chosen to be (i) uninformative within the range [0, 0.01] and (ii) to prevent standard deviations on permanent shocks to become excessively large. The upper bound of 1% per quarter is sufficiently high to include the relevant range of (quarterly) standard deviations σ_{π^*} and σ_ρ.[27] For the parameters related to the Phillips curve, the following priors were used. First, significant 'rule of thumb indexation' is assumed, as embodied in the prior on the inflation indexation parameter, δ_π. We use a beta prior with mean 0.7 and standard deviation 0.05. This prior on inflation indexation attributes a significant role to the (endogenous) backward-looking

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[26] Additional evidence supporting the introduction of a time-varying real rate in the Macro-Finance models comes from the TIPS market. As shown by Gürkaynak, Sack and Wright (2008), the long-run real yields (or forward rates) show significant and persistent time variation.

[27] In previous research, we imposed a stronger prior with respect to the (quarterly) variability of the interest target and/or the neutral real rate (see Dewachter and Lyrio (2008)). In particular, the upper bound of the quarterly variability of the inflation target and the neutral real rates was fixed to 0.1% per quarter (in per annum terms). This restriction reflected the prior belief in smooth changes in either the inflation target or the neutral real rate. Both restrictions turned out to be binding. In this paper, the upper bound is significantly increased and fixed at a value of 1% per quarter.

component in inflation. We use a strict prior for the output sensitivity of inflation. In particular, the prior for κ is normal with a mean of 0.12 and standard deviation of 0.03.[28] Finally, next to allowing for endogenous inertia (through inflation indexation), we also incorporate exogenous inertia induced by the autocorrelation of the supply shock, φ_π. The (loose) prior, i.e. a normal distribution with mean and standard deviation equal to 0.5, for φ_π corresponds to the one commonly proposed in the literature. The IS curve is modeled analogously. Both endogenous and exogenous inertia is allowed for in the form of respectively (external) habit persistence, h, with a beta prior (mean 0.7 and standard deviation 0.05) and in the form of an autoregressive parameter, φ_y, with a normal prior (mean 0.5 standard deviation 0.5). For the curvature parameter σ, the macroeconomic view is imposed by using a prior with mass concentrated on the lower values of σ. In particular, a Gamma distribution is used with mean 1.5 and standard deviation 0.4.[29] Finally, the priors for monetary policy are obtained from the Taylor rule literature: the inflation gap and output gap parameter, γ_π and γ_y have normal priors with mean 0.5. The differences in the standard deviations (i.e. $\sigma_{\gamma_\pi} = 0.25$ and $\sigma_{\gamma_y} = 0.4$) reflect the differences in (estimated) uncertainty for these parameters, as reported in the empirical literature on Taylor rules. The prior on the interest rate smoothing parameter, γ_i, is set to a normal distribution with mean 0.8 and standard deviation 0.2. The high standard deviation of γ_i reflects the ongoing debate concerning the degree of interest smoothing, see Rudebusch (2002), Gerlach-Kristen (2004) and English et al. (2002).

Table 4 also presents the prior distributions for the parameters related to the learning model. The priors for the size of the belief shocks, $\sigma_{\pi^{*b}}$ and σ_{ρ^b}, are uniform distributions with support $[0, 0.02]$. The prior distributions for the gains, g_π and g_ρ, in the learning rule are uniform on the interval $[0, 0.25]$. This support is sufficiently large to contain most of the gain estimates reported in the literature (e.g. Milani (2007), Kozicki and Tinsley (2005a)). Finally, we impose relatively strict priors on the parameters w_g and w_π. These parameters control the degree of information asymmetry allowed for in the model. The prior distributions assumed for w_g and w_π are beta distributions with support on [0,1] with mean 0.85 and standard deviation 0.10. The prior is thus biased towards the full information rational expectations model.[30]Summarizing, the set of priors on the learning parameters (i) gives a large weight to the RE solution ($w = 0.85$), (ii) allows for belief shocks with total impact (at the mean of the prior) of $(1 - w_j)\sigma_{j^b} = 0.15 \times 0.01 = 0.0015$ and (iii) introduces adaptive learning with (mean) gain $(1 - w_j)g_j = 0.15 \times 0.125 = 0.01875$.

Finally, additional prior distributions (listed in Table 5) are required for the (non-structural) prices of risk, the average mispricing (liquidity premiums) and the measurement errors. For the prices of risk (both the constant, Λ_0 and the time-varying prices of risk Λ_1) relatively uninformative priors were used. The priors are set such that at the mean the model implies (i) a positive constant risk premium ($E\Lambda_0 < 0$) and

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[28] The value for the mean of κ (0.12) corresponds to the values found by Bekaert et al. (2005). Despite this strict prior, the data move the posterior distribution for κ to ranges of significantly lower values. This bias towards lower estimates is in line with estimation results using GMM or ML techniques (e.g. Cho and Moreno, 2006).

[29] This prior contrasts with the estimates obtained from the finance literature. In this literature, estimates of σ between 20 and 100 are common.

[30] Note that we impose strict priors because the data set does not contain much information identifying the difference between private sector and central bank assessments of the neutral real rate. By imposing a strict prior, we effectively penalize real rate processes deviating too much from the one implied by the restrictions of a RE equilibrium.

(ii) a risk premium increasing with the inflation and the interest rate gaps, $(\pi_t - \pi_t^*)$ and $(i_t - \pi_t^* - \rho_t)$, while decreasing with the output gap, y_t. Note, however, that given the large standard deviations, the priors for the prices of risk are 'loose'. For the liquidity premiums $\phi(\tau)$, the priors are normal with mean 0 and standard deviation 0.005. This specification reflects the a priori belief of relatively small average mispricing errors (liquidity premiums). Finally, we use the same prior distribution for all the standard deviations of the measurement errors (both on yields and inflation expectations). In particular, an Inverted Gamma distribution is used with mean 0.005 and standard deviation 0.003.[31]

4.3 Estimation results

We present the estimation results in two parts. First, we compare the model versions by means of the marginal likelihood and the BIC statistics. Based on these measures, we select the best performing model. Anticipating results, we find that the encompassing model outperforms all other versions of the Macro-Finance models. Subsequently, in the second part, we discuss in detail the posterior density of the encompassing model.

4.3.1 Relative performance of models

We use the (log) marginal likelihood of the data to assess the relative performance of the respective versions of the model. Furthermore, we complement the marginal likelihood with the BIC statistic. Given that the BIC is a likelihood based statistic, independent of priors, it serves the role of the goodness-of-fit measure (up to a penalty for model dimensionality). The log of the marginal likelihood of the data and the BIC statistics are reported in Table 6.

Insert Table 6

Table 6 performs two types of model comparisons. First, we relate the alternative model versions estimated in this paper (MFS, MFM, MFF and MFE) to the performance of standard econometric models employed in the literature. The latter models come in two forms: either as a standard New-Keynesian model (NK0) or a standard Macro-Finance model (MF1). The standard New-Keynesian model features a constant inflation target and neutral real rate (no stochastic endpoints), while the standard econometric version of the Macro-Finance model allows for one stochastic endpoint, i.e. a time-varying inflation target. In contrast, all model versions estimated in this paper include two stochastic endpoints, i.e. for inflation and the neutral real rate. As can be observed, all model versions (MFS, MFM, MFF and MFE) imply significantly higher marginal likelihoods than the NK0 and the MF1 models. The significantly lower marginal likelihood of both the New-Keynesian model (Marg. Lik NK0 = 6124) and the standard Macro-Finance model (Marg. Lik. MF1 = 7240) establish the empirical relevance of a time-varying neutral real interest rate. The reported BIC statistics corroborate the above analysis. Second, Table 6 can be used to assess the empirical relevance of the learning dynamics and liquidity effects introduced in the encompassing model. Allowing for learning dynamics (imperfect information) and liquidity effects

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[31] To prevent singularity problems in the Kalman filter estimation, a lower bound of 5 basis points was imposed on all measurement errors.

(mispricing), as is done in the encompassing model, significantly improves the overall model fit. Both the marginal likelihood and the BIC statistics favor the encompassing model over *all* alternatives. The (log) marginal likelihood of the MFE model (7741) is substantially higher than those reported for the alternative model versions MFS (7381), MFM (7628) and MFF (7637). Assuming a uniform prior over the alternative model versions, the posterior odds ratio of the MFE version equals its Bayes factor of (approximately) 1. Such a strong bias of the posterior odds ratio in favor of the encompassing model clearly suggest the superiority of this version relative to all other versions of the model. Results for the BIC statistics in Table 6 lead to a similar conclusion. Despite the fact that the encompassing model is the largest model, it is clearly preferred by the BIC statistic, i.e. -15442 for the MFE version compared to -15384 (MFF), -15333 (MFM) and -14815 (MFS). It follows from the BIC statistics that the MFE outperforms the other versions in log-likelihood (of the prediction errors), i.e. in 'fit'.

Table 6 also decomposes the performance of the models in three dimensions, i.e. the macroeconomic, the yield curve and the inflation expectations dimensions. We use the likelihood of the prediction errors of the respective data subsets as performance measure. This decomposition shows that the encompassing model outperforms all other model versions (in likelihood) in each dimension. Note also that the largest improvements in likelihood are observed relative to the NK0 and MF1 models. This observation shows that introducing stochastic endpoints (either in the form of a time-varying inflation target and/or neutral real rate) improves significantly the fit of the model. This observation is especially relevant for the yield curve and the inflation expectations dimensions of the model.

Finally, we use Table 6 to assess the empirical relevance of the (extended) expectations hypothesis. Models MFS and MFM incorporate (a form of) the expectations hypothesis, by imposing constant term premiums. Model versions MFF and MFE allow for deviations from the (extended) expectations hypothesis by introducing time-varying risk premiums. Three observations can be made. First, in line with the extant literature, the pure expectations hypothesis, imposing consistent prices of risk and full-information rational expectations, is clearly rejected. The marginal likelihood of the version imposing the pure expectations hypothesis, i.e. MFS, is significantly lower than those implied by either the (extended) expectations hypothesis (MFM) or the version implying time-varying risk premiums (MFF). Second, two approaches have been suggested in the literature to improve upon the pure expectations model: (i) allowing for ad hoc time-invariant risk premiums leading to the extended expectations hypothesis and (ii) allowing for time-varying prices of risk. The results in Table 6 clearly favor the second approach. That is, allowing for time variation in the risk premiums (MFF) is, from a yield curve perspective, preferred over the extended expectations hypothesis (MFM). Third, it has been argued that deviations from full-information rational expectations can account for the rejection of the expectations hypothesis, e.g. Froot (1989). Specifically, models containing asymmetric information and learning dynamics have been shown to generate substantial deviations from the expectations hypothesis, Kozicki and Tinsley (2005b). The estimation results of the MFE model, discussed in the next section, suggest that learning dynamics alone cannot account for the rejection of the expectations hypothesis. In the MFE version, the time-varying prices of risk remain significant, even after allowing for imperfect information and learning.

4.3.2 Posterior distributions in the MFE specification

In this section, we discuss the posterior moments of the parameters as implied by the MFE version of the model. Tables 7 and 8 report the mean, standard deviation, and confidence interval (90 percent) for the posterior distribution.[32] We focus the discussion on three sets of parameters: (i) the structural parameters determining the NK model, (ii) the (relative sizes) of the respective shocks, including belief shocks, and finally (iii) the learning parameters.

Insert Tables 7, 8 and 9

(i) Structural parameters. The obtained posterior moments of the parameters of the structural model (equations (1), (5), (8)) are in line with findings and estimates previously reported in the empirical macro literature. In particular, the data reject the purely forward-looking NK model in favor of the hybrid version, containing both forward-looking and backward-looking components. Although the purely forward-looking model is clearly rejected, we do find that the forward-looking components dominate. This observation is in line with parameter estimates reported in the literature, e.g. Galí and Gertler (1999). Several specific remarks can be made with respect to the structural parameters.

First, in the Phillips curve, the inflation indexation parameter, δ_π, with mode $\delta_\pi = 0.52$ suggests a relatively minor role for inflation indexation, implying a relatively high weight on the forward-looking inflation component, $\mu_{1,\pi} = 0.66$. This degree of forward-lookingness is typically not recovered in the Macro-Finance literature. For instance, Bekaert et al. (2005) report implied values in the range of [0.53, 0.63]. However, it is aligned with estimates reported in the macroeconomic literature, e.g. Galí and Gertler (1999) and Galí et al. (2005). The inflation sensitivity to the output gap, measured by the parameter κ, is estimated to be relatively small. Despite the strict prior around a mean value of $\kappa = 0.12$, the posterior mode is significantly lower, i.e. $\kappa = 0.012$. This low value indicates a weak link between detrended output and inflation and reflects the mismatch in the persistence of the two variables. Although lower than theoretically expected, the mode of $\kappa = 0.012$ is high compared to other GMM or ML based studies, reporting parameters estimates several orders of magnitude lower (e.g. Cho and Moreno (2006) report a value of 0.001).

Second, the parameters of the IS-curve conform to estimates previously reported in the literature. The habit persistence parameter, h, is estimated precisely with mode 0.76. More importantly, the model imposes sufficient structure to generate a reasonable posterior distribution for the risk aversion parameter. The mode for σ is estimated at 2.55, with 90% of the posterior support contained in the interval [1.9, 3.3]. This range of curvature parameters seems reasonable from a macroeconomic perspective. Combined, the values for σ and h result in a relatively strong forward-looking component. Specifically, evaluated at the modes of h and σ, the weight on the expected future output gap in the IS equation becomes $\mu_y = 0.69$.

Finally, the posterior densities characterize the monetary policy rule as an activist rule both in the inflation and the output gap. Both γ_π and γ_y (with respective modes at $\gamma_\pi = 0.44$ and $\gamma_y = 0.63$) are statistically significant and positive and correspond to values implied by the standard Taylor rule.

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[32] The Appendix contains the tables for the posterior distributions of the parameters for the other model versions.

Also, relatively low values for the interest rate smoothing parameter, γ_i, are found. The interest rate smoothing parameter (with mode at $\gamma_i = 0.69$) is contained in the confidence interval [0.64, 0.78]. This posterior for γ_i clearly indicates lower interest rate inertia than previously suggested and implies a more plausible description of monetary policy. For example, values for interest rate smoothing parameter γ_i of the order of $\dot{0}.9$ (on a quarterly frequency) are commonly reported in the literature. These values suggest, irrealistically, that it would take the FED more than six quarters to halve the gap between the actual and the target interest rate. In contrast, the estimate reported in this paper ($\gamma_i = 0.69$) implies, more realistically, a halving time of less than two quarters. These findings are in line with Trehan and Wu (2007) and underscore the importance of omitted variable bias in Taylor rule estimation.[33]

(ii) Structural shocks. The MFE version of the model features seven shocks: Three temporary structural macroeconomic shocks (the supply, demand and policy rate shock), two belief shocks (related to the inflation target and the natural real rate) and, finally, two additional and permanent shocks associated to movements in the inflation target and the neutral real rate. As far as sizes and autocorrelations of the temporary shocks are concerned, the reported estimates are quite similar across model versions and suggest that both the supply and policy rate shocks are relatively large but not persistent. Across model specifications, we find evidence of negative autocorrelation for supply and policy rate shocks.[34] The demand shock is positively autocorrelated with mode at 0.65. An important feature of the learning versions of the model is the introduction of belief shocks for inflation and the neutral real rate. For inflation, exogenous belief shocks dominate, i.e. $\sigma_{\pi^{*b}} > \sigma_{\pi^*}$. For instance, at the mode of the posterior, the difference in estimated sizes is very pronounced, i.e. $\sigma_{\pi^*} = 0.04\%$ while $\sigma_{\pi^{*b}} = 0.58\%$. For the neutral real rate, we find a relatively high value for the standard deviation of the neutral real rate, i.e. $\sigma_\rho = 0.73\%$ (at the mode). In contrast, belief shocks in the real rate are relatively small. Both findings, i.e. the significant sizes of belief shocks in inflation expectations and the 'true' shocks to the neutral real rate, are important departures from the findings of standard Macro-Finance models. First, the large standard deviation of the neutral real rate (around 0.73%) highlights the importance of the neutral real rate shocks in yield curve dynamics, especially for long-term yields. This source of variation is typically ignored in standard Macro-Finance models by assuming a constant equilibrium real rate. Second, the learning model clearly differentiates between actual inflation target shocks and inflation belief shocks. Belief shocks are substantially larger than actual inflation target shocks. This finding implies a relatively smooth inflation target dynamics while still allowing for substantial variation in the perceived long-run

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[33] The argument of omitted variable bias in the interest rate smoothing parameter has been put forward by Rudebusch (2002). Subsequent studies, introducing latent factors in the Taylor rule, found that a substantial part of interest rate inertia could be attributed to omitted variables. For instance, Gerlach-Kristen (2004) estimates interest rate smoothing parameters around 0.6 with standard errors 0.2. Also, English et al (2002) report in a similar type of study values for the interest rate smoothing around 0.6 with standard error 0.15. For both studies, the confidence intervals significantly overlap the confidence interval for γ_i reported in this study.

[34] The finding of a negatively autocorrelated supply and policy rate shock may be somewhat surprising. It is important to note, however, that the model incorporates two additional channels, modeling persistence. First, the implied endogenous persistence due to interest rate smoothing or inflation indexation. Second, and importantly, both the interest rate and the inflation depend on the exogenous processes modeling the (perceived) inflation target and natural real rate. The latter two processes are persistent, picking up a substantial part of the persistence of interest rates or inflation. Finally, low first order correlation for supply and policy rate shocks has also been reported by Ireland (2005). Smets and Wouters (2007) find a much higher AR(1) parameter. However, their model includes a significantly negative MA(1) term as well.

inflation expectations (through the belief shocks).

(iii) Learning parameters. Finally, the learning parameters indicate substantial deviations from the full-information rational expectations model. Noting that the full information RE model is embedded in the MFE model, i.e. for $w_\pi = w_\pi = 1$, it is clearly rejected in favor of the alternative of learning, i.e. $w_\pi < 1, w_\rho < 1$. The deviation from the full information case is especially pronounced for the perceived long-run inflation expectations and suggests only weakly anchored inflation expectations. For inflation expectations, we observe significant learning effects given (i) the relatively large factor $(1 - w_\pi)$, (ii) the relatively large size of the belief shock, $\sigma_{\pi^{*b}}$, and (iii) the significant constant gain g_π (mode estimated around 0.21). Multiplying the gain by $(1 - \omega_\pi)$ yields the impact of the subjective forecast error on the perceived long-run inflation rate. At the mode, this impact is approximately equal to 0.07. This value corresponds to constant gain estimates reported in the literature. For instance, Milani (2007) finds constant gains around 0.02-0.03 while Kozicki and Tinsley (2005a) using a similar learning model find higher values (around 0.10). Note that the mode of the posterior implies only marginal effects of learning on the real rate dynamics, given the high weight $\omega_\rho = 0.97$.

4.4 Implied macro-factors

The macro factors as implied by the mode of the posterior of the encompassing model (MFE specification) are displayed in Figure 2. This figure contains the filtered time series for all ten macroeconomic factors: the observed factors (inflation, output gap and the effective federal funds rate), the exogenous supply, demand and policy rate shocks and, finally, the four (perceived and actual) stochastic endpoints for inflation and the neutral real rate, respectively. Figure 3 zooms in on the perceived and actual stochastic endpoints by relating the endpoints to observed macroeconomic variables and by providing the respective 90% confidence intervals of the endpoints.

Insert Figures 2 and 3

Figure 2 illustrates some important new findings implied by the MFE model. First, the encompassing model introduces, unlike standard New-Keynesian or Macro-Finance models, a second type of stochastic endpoint, i.e. the neutral real rate. As can be observed from Figure 2, the (perceived) neutral real interest rate displays significant volatility and persistence. The filtered neutral real rate is typically contained within the 0%-5% p.a. region (with an historical average close to 2.5% p.a.) and shows significant persistence with relatively low rates in the 1970s and substantially higher rates in the 1980s. These findings of persistent and volatile neutral real rates concur with recent macroeconomic research. In particular, recent studies, estimating natural real rate dynamics, also find significant persistence and volatility in the equilibrium real rate (e.g. Laubach and Williams (2003), Clarck and Kozicki (2004) or Bjornland et al. (2007)). Note, however, that Figure 3 implies substantial differences in the uncertainty surrounding the estimated time paths of the actual and perceived neutral real rates. More specifically, only the *perceived* neutral real rate is identified accurately (primarily from yield curve dynamics). The dynamics of the *actual* neutral real rate is only weakly identified (by the ALM), as can be observed from the large confidence interval. In fact, the 90% confidence interval reported in Figure 3 does not exclude a constant (or smooth) *actual* neutral real rate.

Second, the filtered stochastic endpoints of inflation, as displayed in Figure 2, imply significant and persistent differences between the perceived long-run inflation expectations and the actual inflation target. The perceived long-run inflation expectations display substantial time variation, while the time path of the inflation target is mostly contained within the [1%, 3.8%] interval (see Figure 3). A similar type of disconnection between subjective inflation expectations and the inflation target is found in Dewachter and Lyrio (2008) or Kozicki and Tinsley (2005a). The marked differences between perceived inflation expectations and the actual inflation target suggest that subjective inflation expectations were not well-anchored, especially over the first part of the sample. This interpretation is also implicit in the parameter estimates (Table 7) reporting significant imperfect information (credibility) problem, $w_\pi << 1$. This imperfect information channel allows alternative information sources to drive a wedge between inflation target and subjective inflation expectations. Both belief shocks ($\sigma_{\pi^{*b}} = 0.58\%$) as well as adaptive learning ($g_\pi = 0.21$) contribute substantially to the observed wedge. Note that a strong similarity is observed for the perceived and actual neutral real rate. This similarity is obviously implied by the parameter estimates marginalizing the learning effects for the real rate, i.e. w_ρ is close to 1.

Finally, the encompassing model, introducing a neutral real rate factor and allowing for learning dynamics, solves an important interpretation puzzle of standard Macro-Finance models. This puzzle relates to the excess volatility and the timing of the inflation target as implied by standard Macro-Finance models. As already stated, a major challenge of standard Macro-Finance models is to 'explain' the substantial time variation of the long end of the yield curve. Following the suggestion of Kozicki and Tinsley (2001), Macro-Finance models accommodate this time variability by introducing stochastic endpoints, typically in the form of a time-varying central bank inflation target (or long-run inflation expectations). However, standard Macro-Finance models attributing *all* variation in long-term yields to the inflation target (which under RE matches long-run inflation expectations) face two interpretation problems. First, to account for the time variation in yields, the standard models have to assume an excessively large standard deviation for the inflation target of the central bank. For instance, Doh (2006) reports standard deviations between 30 and 35 basis points (per quarter) for US using data covering 1960-2005. Bekaert et al. (2005) or Dewachter and Lyrio (2006) find values ranging from 30 to more than 73 basis points (per quarter). Such values seem relatively large given that it is reasonable to expect the inflation target to move slowly and smoothly over time. Second, it is often found that the filtered inflation target, as implied by the standard Macro-Finance models, *lags* observed inflation. Typically, the implied inflation target peaks long after the start of the disinflation policy of Volcker. Obviously, the fact that the inflation target *lags* actual inflation is more difficult to reconcile with intuition and the historical record. The encompassing model, in contrast, generates smooth inflation targets dynamics with mode around two percent and also generates inflation expectations in line with survey data (see Figure 3). This resolution of the puzzle is due to the fact that the encompassing model (i) attributes a substantial part of the time variation in long-term yields to the perceived output-neutral real rate factor and (ii) allows for substantial differences between the inflation target and the subjective inflation expectations.

4.5 The fit of the yield curve

The yield curve model implied by the MFE version contains eight factors: Three observable macroeconomic factors, three exogenous shocks and two latent factors, tracking respectively the perceived inflation target (stochastic endpoint) and the output-neutral real interest rate.[35]

Insert Figure 4

The factor loadings of the yield curve are depicted in Figure 4. The factor loadings measure the sensitivity of the yield curve to each of the macroeconomic factors. Figure 4 establishes the relevance of the neutral rate dynamics for the long-term yields. As can be observed, long-term yields are affected by both stochastic endpoints. Changes in long-run inflation expectations or the neutral real interest rate, are transmitted almost one-to-one in the long end of the yield curve. The factor loadings on the policy rate reveal a slope factor response, while other macroeconomic variables, i.e. inflation and demand shocks, primarily affect the intermediate maturities.

The performance of the model in fitting the yield curve can be assessed using the posterior densities for the measurement errors, as reported in Table 8. Evaluated at the posterior mode, the size of the measurement errors (as measured by $\sigma_{\eta,y}(\tau)$) are well below 40 basis points for yields with maturity beyond one year.[36] Moreover, the 90% confidence bounds of the posterior of the measurement errors indicate that the MSE of the model is below 50 basis points for maturities beyond one year. These values are small relative to the total variation of the yields, exceeding 240 basis points (see Table 2). Comparing these statistics, the overall success of the MFE model in explaining the yield curve variation in terms of macroeconomic shocks is obvious. More than 95 % of all variation (measured as the unconditional variance) in yields (with maturities beyond one year) is explained by the model. Figure 5 illustrates the fit of the MFE version of the model across the maturity spectrum. Figure 6 decomposes the fit of the ten year maturity yield into the expected real rate, expected inflation and risk premium dynamics.

Insert Figures 5 and 6

Two additional observations can be made with respect to the yield curve fit. First, despite the large number of factors included in the model, average mispricing seems economically important (see Table 8). Specifically, the posterior for average mispricing, $\phi(\tau)$, suggests an increasing pattern of mispricing with model-implied yields being too high at the short end and too low at the long end of the yield curve. Note, however, that only the (negative) mispricing term at the short end of the yield curve, $\phi(1/2)$

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[35] The MFE model features a total of ten factors. However, only the factors entering the Perceived Law of Motion are relevant for the yield curve, given that the yields are formed under the subjective expectations operator.

[36] A remarkable aspect of the data is the bad fit of the short end of the yield curve with fitting errors around one percent. This finding is due to the choice of policy rate. With the federal fund rate representing the policy rate, there is an obvious tension with short-term treasury rates, given that on average treasury rates have been selling at yields below the federal fund rate. This persistent gap between the fed fund rate and the short-term treasury yield is picked up in the measurement error.

is statistically significant. Given that the observed mispricing terms are structural and not related to macroeconomic factors we interpret them in terms of liquidity preferences or preferred habitat. The observed pattern of mispricing thus suggests positive (although insignificant) liquidity premiums at the long end of the yield curve and a preferred habitat with negative liquidity premiums for maturities below one year.[37] Second, disregarding average mispricing, the standard deviations of the measurement errors (estimated under 40 basis points for maturities beyond 1 year) are in line with estimates reported in the Macro-Finance literature. Studies using small-scale structural NK models over a similar time period typically find measurement errors between 10 and 50 basis points. For instance, Bekaert et al. (2005) report measurement errors of 45 and 54 basis points for the one and ten year yield. Cogley (2005) reports absolute mean pricing errors in between 50 and 60 for maturities in between one to four years. Finally, standard deviations for the measurement errors based on comparable models presented in Dewachter and Lyrio (2008) hover around 50 basis points.[38] Interestingly, De Graeve et al. (2007), using a medium-scaled NK model, i.e. the Smets and Wouters (2007) model, report significantly lower measurement errors (of the order of 10 to 20 basis points).

Insert Figure 7

Finally, the affine yield curve model also generates an affine representation for the risk premiums. Specifically, expected excess holding returns (risk premium) are linearly related to the macroeconomic state vector (see equation (29)). The expected holding returns (per annum for a quarterly holding period), as implied by the posterior mode of the distribution are displayed together with the NBER recession dates in Figure 7.[39] As can be observed, the holding returns have an important time-varying component ranging (for a ten year maturity bond) from -2% p.a. in 1965 to more than 6% in 1984. Risk premiums with broadly similar time patterns and order of magnitudes have been reported by Duffee (2002) or Campbell et al. (2007). In line with intuition, the observed risk premiums are countercyclical, generating large and positive risk premiums during recessions and smaller and even negative holding returns during expansions. This observation is born out by the estimates of the prices of risk related to the output gap ($\Lambda_{.,y}$). All the output gap related prices of risk are positive at the mode (see Table 9). Positivity of these prices of risk implies that risk premiums tend to increase in recessions and decrease during economic booms. Furthermore, risk premiums are significantly related to the level of the federal funds rate as both the price of inflation (supply shock) risk ($\Lambda_{\pi,i}$) and the price of interest rate risk ($\Lambda_{i,i}$) are significantly different from zero. Typically, tougher monetary policy is linked to a higher inflation risk premium and a lower interest rate risk premium. The price of inflation risk decreases (risk premium increases) with the level of federal funds rate, $\Lambda_{\pi,i} < 0$, while the price of interest rate risk increases (risk

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[37] The negative liquidity premium at the short end of the yield curve should not come as a surprise. The positive spread between the federal funds rate and the short term treasuries is well documented and is typically attributed to a risk premium in the federal funds rate reflecting private banks' uncertainty over reserve management.

[38] Taking into account that relative to most of the above mentioned studies, this model has two additional variables to fit, i.e. the subjective inflation expectations, one can consider the fit of the yield curve as comparable to the extant literature.

[39] The average holding risk premiums and the standard deviation, reported in barckets, implied by the data are: 1.1% (0.2%), 1.5% (0.7%), 1.7% (1%) and 2% (1.7%) for the 1, 3, 5 and 10 year bonds respectively. The average holding premiums implied by the model (mode) are respectively: 0.4%, 1.2%,1.6% and 2.1%.

premium decreases) with the interest rate level, $\Lambda_{i,i} > 0$.

An interesting hypothesis due to Kozicki and Tinsley (2007) is that the expectations hypothesis puzzle may be generated by learning dynamics (of the type studied in this paper). Learning dynamics generate substantial deviations from full information rational expectations and hence generate an expectations hypothesis puzzle. The reported prices of risk suggest however that learning dynamics by themselves cannot fully account for the expectations hypothesis puzzle. Even after allowing for learning dynamics, we find statistically and economically significant time variation in the prices of risk.

4.6 What drives the yield curve?

We turn to the identification of the macroeconomic factors driving monetary policy and the yield curve. We use the federal funds rate as the variable identifying the monetary policy. The yield curve is decomposed into its level, slope and curvature factors. We follow the literature in identifying the level factor as the average (across maturity) yield, the slope factor as the ten year maturity yield spread (relative to the one quarter yield) and the curvature as the sum of the 10 year and 1 quarter yields minus two times the one year maturity yield. Table 10 presents the results of the variance decomposition for the federal funds rate, the level, slope and curvature factor for three horizons, i.e. one quarter, one year and five years. Figure 8 complements the variance decomposition by displaying the contemporaneous impulse response functions of the yield curve to the respective macroeconomic shocks.

Insert Table 10

The *variance decomposition of the federal funds rate* identifies monetary policy actions and reactions to each of the macroeconomic shocks. The decomposition implies that the high frequency variation in the monetary policy is largely due to independent monetary policy shocks. Monetary policy shocks account for over 80% of all short-term variation in the federal funds rate. Macroeconomic shocks contribute only marginally to the high frequency movements in the federal funds rate: real rate shocks, supply shocks and demand shocks account respectively for 11%, 4% and 4%. The finding that the high frequency component of monetary policy is not determined by macroeconomic shocks is in line with e.g. Bekaert et al. (2005). Supply and demand shocks become more important at the intermediate frequencies. The fact that these shocks only become relevant at the intermediate frequencies is explained by interest rate smoothing. The gradual response of interest rates to macroeconomic shocks generates a smoothed and delayed interest rate response. For instance, at the yearly frequency, supply and demand shocks account for more than 24% of the total variation in the policy rate. Low frequencies dynamics of monetary policy are dominated by long-term equilibrium forces. Table 10 shows that low frequency movements in the federal funds rate is aligned with the long-run inflation expectations and the output-neutral real rate. At a frequency of five years for instance, the movements in the federal funds rate are mostly due to movements in the neutral real rate (more than 75%), while shocks to inflation expectations, either in the form of supply shocks and belief shocks account for another 14%. Neither inflation target shocks (due to the low standard deviation) nor belief shocks w.r.t. to the real rate (due to the marginal contribution of learning, i.e. $1 - w_\rho \approx 0$)

play a role in the federal funds rate dynamics.

The *variance decomposition of the level factor* shows that the standard interpretation of the level factor is not corroborated by the encompassing model. In standard Macro-Finance models, all variation in the level factor is attributed to long-run inflation expectations, e.g. Doh (2006), De Graeve et al. (2007). The variance decomposition in Table 10 suggests that this standard interpretation is not robust. The encompassing model identifies three types of macroeconomic factors impacting on the level factor. Specifically, shocks to the neutral real rate, subjective belief shocks for inflation and supply shocks account for almost all variation in the level factor at the intermediate and low frequencies. Within the extended framework of the encompassing model, the dominant role of shocks to the long-run real rate (ρ_t) becomes apparent. Shocks to (perceived) long-run inflation expectations, i.e. belief shocks and supply shocks, are significantly less dominating than suggested by standard Macro-Finance model. In particular, while shocks to the neutral real rate account for more than 80%, inflation expectations shocks, combining belief and supply shocks, only account for about 14% of the low frequency variation in the level factor. We thus find that on average the level factor is primarily associated with (perceived) equilibrium real rate movements. Inflation expectations on average play a less prominent role. This conclusion contradicts the standard interpretation of the level factor as an inflation expectations factor.[40]

The *slope and curvature factor decompositions* are more in line with the findings of benchmark Macro-Finance models (e.g. Bekaert et al. (2005)). The variation in the slope factor is dominated at all frequencies by exogenous monetary policy shocks. At intermediate frequencies, also supply and demand shocks impact on the slope factor. Findings for the decomposition of the curvature factor concur with the results for the slope factor, with monetary policy shocks dominating. Again, at the intermediate frequencies, a significant impact of supply and demand shocks is found. The decompositions are in line with the standard interpretation of slope and curvature as factors signalling the monetary policy stance and business cycle conditions.

Insert Figure 8

Finally, the *instantaneous impulse responses* of the yield curve, depicted in Figure 8, show that the model is able to generate excess sensitivity (at least qualitatively). Excess sensitivity of long maturity yields refers to the empirical observation that long forward rates (and thus yields) tend to respond to temporary macroeconomic shocks and surprises. Typically, this finding cannot be replicated by standard macroeconomic models, as discussed in Gürkaynak et al. (2005). Figure 8 shows the contemporaneous response of the yield curve to each of the structural shocks. Supply shocks have significant impact on long maturity yields. With supply shocks dominating inflation surprises, the model implies a direct link between inflation surprises and movements in the long end of the yield curve. Note that unlike supply

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[40] It is important to keep in mind that the variance decomposition presents an unconditional average estimate of the contribution of each shock. Overall, the encompassing model implies that long-run real rates have been more variable than long-run inflation expectations. This finding is supported by historical developments in the bond market since the 1980s. Also note that the variance decomposition does not necessarily contradict the experiences of the Great Inflation in the 1970s, where inflation expectations did dominate the dynamics of the level factor. Such episodes are not excluded by the variance decomposition; only they are not considered the as norm but rather as the exception.

shocks, the model does not generate excess sensitivity with respect to either the demand or the policy rate shock. As already discussed before (see Table 10), shocks to the latent factors have the strongest impact on the long end of the yield curve, as illustrated by the responses to both the inflation belief shock and the neutral real rate shock.

4.7 Historical decomposition of yield movements

An important theoretical corollary of macroeconomic models with (constant gain) learning is the possibility of inflation scares and inflation escapes, e.g. Orphanides and Williams (2005) or Sargent and Williams (2005). Inflation scares or escapes are endogenous events where inflation dynamics are no longer anchored by monetary policy. Instead, inflation dynamics display prolonged and persistent deviations from the long-run inflation target. Theoretically, serially correlated inflation shocks, i.e. supply shocks, (Orphanides and Williams (2005) or inflation belief shocks (Kozicki and Tinsley (2005a)) could trigger such inflation scares/escapes. In this section, we perform historical decompositions of the Great Inflation and Disinflation episodes.[41] These periods are of particular interest as they represent periods characterized by unprecedented un-anchoring (1970-1980) and re-anchoring (1982-1987) of inflation, inflation expectations and yield curve dynamics. We use the historical decomposition of these historical episodes to verify the inflation escape thesis. More specifically, we assess the role and contribution of supply and inflation belief shocks in the observed inflation expectations and yield curve dynamics. Figures 9 and 10 display the historical decompositions of the short-term yield (1 quarter), the five-year yield and the long-run inflation expectations for the Great Inflation and Disinflation periods.

Insert Figure 9

The results of the decomposition for the Great Inflation period are in line with the inflation scare corollary. The decomposition supports the idea that supply shocks were the main driving force behind the un-anchoring of the inflation expectations and the yield curve. Other shocks, including belief shocks, only played a minor role. The identification of supply shocks as the main cause of un-anchoring is in line with Blinder (1982). However, in line with the inflation scare hypothesis, learning dynamics are crucial in transmitting supply shocks into long-run inflation expectations and yields.[42] Under learning, the effects of supply shocks get amplified and prolonged as they feed into long-run inflation expectations. In particular, supply shocks, by generating inflation surprises, lead to revisions of the long-run inflation expectations, which subsequently feed into actual inflation and the yield curve. Figure 9 displays the historical decomposition and highlights the prominent role of supply shocks in the un-anchoring process of inflation and inflation expectations. First, we observe that supply shocks account almost fully for the trend-wise increase in the long-run inflation expectations, increasing long-run inflation expectations from 3% to more than 6%. The un-anchoring of inflation expectations also implied a trend-wise increase of

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[41] Note that we used the parameter as implied by the mode of the posterior distribution.

[42] Although the source of the decoupling of inflation expectations from the target is attributed to supply shocks, our results do not necessarily support the bad-luck hypothesis. Instead, in line with the 'bad policy' literature, monetary policy (as measured by the gap between ex ante and the neutral real rate) was on average weak, i.e. accommodating inflation increases.

both the short-term interest rate and the five-year yield, increasing respectively by more than 4% and 3%. Other shocks did not counterbalance the trend-wise effect of the supply shocks. Only a minor effect of negative natural real rate shocks, proxying for the productivity slow-down, is observed. The inflation scare was thus transmitted almost one-to-one into the level factor of the yield curve.

Insert Figure 10

The historical decomposition of the Great Disinflation period (Figure 10) complements the previous analysis by illustrating the empirical relevance of exogenous inflation belief shocks. The encompassing model attributes the trend-wise decrease of inflation expectations and of the yield curve observed during the Volcker disinflation to two types of shocks: supply shocks and inflation belief shocks. We find that about one third of the trend-wise decrease in inflation expectations and yields can be accounted for by exogenous (inflation) belief shocks. This finding suggests an interpretation of the belief shocks in terms of the credibility of the FED. Goodfriend and King (2005) date the restoration of credibility of the FED around 1982 with a full restoration of credibility only after the FED defeated the third inflation scare of 1983-1984. As can be observed from Figure 10, this dating of the credibility process concurs with the evolution of the inflation belief shocks, starting to decrease mid 1981 and further decreasing with a mild hick up (following the inflation scare) in 1983. Note that while the trend-wise decrease in inflation expectations and yields is fully explained by supply and belief shocks, other shocks explain the transient volatility in the yield curve. In particular, the short-term yield moved in line with the changes in the output neutral real rate, and the negative demand shocks during the recession following the Volcker disinflation. The transient variations in long-term yields are mainly caused by variations in the neutral real rate.

5 Conclusion

In this paper, we have estimated an encompassing Macro-Finance model, incorporating time variation in the neutral real rate, mispricing and learning dynamics. The model builds on a standard benchmark NK model, and augments it with two latent macroeconomic factors, interpreted as the stochastic endpoints for inflation and the neutral real rate, respectively. Additionally, we allow for imperfect information by including belief shocks and (constant gain) learning dynamics.

The main finding of the paper is that this ecompassing model outperforms standard Macro-Finance models. More specifically, standard Bayesian statistics, such as the marginal likelihood or the BIC, unambiguously favor the encompassing model relative to various other Macro-Finance models. These results establish empirically the importance of allowing both for time variation in the perceived neutral real rate and for learning dynamics.

The structural decomposition of the yield curve into its macroeconomic components provides new insights concerning the interpretation of the level, slope and curvature factors. The decomposition generated by the encompassing model concurs with the standard interpretation of the slope and curvature factors. In particular, in line with standard Macro-Finance models, the slope and curvature factors are primarily affected by transient monetary policy shocks, with demand and supply shocks contributing substantially. However, unlike standard Macro-Finance models, attributing (almost) all time variation in the level factor to inflation target shocks, the encompassing model identifies three sources impacting on the level factor: shocks to the perceived equilibrium real rate, subjective inflation belief shocks and cost-push shocks due to adaptive learning dynamics.

Finally, the historical decompositions of the Great Inflation and Disinflation periods illustrate the empirical relevance of the inflation scare hypothesis. The un-anchoring of inflation during the Great Inflation period is explained by serially correlated supply shocks generating, through the learning process, persistent and prolonged deviations between private inflation expectations and the inflation target. Also, we find that a substantial part of the Volcker disinflation is attributed to exogenous belief shocks tracking the restoration of the FED's credibility.

Several extensions of the model could be undertaken. First, in this paper, a short-cut was used to identify the output-neutral real rate. Given the importance of this real rate factor for long-term yields, an important task is to verify further the interpretation of this factor within a learning model. To this end, the learning model could be extended by introducing a complete micro-founded supply side. Such an extension would facilitate the identification of the long-run real interest rate through the supply side constraints and would enrich and refine the set of observable macroeconomic shocks, as in De Graeve et al. (2007). Second, the estimation results establish the statistical significance of the mispricing terms within the setting of a structural Macro-Finance model. This mispricing can be quite substantial, especially at the short end of the yield curve. Further analysis of these mispricing errors and implied liquidity effects seems imperative.

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Tables

Table 1: Definitions and properties of alternative versions of the Macro-Finance model

	Macro Model	**Prices of Risk**	**Expectations**	**Mispricing**
MFS	NK model	Consistent: Λ_0^{IS}	Full-info RE	No
MFM	NK model	Consistent: Λ_0^{IS}	Full-info RE	Yes
MFF	NK model	Free: Λ_0, Λ_1	Full-info RE	No
MFE	NK model	Free: Λ_0, Λ_1	Imp-info RE	Yes

Table 2: Descriptive statistics of the data series

	Infl.	Outp. Gap	Fed rate	Yields						Infl. exp.	
				1/4 yr	1/2 yr	1 yr	3 yr	5 yr	10 yr	1 yr	10 yr
Data	187	187	187	187	187	187	187	187	187	147	109
Mean	0.037	-0.006	0.061	0.055	0.057	0.061	0.065	0.067	0.070	0.040	0.039
Std. Dev.	0.024	0.025	0.034	0.028	0.027	0.028	0.027	0.026	0.024	0.020	0.015
Skewness	1.167	-0.041	1.233	1.031	0.951	0.899	0.891	0.921	0.945	0.909	1.247
Kurtosis	3.881	3.329	5.122	4.460	4.189	4.031	3.756	3.628	3.563	2.958	3.999
	Correlation matrix										
Inflation	1										
Output gap	-0.311	1									
Fed. fund rate	0.737	-0.152	1								
Yield (1/4)	0.717	-0.156	0.992	1							
Yield (1/2)	0.708	-0.169	0.987	0.998	1						
Yield (1)	0.666	-0.192	0.969	0.982	0.989	1					
Yield (3)	0.619	-0.248	0.928	0.950	0.962	0.984	1				
Yield (5)	0.601	-0.283	0.900	0.924	0.938	0.963	0.995	1			
Yield (10)	0.595	-0.327	0.864	0.887	0.901	0.930	0.976	0.992	1		
Infl. exp. (1)	0.873	-0.335	0.896	0.890	0.887	0.863	0.847	0.839	0.836	1	
Infl. exp. (10)	0.835	-0.367	0.876	0.872	0.872	0.857	0.862	0.867	0.875	0.983	1

Table 3: Descriptive statistics of the first difference of the data series

	ΔInfl.	ΔOutp. Gap	ΔFed rate	ΔYields						ΔInfl. exp.	
				1/4 yr	1/2 yr	1 yr	3 yr	5 yr	10 yr	1 yr	10 yr
Data	186	186	186	186	186	186	186	186	186	146	108
Mean($\times 10^2$)	0.014	0.007	0.017	0.007	0.007	0.009	0.006	0.005	0.003	0.001	0.000
Std. Dev.	0.012	0.008	0.013	0.009	0.008	0.010	0.008	0.007	0.006	0.004	0.002
Skewness	-0.111	-0.167	-0.926	-0.893	-0.951	-1.556	-0.727	-0.537	-0.490	0.226	-1.156
Kurtosis	3.695	4.199	16.547	12.906	12.152	14.603	6.863	4.859	5.199	8.191	7.441
	Correlation matrix										
ΔInflation	1										
ΔOutput gap	0.027	1									
ΔFed. fund rate	0.071	0.335	1								
ΔYield (1/4)	0.086	0.370	0.912	1							
ΔYield (1/2)	0.079	0.345	0.862	0.980	1						
ΔYield (1)	0.104	0.153	0.774	0.854	0.894	1					
ΔYield (3)	0.064	0.098	0.604	0.723	0.798	0.940	1				
ΔYield (5)	0.054	0.079	0.518	0.651	0.733	0.875	0.984	1			
ΔYield (10)	0.056	0.031	0.433	0.552	0.644	0.797	0.929	0.968	1		
ΔInfl. exp. (1)	-0.012	0.065	0.102	0.183	0.189	0.173	0.183	0.182	0.187	1	
ΔInfl. exp. (10)	0.098	0.035	0.089	0.122	0.130	0.197	0.229	0.226	0.241	0.470	1

Table 4: Prior distribution of the parameters for the alternative model versions (part I)

Param	**Prior distr**	**Mean**	**Stand.Dev.**	**MFS**	**MFM**	**MFF**	**MFE**
	Standard deviations structural shocks						
σ_π	IG	0.010	0.003	y	y	y	y
σ_y	IG	0.010	0.003	y	y	y	y
σ_i	IG	0.010	0.003	y	y	y	y
σ_{π^*}	U	0.005	0.003	y	y	y	y
σ_ρ	U	0.005	0.003	y	y	y	y
$\sigma_{\pi^{*b}}$	U	0.010	0.006	no	no	no	y
σ_{ρ^b}	U	0.010	0.006	no	no	no	y
	Structural parameters						
δ_π	B	0.700	0.050	y	y	y	y
κ	N	0.120	0.030	y	y	y	y
h	B	0.700	0.050	y	y	y	y
σ	G	1.500	0.335	y	y	y	y
γ_π	N	0.500	0.250	y	y	y	y
γ_y	N	0.500	0.400	y	y	y	y
γ_i	N	0.800	0.200	y	y	y	y
	Initial values						
π_0^*	N	0.020	0.010	y	y	y	y
ρ_0	N	0.020	0.010	y	y	y	y
π_0^{*P}	U	0.02	0.012	no	no	no	y
ρ_0^P	U	0.02	0.012	no	no	no	y
	Autocorrelation parameters						
φ_π	N	0.500	0.500	y	y	y	y
φ_y	N	0.500	0.500	y	y	y	y
φ_i	N	0.500	0.500	y	y	y	y
	Learning						
w_π	B	0.850	0.100	no	no	no	y
w_ρ	B	0.850	0.100	no	no	no	y
g_π	U	0.125	0.075	no	no	no	y
g_ρ	U	0.125	0.075	no	no	no	y

Notes: This table reports the prior distributions used in the estimation. Column 1 presents the parameters. The second column specifies the type of distribution function: B=Beta, G= Gamma, IG= Inverted Gamma, N= Normal and U= Uniform. Columns 3 and 4 report the mean and standard deviation as implied by the respective prior distributions. Columns 5 till 8 indicate whether or not the prior is used (and the parameter is estimated) in the respective models. The entry 'y' implies the parameter is estimated and the prior is used; The entry 'no' indicates the prior (nor the parameter) is used in the version of the model.

Table 5: Prior distribution of the parameters for the alternative model versions (part II)

Param	**Prior distr**	**Mean**	**Stand.Dev.**	**MFS**	**MFM**	**MFF**	**MFE**
	Average mispricing: ϕ						
$\phi(1/2)$	N	0.000	0.005	no	y	no	y
$\phi(1)$	N	0.000	0.005	no	y	no	y
$\phi(3)$	N	0.000	0.005	no	y	no	y
$\phi(5)$	N	0.000	0.005	no	y	no	y
$\phi(10)$	N	0.000	0.005	no	y	no	y
	Standard deviation measurement errors yields						
$\sigma_{\eta,y}(1/4)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,y}(1/2)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,y}(1)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,y}(3)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,y}(5)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,y}(10)$	IG	0.005	0.003	y	y	y	y
	Standard deviation measurement errors inflation expectations						
$\sigma_{\eta,\pi}(1)$	IG	0.005	0.003	y	y	y	y
$\sigma_{\eta,\pi}(10)$	IG	0.005	0.003	y	y	y	y
	Prices of risk, $\Lambda_0(\times 10^{-2})$						
$\Lambda_{0,\pi}$	N	-0.050	0.150	imp	imp	y	y
$\Lambda_{0,y}$	N	-0.050	0.150	imp	imp	y	y
$\Lambda_{0,i}$	N	-0.050	0.150	imp	imp	y	y
Λ_{0,π^*}	N	-0.050	0.150	imp	imp	y	y
$\Lambda_{0,\rho}$	N	-0.050	0.150	imp	imp	y	y
	Prices of risk: $\Lambda_1(\times 10^{-2})$						
$\Lambda_{1,\pi\pi}$	N	-0.050	0.150	no	no	y	y
$\Lambda_{1,\pi y}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,\pi i}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,y\pi}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,yy}$	N	0.050	0.150	no	no	y	y
$\Lambda_{1,yi}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,i\pi}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,iy}$	N	0.000	0.500	no	no	y	y
$\Lambda_{1,ii}$	N	-0.050	0.150	no	no	y	y

Notes: This table reports the prior distributions used in the estimation. Column 1 presents the parameters. The second column specifies the type of distribution function: B=Beta, G= Gamma, IG= Inverted Gamma, N= Normal and U= Uniform. Columns 3 and 4 report the mean and standard deviation as implied by the respective prior distributions. Columns 5 till 8 indicate whether or not the prior is used (and the parameter is estimated) in the respective models. The entry 'y' implies the parameter is estimated and the prior is used; The entry 'no' indicates the prior (nor the parameter) is used in the version of the model. Finally 'imp' refers to the fact that the parameter (and the prior) is implied by other structural parameters.

Table 6: Model performance: Marginal likelihood and BIC

Log Marginal likelihood and BIC						
Model	NK0	MF1	MFS	MFM	MFF	MFE
Marg. Lik	6124	7240	7381	7628	7638	7741
BIC	-12387	-14591	-14815	-15333	-15384	-15442

Decomposition of BIC						
Model	NK0	MF1	MFS	MFM	MFF	MFE
Macro (-2lnlik)	-3414	-3495	-3760	-3711	-3745	-3772
Yields (-2lnlik)	-7413	-9187	-9037	-9392	-9504	-9538
Infl. exp.(-2lnlik)	-1683	-1447	-2227	-2197	-2182	-2256
Penalty	131	136	131	157	194	272

Notes: The marginal likelihood was computed using the modified harmonic mean procedure of Geweke. The findings are robust to alternative cut-off levels. The BIC refers to the standard Bayesian Information Criterion and was computed at the mode of the posterior distribution. The decomposition of the likelihood and the BIC are based on the likelihood of the prediction errors of the respective data series. The acronyms refer to the following model specifications. NK0: the New-Keynesian model without stochastic endpoints, MF1: the standard Macro-Finance model with a stochastic endpoint for inflation. The acronyms MFS, MFM, MFF and MFE refer to the alternative versions of the model, i.e. respectively the structural version, the version with liquidity effects, the flexible version and the encompassing model.

Table 7: Posterior density estimates I: Encompassing model (MFE)

Param	**Mean**	**Std. Dev**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Standard deviations structural shocks				
σ_π	0.0118	0.0010	0.0120	0.0101	0.0136
σ_y	0.0032	0.0003	0.0031	0.0027	0.0038
σ_i	0.0117	0.0007	0.0119	0.0110	0.0131
σ_{π^*}	0.0015	0.0010	0.0004	0.0001	0.0034
σ_ρ	0.0074	0.0014	0.0073	0.0039	0.0083
$\sigma_{\pi^{*b}}$	0.0045	0.0014	0.0058	0.0015	0.0067
σ_{ρ^b}	0.0121	0.0058	0.0050	0.0017	0.0194
	Structural parameters				
δ_π	0.5337	0.0327	0.5288	0.4829	0.5891
κ	0.0137	0.0042	0.0117	0.0078	0.0212
h	0.7512	0.0445	0.7566	0.6741	0.8179
σ	2.6779	0.4004	2.5551	1.9344	3.2328
γ_π	0.3707	0.1081	0.4389	0.2296	0.5824
γ_y	0.6673	0.1638	0.6341	0.4931	1.0214
γ_i	0.6827	0.0406	0.6896	0.6462	0.7849
	Initial values				
π_0^*	0.0182	0.0085	0.0184	0.0051	0.0332
ρ_0	0.0204	0.0072	0.0197	0.0061	0.0308
π_0^{*b}	0.0057	0.0039	0.0037	0.0006	0.0125
ρ_0^b	0.0240	0.0087	0.0231	0.0078	0.0366
	Autocorrelation parameters				
φ_π	-0.3657	0.0839	-0.3781	-0.5056	-0.2332
φ_y	0.6285	0.0472	0.6489	0.5675	0.7230
φ_i	-0.1609	0.0603	-0.1531	-0.2175	-0.0228
	Learning parameters				
w_π	0.6083	0.0645	0.6550	0.4897	0.6907
w_ρ	0.7789	0.1441	0.9746	0.5858	0.9872
g_π	0.2191	0.0255	0.2190	0.1668	0.2479
g_ρ	0.1200	0.0746	0.0443	0.0117	0.2392

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 8: Posterior density estimates II: Encompassing model (MFE)

Param	**Mean**	**Std. Dev**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Average mispricing yields				
$\phi(1/2)$	-0.0034	0.0016	-0.0032	-0.0054	-0.0001
$\phi(1)$	-0.0001	0.0019	0.0003	-0.0023	0.0038
$\phi(3)$	0.0010	0.0021	0.0017	-0.0010	0.0058
$\phi(5)$	0.0011	0.0022	0.0018	-0.0008	0.0063
$\phi(10)$	0.0010	0.0038	0.0013	-0.0023	0.0095
	Standard deviation measurement errors yield curve				
$\sigma_{\eta,y}(1/4)$	0.0103	0.0005	0.0101	0.0094	0.0111
$\sigma_{\eta,y}(1/2)$	0.0044	0.0003	0.0044	0.0040	0.0049
$\sigma_{\eta,y}(1)$	0.0040	0.0002	0.0040	0.0037	0.0043
$\sigma_{\eta,y}(3)$	0.0020	0.0001	0.0019	0.0018	0.0022
$\sigma_{\eta,y}(5)$	0.0008	0.0001	0.0008	0.0006	0.0010
$\sigma_{\eta,y}(10)$	0.0035	0.0002	0.0034	0.0032	0.0039
	Standard deviation measurement errors inflation expectations				
$\sigma_{\eta,\pi}(1)$	0.0052	0.0004	0.0051	0.0046	0.0058
$\sigma_{\eta,\pi}(10)$	0.0010	0.0001	0.0010	0.0008	0.0012

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 9: Posterior density estimates III: Encompassing model (MFE)

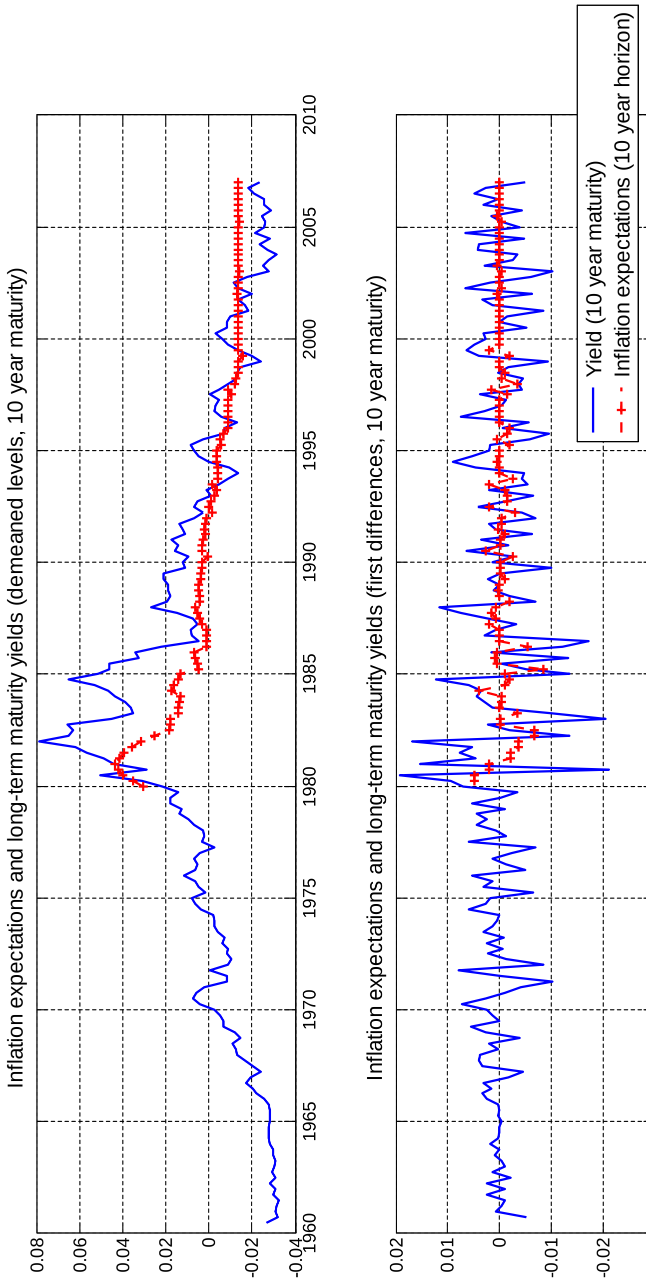
Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Price of risk: $\Lambda_0(\times 10^{-2})$				
$\Lambda_{0,\pi}$	-0.0700	0.1379	-0.1257	-0.3218	0.1365
$\Lambda_{0,y}$	-0.0675	0.1359	0.0432	-0.2558	0.1843
$\Lambda_{0,i}$	-0.0844	0.1456	-0.0193	-0.3322	0.1576
Λ_{0,π^*}	-0.0576	0.1680	-0.0559	-0.2970	0.2323
$\Lambda_{0,\rho}$	-0.1026	0.0796	-0.1144	-0.2128	0.0385
	Price of risk: $\Lambda_1(\times 10^{-4})$				
$\Lambda_{1,\pi\pi}$	0.0728	0.0779	-0.0030	-0.0456	0.1973
$\Lambda_{1,\pi y}$	0.3139	0.0944	0.2782	0.1737	0.4842
$\Lambda_{1,\pi i}$	-1.1067	0.2303	-0.9401	-1.5123	-0.7743
$\Lambda_{1,y\pi}$	-0.1302	0.3327	-0.0016	-0.8500	0.2734
$\Lambda_{1,yy}$	0.1051	0.1363	0.0575	-0.1463	0.3112
$\Lambda_{1,yi}$	-0.4329	0.4250	-0.5493	-1.0148	0.3596
$\Lambda_{1,i\pi}$	-0.0445	0.0469	-0.0382	-0.1268	0.0337
$\Lambda_{1,iy}$	-0.0274	0.0363	-0.0198	-0.0887	0.0326
$\Lambda_{1,ii}$	0.5592	0.0718	0.5353	0.4471	0.6808

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 10: Variance decomposition of the yield curve: Encompassing model (MFE)

Type of shock	Fed fund rate	Level	Slope	curvature	Infl exp 1y	Infl exp 10y
Frequency: 1 quarter						
Supply (ε_π)	0.04	0.08	0.01	0.13	0.90	0.35
Demand (ε_y)	0.04	0.04	0.03	0.22	0.02	0.00
Policy rate(ε_i)	0.81	0.33	0.88	0.63	0.00	0.00
Belief inflat.(η_π)	0.00	0.03	0.01	0.00	0.08	0.64
Belief real rate(η_ρ)	0.00	0.00	0.00	0.00	0.00	0.00
Infl. target (ε_{π^*})	0.00	0.00	0.00	0.00	0.00	0.00
Neutral real rate(ε_ρ)	0.11	0.52	0.06	0.02	0.00	0.00
Frequency: 4 quarters						
Supply (ε_π)	0.09	0.08	0.05	0.13	0.73	0.27
Demand (ε_y)	0.16	0.07	0.24	0.23	0.03	0.00
Policy rate(ε_i)	0.43	0.12	0.65	0.62	0.00	0.00
Belief inflat.(η_π)	0.02	0.05	0.01	0.00	0.24	0.73
Belief real rate(η_ρ)	0.00	0.00	0.00	0.00	0.00	0.00
Infl. target (ε_{π^*})	0.00	0.00	0.00	0.00	0.00	0.00
Neutral real rate(ε_ρ)	0.30	0.68	0.05	0.02	0.00	0.00
Frequency: 20 quarters						
Supply (ε_π)	0.06	0.04	0.06	0.13	0.42	0.21
Demand (ε_y)	0.07	0.02	0.32	0.24	0.03	0.00
Policy rate(ε_i)	0.12	0.03	0.55	0.60	0.00	0.00
Belief inflat.(η_π)	0.09	0.09	0.03	0.01	0.57	0.79
Belief real rate(η_ρ)	0.00	0.00	0.00	0.00	0.00	0.00
Infl. target (ε_{π^*})	0.00	0.00	0.00	0.00	0.00	0.00
Neutral real rate(ε_ρ)	0.66	0.82	0.04	0.02	0.00	0.00
Frequency: 40 quarters						
Supply (ε_π)	0.04	0.03	0.06	0.13	0.32	0.20
Demand (ε_y)	0.04	0.01	0.32	0.23	0.02	0.00
Policy rate(ε_i)	0.07	0.02	0.53	0.59	0.00	0.00
Belief inflat.(η_π)	0.10	0.09	0.05	0.03	0.66	0.80
Belief real rate(η_ρ)	0.00	0.00	0.00	0.00	0.00	0.00
Infl. target (ε_{π^*})	0.00	0.00	0.00	0.00	0.00	0.00
Neutral real rate(ε_ρ)	0.75	0.85	0.04	0.02	0.00	0.00

Figure 1: Long-term yields and long-term inflation expectations



Inflation expectations and long-term maturity yields (demeaned levels, 10 year maturity)

Inflation expectations and long-term maturity yields (first differences, 10 year maturity)

Figure 2: Macroeconomic factors implied by the mode of the posterior distribution of the MFE model

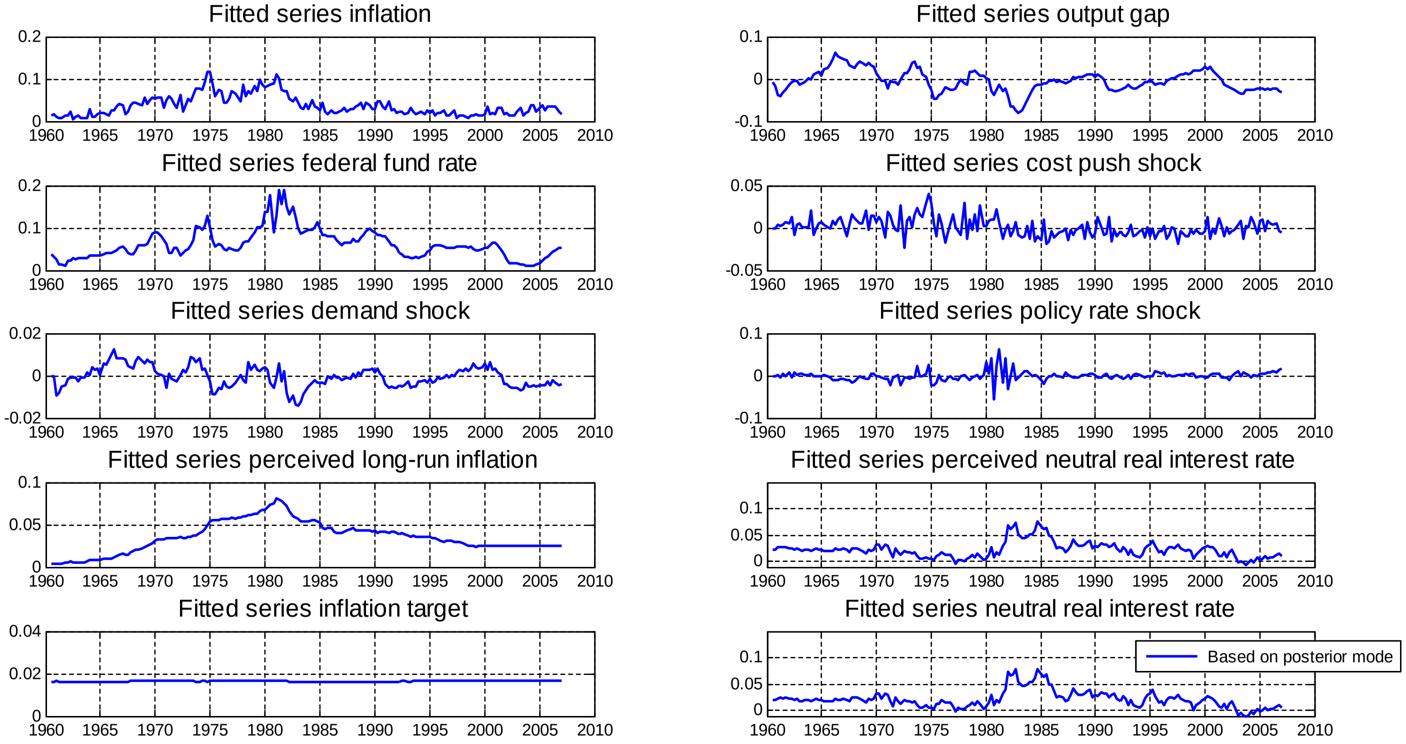


Figure 3: Perceived and actual stochastic endpoints for inflation and the real interest rate (MFE version)

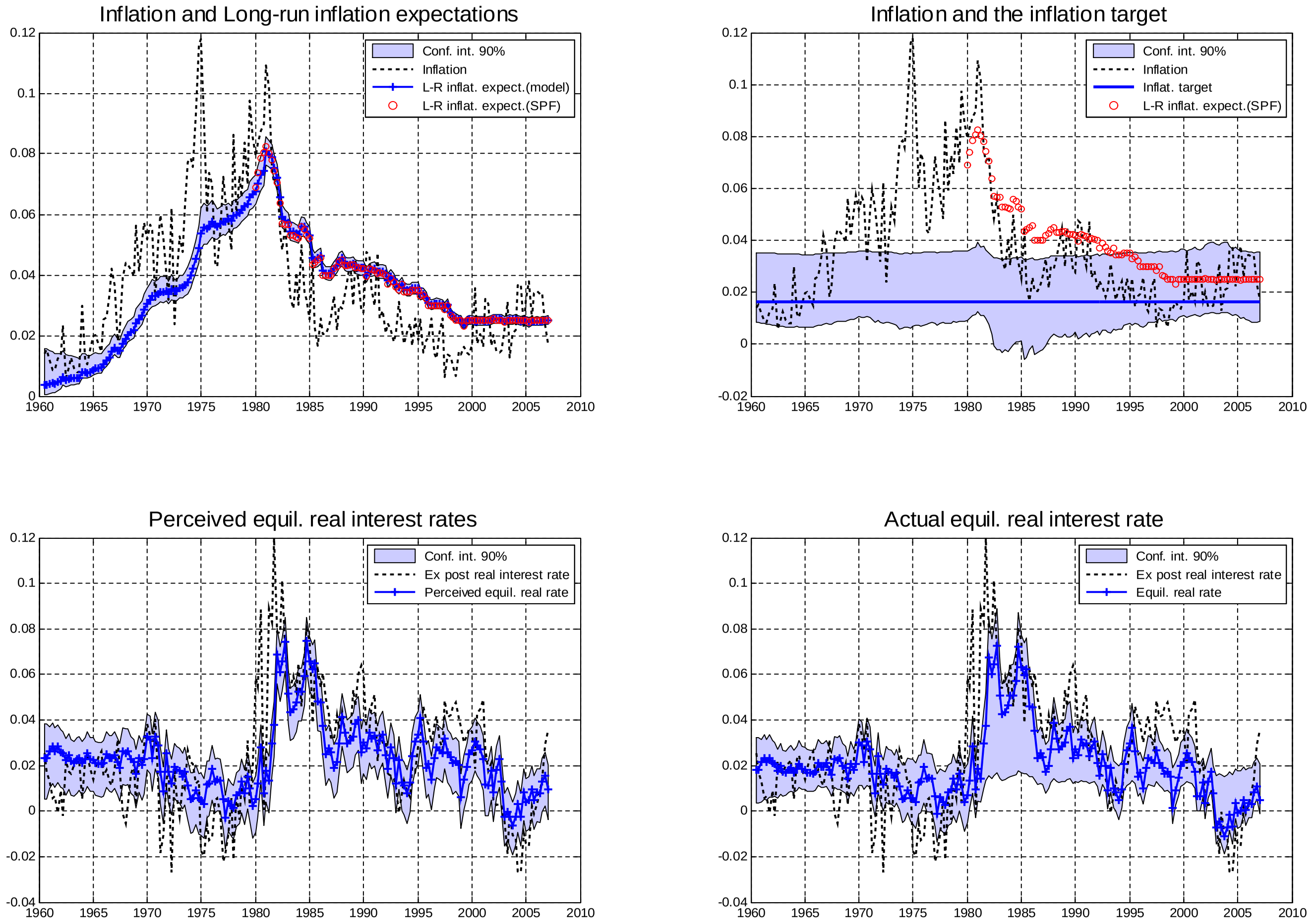


Figure 4: Factor loadings of the Yield Curve (MFE version)

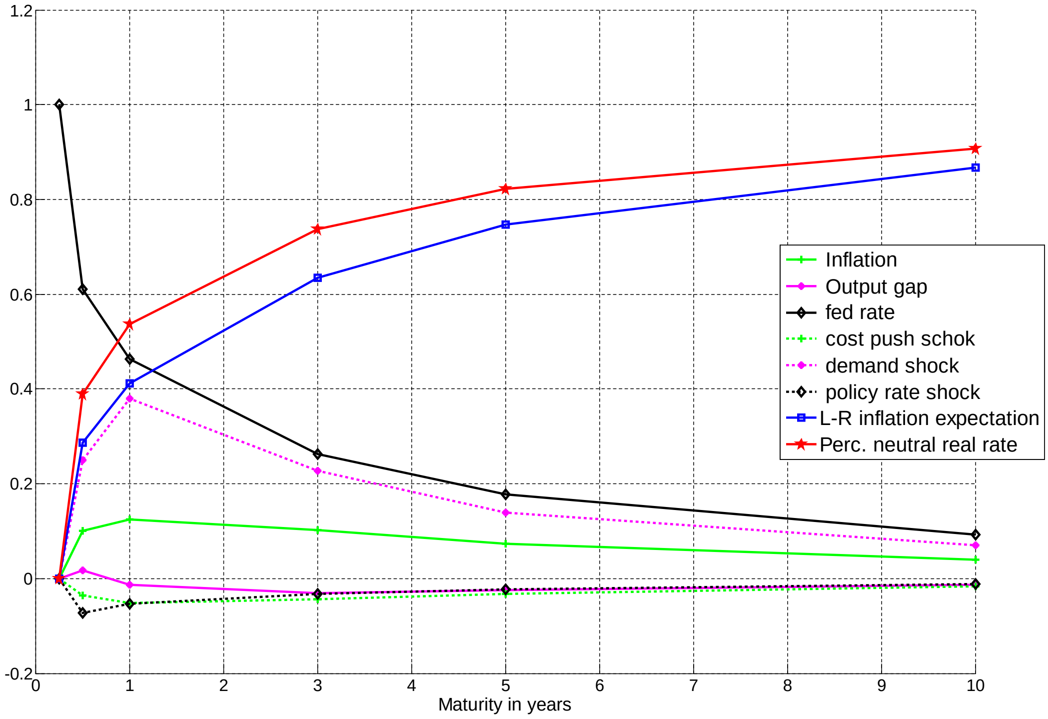


Figure 5: Observed and fitted yield curve of the MFE model

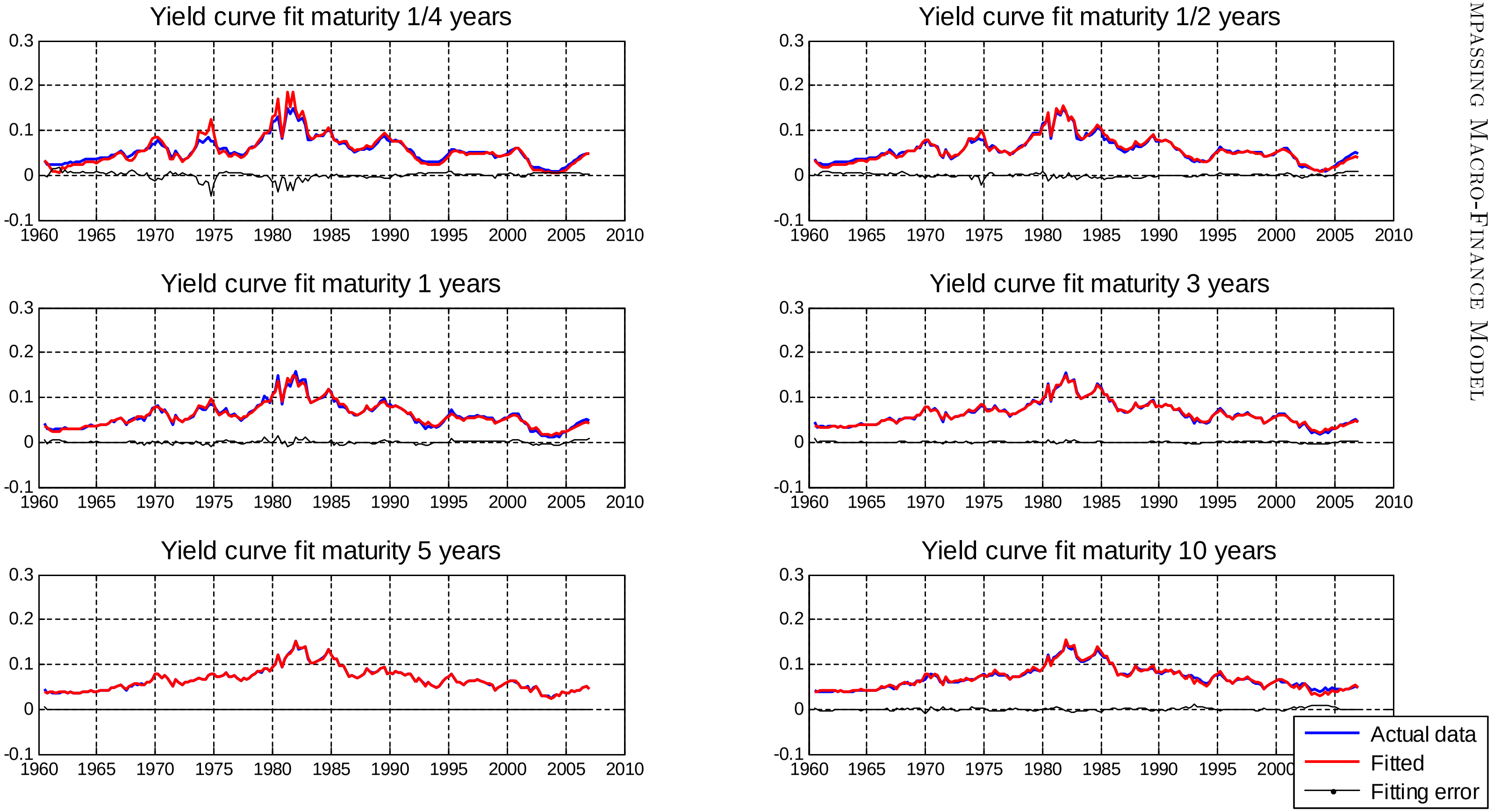


Figure 6: Decomposition of the variation in the long-term yield and inflation expectatiions (10 yr.)

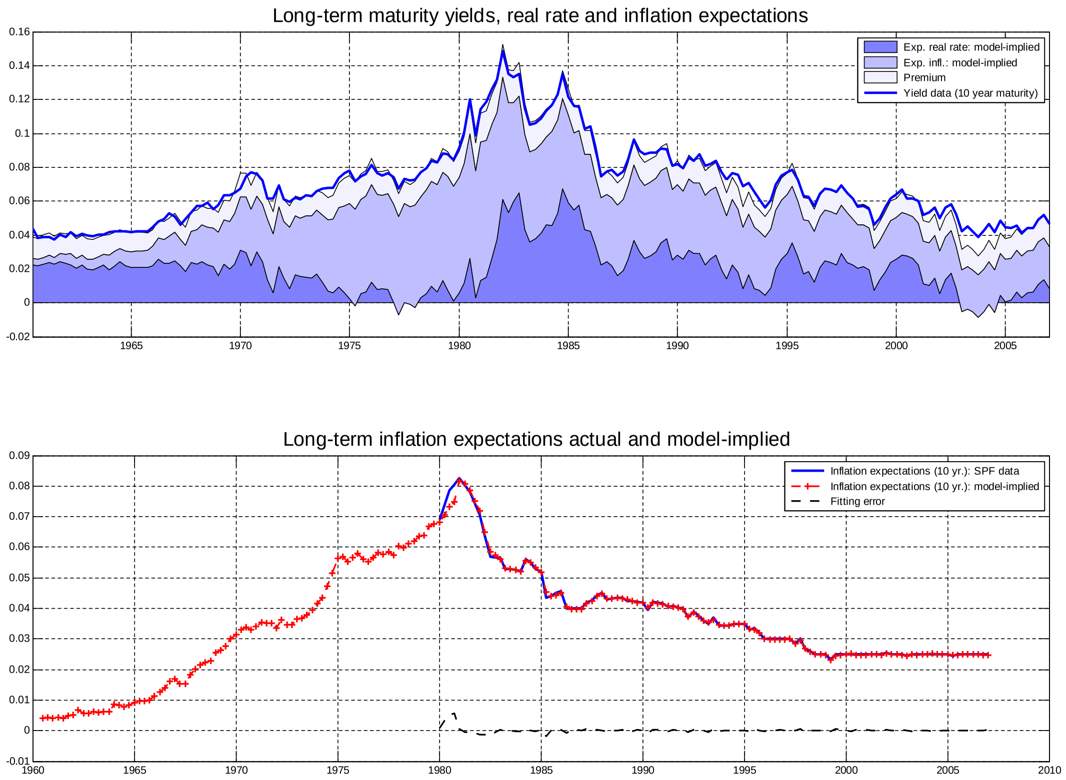


Figure 7: Expected excess holding return across the maturities based on MFE model

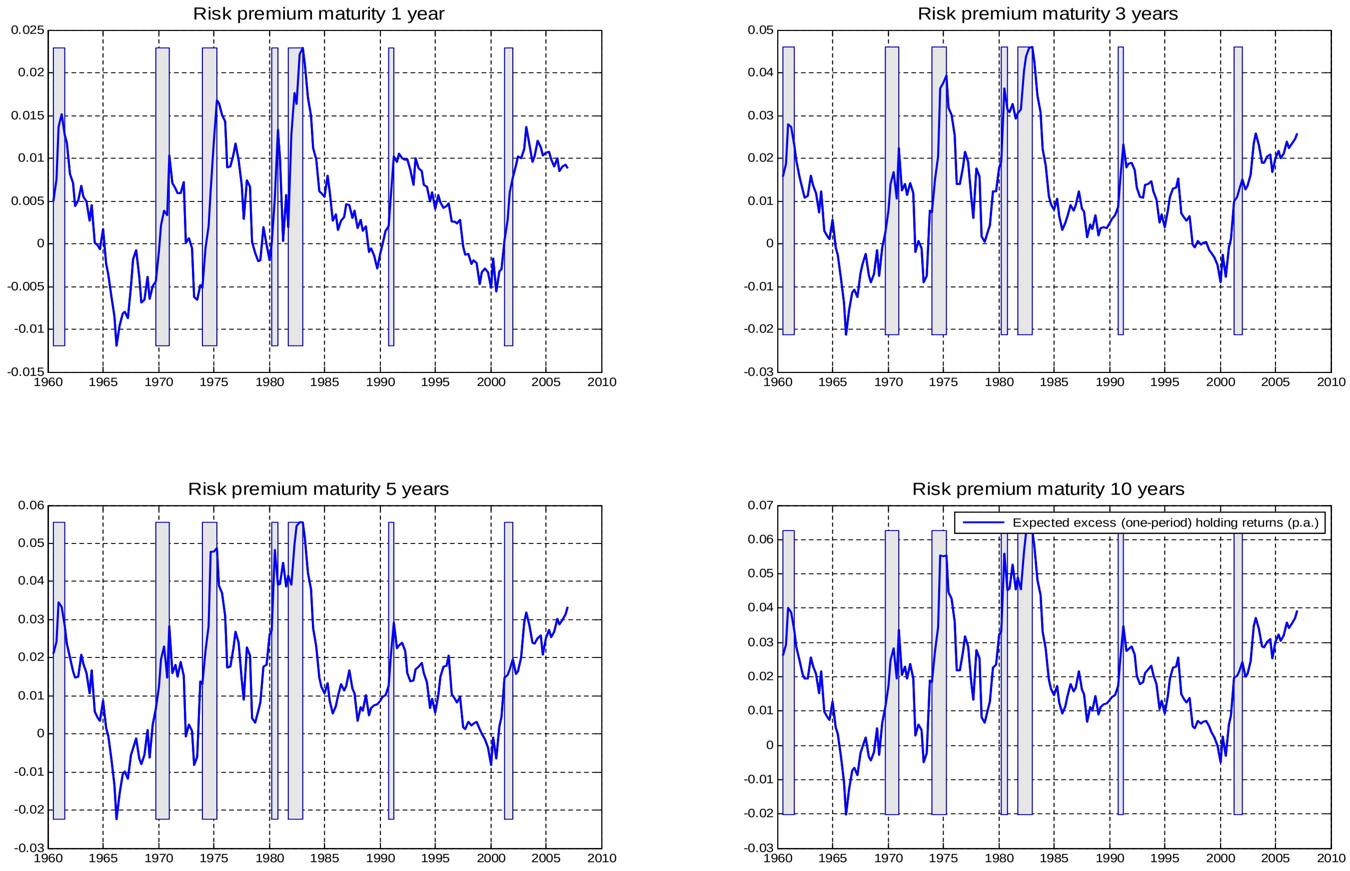


Figure 8: Instantaneous impulse response functions of the yield curve MFE model

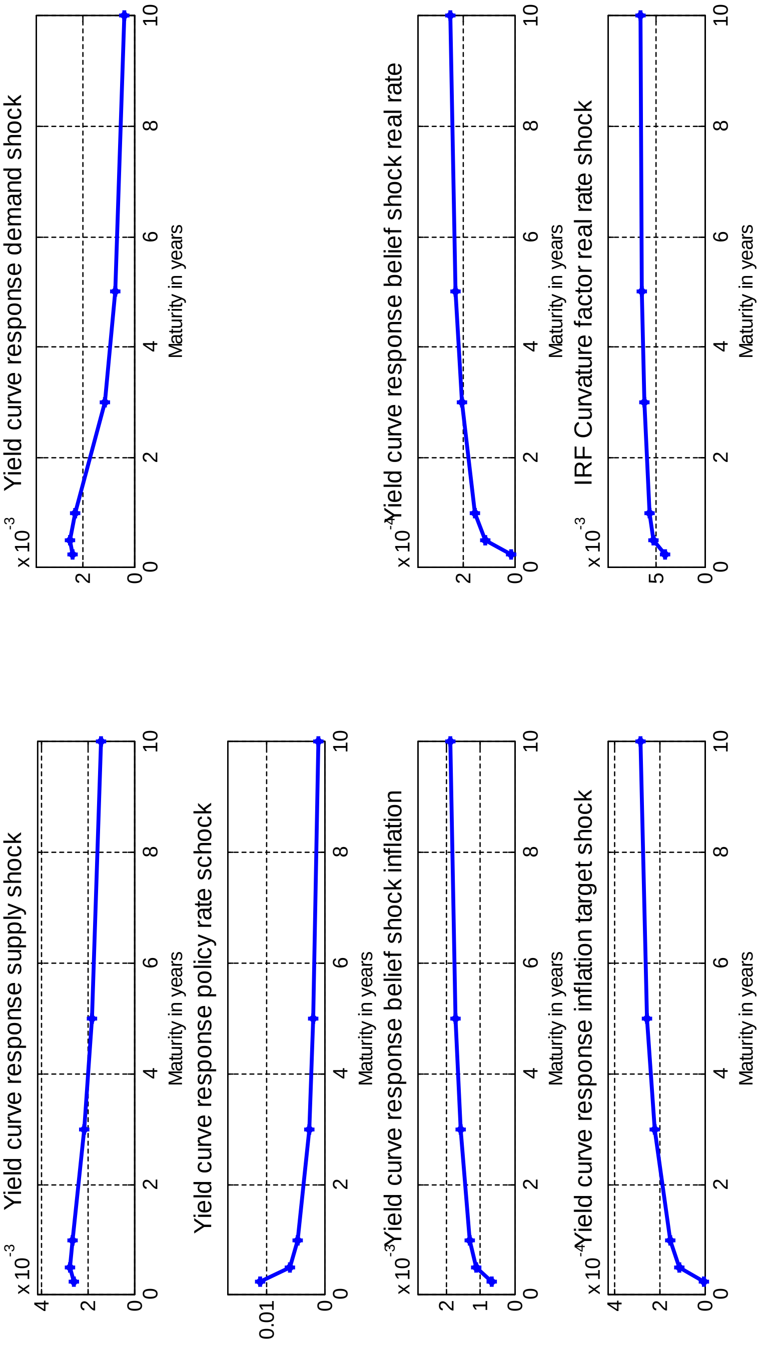


Figure 9: Historical decomposition based on the MFE model of the Great Inflation period

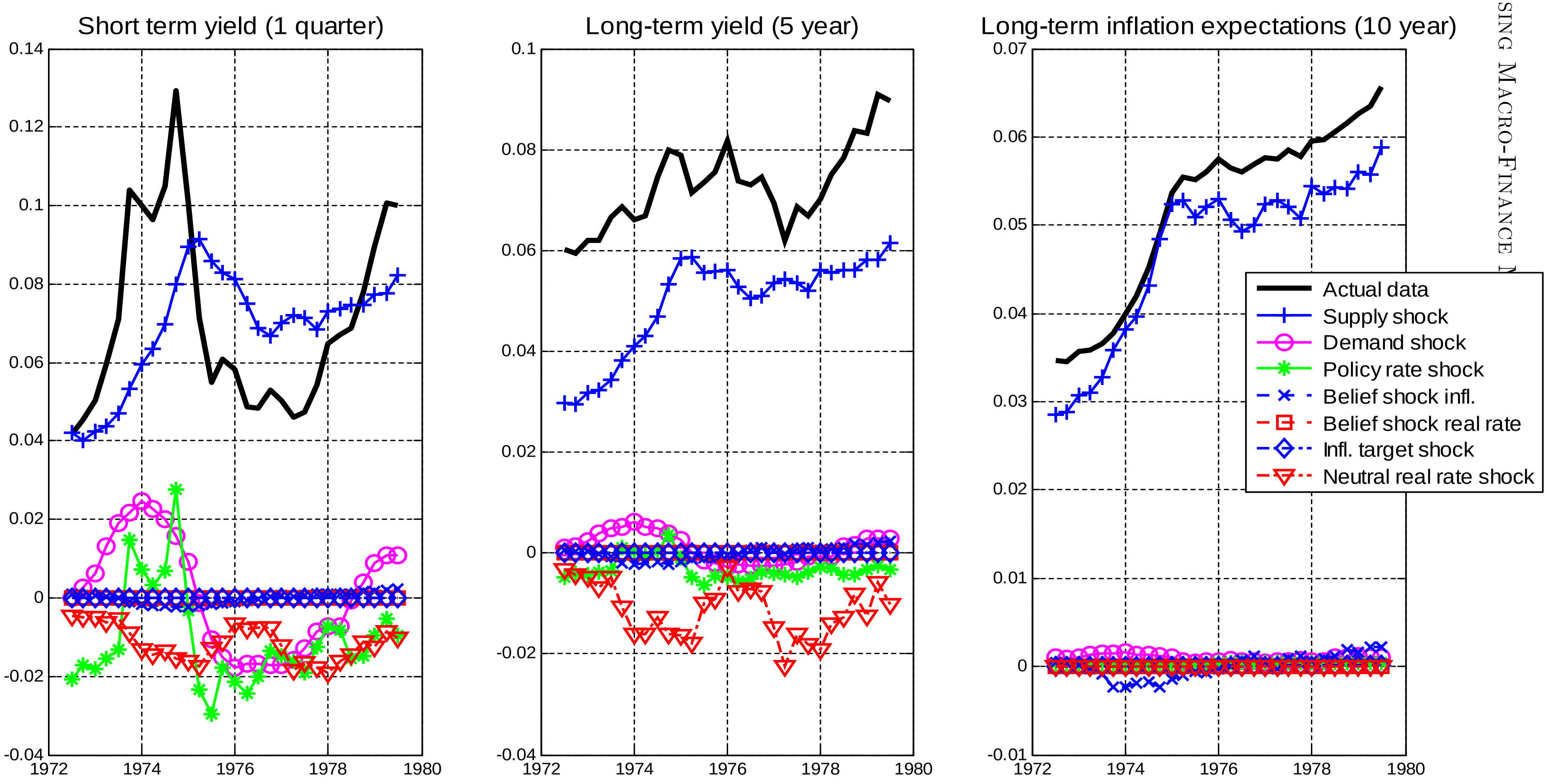
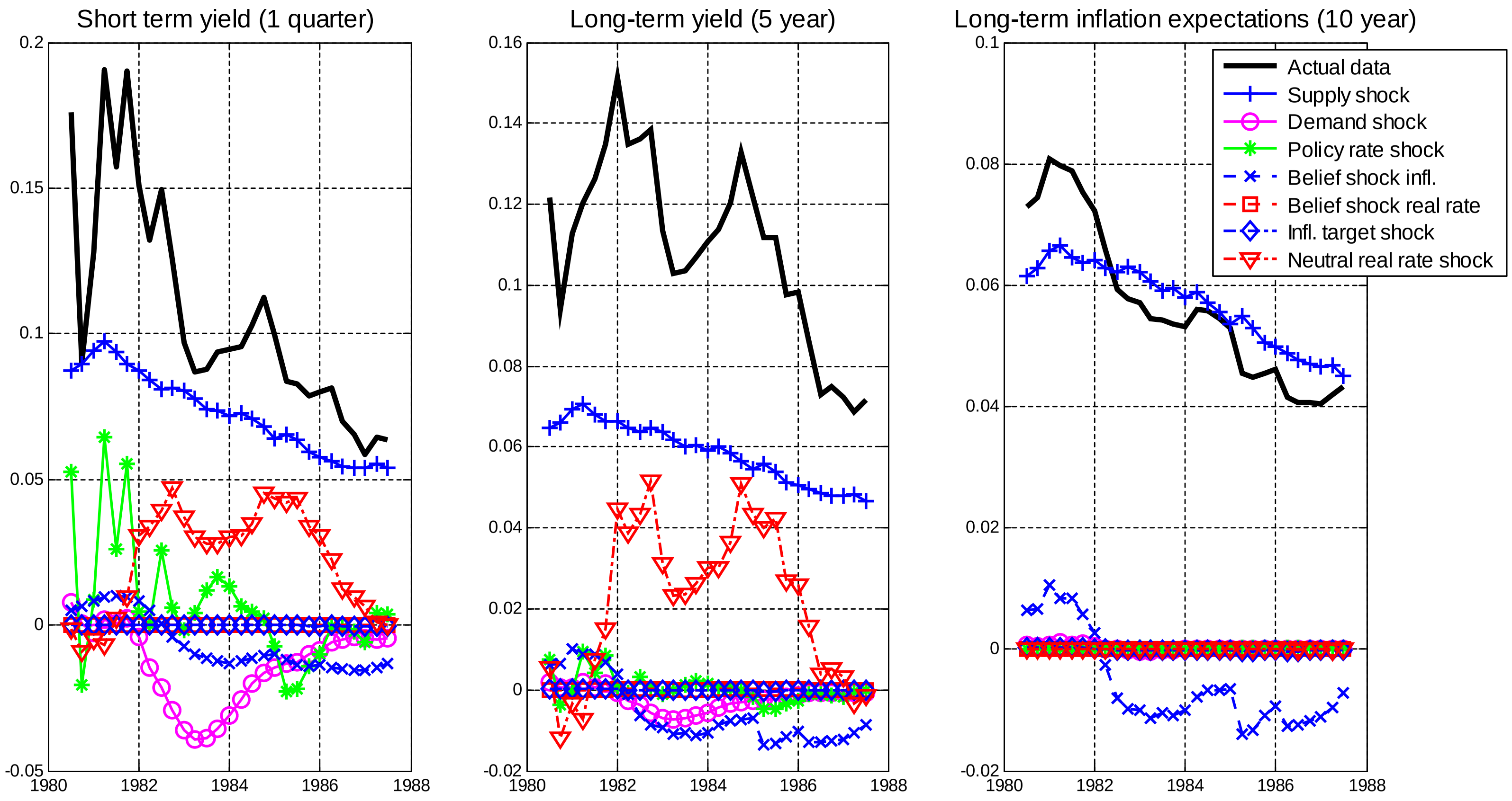


Figure 10: Historical decomposition based on the MFE model of the Great Disinflation period



6 Appendix: Tables posterior of parameters for alternative model versions

In this appendix tables for the posterior of the alternative model versions are presented. We restrict the reporting to the posteriors of the models allowing for autocorrelation. In particular, we present results for the MFS, the MFM and the MFF versions of the model.

Table 11: Posterior density estimates I: Structural rational expectation model, MFS

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Standard deviations structural shocks				
σ_π	0.0097	0.0009	0.0096	0.0085	0.0113
σ_y	0.0031	0.0003	0.0031	0.0027	0.0037
σ_i	0.0111	0.0006	0.0111	0.0102	0.0121
σ_{π^*}	0.0029	0.0001	0.0029	0.0027	0.0032
σ_ρ	0.0049	0.0002	0.0049	0.0045	0.0052
$\sigma_{\pi^{*b}}$	-	-	-	-	-
σ_{ρ^b}	-	-	-	-	-
	Structural parameters				
δ_π	0.5343	0.0703	0.5063	0.4405	0.6723
κ	0.0085	0.0037	0.0072	0.0035	0.0153
h	0.7022	0.0483	0.7057	0.6185	0.7764
σ	2.1574	0.354	2.0651	1.6220	2.7805
γ_π	0.4105	0.1098	0.4056	0.2295	0.596
γ_y	0.3765	0.0575	0.3697	0.2855	0.4729
γ_i	0.6205	0.0429	0.6296	0.5450	0.6817
	Initial values				
π_0^*	0.0291	0.0042	0.0291	0.0222	0.0359
ρ_0	0.0158	0.0050	0.0161	0.0074	0.0241
π_0^{*b}	-	-	-	-	-
ρ_0^b	-	-	-	-	-
	Autocorrelation parameters				
φ_π	-0.2476	0.0858	-0.2326	-0.395	-0.1189
φ_y	0.666	0.0384	0.6702	0.6009	0.7282
φ_i	-0.2417	0.0653	-0.2409	-0.3473	-0.1357
	Learning parameters				
w_π	-	-	-	-	-
w_ρ	-	-	-	-	-
g_π	-	-	-	-	-
g_ρ	-	-	-	-	-

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 12: POSTERIOR DENSITY ESTIMATES II: STRUCTURAL RATIONAL EXPECTATION MODEL, MFS

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Average mispricing yields				
$\phi(1/2)$	-	-	-	-	-
$\phi(1)$	-	-	-	-	-
$\phi(3)$	-	-	-	-	-
$\phi(5)$	-	-	-	-	-
$\phi(10)$	-	-	-	-	-
	Standard deviation measurement errors yield curve				
$\sigma_{\eta,y}(1/4)$	0.0102	0.0005	0.0102	0.0094	0.0112
$\sigma_{\eta,y}(1/2)$	0.0080	0.0004	0.008	0.0073	0.0088
$\sigma_{\eta,y}(1)$	0.0054	0.0003	0.0054	0.005	0.0059
$\sigma_{\eta,y}(3)$	0.0027	0.0002	0.0026	0.0024	0.0029
$\sigma_{\eta,y}(5)$	0.0010	0.0002	0.001	0.0008	0.0013
$\sigma_{\eta,y}(10)$	0.0059	0.0003	0.0058	0.0053	0.0065
	Standard deviation measurement errors inflation expectations				
$\sigma_{\eta,\pi}(1)$	0.0060	0.0010	0.0055	0.0047	0.0081
$\sigma_{\eta,\pi}(10)$	0.0011	0.0002	0.0010	0.0008	0.0014

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 13: Posterior density estimates I: Rational expectation model, allowing for mispricing, MFM

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Standard deviations structural shocks				
σ_π	0.0104	0.0009	0.0102	0.0089	0.0119
σ_y	0.0031	0.0003	0.0031	0.0026	0.0037
σ_i	0.0117	0.0006	0.0117	0.0107	0.0127
σ_{π^*}	0.0033	0.0002	0.0033	0.0030	0.0037
σ_ρ	0.0065	0.0004	0.0065	0.0059	0.0073
$\sigma_{\pi^{*b}}$	-	-	-	-	-
σ_{ρ^b}	-	-	-	-	-
	Structural parameters				
δ_π	0.4453	0.0292	0.4468	0.4006	0.4956
κ	0.0098	0.0039	0.0089	0.0042	0.0167
h	0.7559	0.046	0.7589	0.6772	0.8300
σ	2.7394	0.411	2.7017	2.1159	3.4655
γ_π	0.1963	0.0801	0.1692	0.0762	0.3359
γ_y	0.2019	0.0384	0.2015	0.1358	0.2637
γ_i	0.5232	0.0404	0.5326	0.4496	0.5822
	Initial values				
π_0^*	0.0202	0.0043	0.0205	0.013	0.0273
ρ_0	0.0191	0.0059	0.0188	0.0097	0.0293
π_0^{*b}	-	-	-	-	-
ρ_0^b	-	-	-	-	-
	Autocorrelation parameters				
φ_π	-0.1676	0.0634	-0.1618	-0.2736	-0.0648
φ_y	0.5894	0.0494	0.5923	0.5022	0.6624
φ_i	-0.1800	0.0762	-0.1887	-0.2984	-0.0521
	Learning parameters				
w_π	-	-	-	-	-
w_ρ	-	-	-	-	-
g_π	-	-	-	-	-
g_ρ	-	-	-	-	-

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 14: Posterior density estimates II: Rational expectation model, allowing for mispricing, MFM

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Average mispricing yields				
$\phi(1/2)$	-0.0048	0.0006	-0.0047	-0.0057	-0.0038
$\phi(1)$	-0.0016	0.0007	-0.0016	-0.0028	-0.0004
$\phi(3)$	0.0014	0.0011	0.0014	-0.0003	0.0034
$\phi(5)$	0.0041	0.0011	0.0041	0.0022	0.0062
$\phi(10)$	0.0106	0.0013	0.0106	0.0085	0.0130
	Standard deviation measurement errors yield curve				
$\sigma_{\eta,y}(1/4)$	0.0102	0.0005	0.0101	0.0094	0.0111
$\sigma_{\eta,y}(1/2)$	0.0057	0.0003	0.0057	0.0052	0.0063
$\sigma_{\eta,y}(1)$	0.0041	0.0002	0.0041	0.0038	0.0045
$\sigma_{\eta,y}(3)$	0.0021	0.0001	0.0021	0.0019	0.0023
$\sigma_{\eta,y}(5)$	0.0008	0.0001	0.0008	0.0006	0.0010
$\sigma_{\eta,y}(10)$	0.0036	0.0002	0.0036	0.0033	0.0040
	Standard deviation measurement errors inflation expectations				
$\sigma_{\eta,\pi}(1)$	0.0047	0.0003	0.0047	0.0042	0.0052
$\sigma_{\eta,\pi}(10)$	0.0010	0.0002	0.0010	0.0008	0.0013

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 15: Posterior density estimates I: Rational expectation model allowing for flexible prices of risk, MFF

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Standard deviations structural shocks				
σ_π	0.0115	0.0010	0.0118	0.0098	0.0132
σ_y	0.0032	0.0004	0.0033	0.0026	0.0038
σ_i	0.0128	0.0006	0.0138	0.0121	0.0141
σ_{π^*}	0.0035	0.0002	0.0034	0.0031	0.0039
σ_ρ	0.0069	0.0003	0.0069	0.0064	0.0075
$\sigma_{\pi^{*b}}$	-	-	-	-	-
σ_{ρ^b}	-	-	-	-	-
	Structural parameters				
δ_π	0.5010	0.0298	0.4726	0.4440	0.5373
κ	0.0117	0.0038	0.0095	0.0057	0.0184
h	0.7574	0.0444	0.7136	0.6770	0.8285
σ	2.9300	0.3636	3.4849	2.4057	3.6332
γ_π	0.3493	0.1138	0.3273	0.2161	0.5339
γ_y	0.7060	0.1433	0.7402	0.4649	0.9662
γ_i	0.7529	0.0262	0.7798	0.7047	0.7976
	Initial values				
π_0^*	0.0194	0.0048	0.0241	0.0123	0.0268
ρ_0	0.0133	0.0033	0.0179	0.0085	0.0193
π_0^{*b}	-	-	-	-	-
ρ_0^b	-	-	-	-	-
	Autocorrelation parameters				
φ_π	-0.2552	0.0701	-0.2370	-0.3501	-0.1186
φ_y	0.6202	0.0501	0.5858	0.5300	0.6845
φ_i	-0.1275	0.0423	-0.0864	-0.1891	-0.0494
	Learning parameters				
w_π	-	-	-	-	-
w_ρ	-	-	-	-	-
g_π	-	-	-	-	-
g_ρ	-	-	-	-	-

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 16: Posterior density estimates II: Rational expectation model allowing for flexible prices of risk, MFF

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Average mispricing yields				
$\phi(1/2)$	-	-	-	-	-
$\phi(1)$	-	-	-	-	-
$\phi(3)$	-	-	-	-	-
$\phi(5)$	-	-	-	-	-
$\phi(10)$	-	-	-	-	-
	Standard deviation measurement errors yield curve				
$\sigma_{\eta,y}(1/4)$	0.0103	0.0005	0.0099	0.0095	0.0113
$\sigma_{\eta,y}(1/2)$	0.0049	0.0003	0.0050	0.0045	0.0053
$\sigma_{\eta,y}(1)$	0.0042	0.0002	0.0041	0.0037	0.0045
$\sigma_{\eta,y}(3)$	0.0020	0.0001	0.0019	0.0018	0.0022
$\sigma_{\eta,y}(5)$	0.0008	0.0001	0.0007	0.0006	0.0010
$\sigma_{\eta,y}(10)$	0.0035	0.0002	0.0034	0.0032	0.0038
	Standard deviation measurement errors inflation expectations				
$\sigma_{\eta,\pi}(1)$	0.0046	0.0003	0.0043	0.0042	0.0051
$\sigma_{\eta,\pi}(10)$	0.0011	0.0002	0.0010	0.0008	0.0014

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

Table 17: POSTERIOR DENSITY ESTIMATES III: RATIONAL EXPECTATION MODEL ALLOWING FOR FLEXIBLE PRICES OF RISK, MFF

Param	**Mean**	**Std. Dev.**	**Mode**	**Crit.val.** 5%	**Crit. val.** 95%
	Price of risk: $\Lambda_0(\times 10^{-2})$				
$\Lambda_{0,\pi}$	-0.2077	0.0906	-0.0889	-0.2905	-0.0447
$\Lambda_{0,y}$	-0.1883	0.1236	-0.1213	-0.3643	0.0026
$\Lambda_{0,i}$	-0.1716	0.0609	-0.2017	-0.2918	-0.1126
Λ_{0,π^*}	-0.1159	0.0930	-0.1315	-0.2956	0.0751
$\Lambda_{0,\rho}$	-0.0759	0.0266	-0.0620	-0.1304	-0.0325
	Price of risk: $\Lambda_1(\times 10^{-4})$				
$\Lambda_{1,\pi\pi}$	0.1580	0.0661	0.1628	0.0673	0.2798
$\Lambda_{1,\pi y}$	0.4902	0.0904	0.4948	0.3262	0.6406
$\Lambda_{1,\pi i}$	-1.3656	0.2117	-1.5239	-1.7286	-1.0038
$\Lambda_{1,y\pi}$	-0.4380	0.3497	-0.1740	-0.9482	0.2550
$\Lambda_{1,yy}$	-0.0363	0.0820	0.0768	-0.1751	0.0592
$\Lambda_{1,yi}$	-0.3625	0.5253	-0.5938	-1.6301	0.1121
$\Lambda_{1,i\pi}$	-0.0557	0.0339	-0.0749	-0.1232	-0.0194
$\Lambda_{1,iy}$	-0.0797	0.0314	-0.0802	-0.1317	-0.0319
$\Lambda_{1,ii}$	0.5578	0.0539	0.5094	0.4805	0.6577

Notes: This table reports the results of the posterior density estimates for the parameters of the MFE model. Estimates reported are the following: Mean refers to the mean of the posterior density, Std. Dev. refers to the standard deviation of the posterior while Mode refers to the mode of the posterior distribution. Finally the 95% (posterior) confidence interval is implied by the 5-the and the 95-th percentile reported in respectively Crit. val. 5% and Crit. val. 95%. All results were obtained using the Metropolis Hastings algorithm.

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