Working Paper Research

July 2024 No 450

Managing the inflation-output trade-off with public debt portfolios

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NationalBank OF BELGIUM
Eurosystem

Publisher

Pierre Wunsch, Governor of the National Bank of Belgium

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ISSN: 1375-680X (print)
ISSN: 1784-2476 (online)

Abstract

When taxes do not sufficiently adjust to government debt levels, the Fiscal Theory of the Price Level predicts that other variables, such as inflation and output gap, must adjust to ensure the solvency of public finances. We study the role of optimal debt maturity portfolios in this context, using a New Keynesian model with both demand and supply-side shocks. Our paper offers new analytical insights into the mechanisms through which debt maturity composition affects the trade-off between inflation and output gap: The Persistence, Discounting and Hedging channels. Our findings, based on a rich prior predictive analysis indicate that the key driving force behind optimal portfolio decisions is the Hedging channel. Moreover, the optimal maturity composition of debt is driven primarily by the supply side shocks, rather than by demand shocks. Finally, our results indicate that optimal debt management is a significant margin to complement monetary policy in stabilizing inflation when debt solvency is an important constraint.

Keywords: Monetary and fiscal policy, Government debt management, Fiscal theory.

JEL Codes: E31, E52, E58, E62, C11.

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We thank Harris Dellas, Tasos Karantounias, Jochen Mankart, Ricardo Nunes, Tim Willems, Raf Wouters as well as seminar participants at the Bank of Slovakia, the National Bank of Belgium and the University of Surrey for useful comments and suggestions. The views expressed in this paper are those of the authors and should not be attributed to the National Bank of Belgium, the Eurosystem, the Bank of England or its committees.

Non-technical summary

This research paper examines the impact of public debt maturity structures on the trade-off between inflation and economic output within the context of the Fiscal Theory of the Price Level (FTPL). This economic theory suggests that if government debt is not sufficiently backed by future surpluses (tax revenues), other variables like inflation and output must adjust to maintain debt solvency. The paper is thus particularly relevant in the current economic climate, where government debt levels have risen significantly post-COVID-19, and economies face frequent supply-side shocks leading to inflation volatility.

1. **Key Findings:**
   a) **Hedging Against Supply-Side Shocks**: The primary factor influencing the optimal maturity structure of public debt is the need to hedge against supply-side shocks. These shocks pose significant risks to the government's budget constraint, more so than demand shocks.

   b) **Optimal Debt Maturity**: At the optimal maturity structure, the trade-off induced by the debt constraint is primarily due to demand shocks. This optimal structure allows for a significant improvement in managing the balance between inflation and output.

   c) **Complementing Monetary Policy**: Proper management of debt maturity can serve as a crucial policy tool alongside monetary policy, particularly in scenarios where government debt and surpluses are not balanced.

2. **Analytical and Numerical Insights:**
   - The study identifies three critical dimensions affecting the optimal policy: Persistence, Discounting, and Hedging. Among these, the Hedging channel is found to be the most influential.
   - Numerical experiments reveal that supply-side (cost-push) shocks have a more substantial impact on the optimal debt portfolio compared to demand-side shocks.

3. **Practical Implications:**
   - Policymakers can enhance economic stability by targeting a debt portfolio that mitigates the effects of supply-side shocks, thereby reducing the volatility in the government's budget.
   - The findings suggest that optimal debt maturity management can effectively complement traditional monetary policies to achieve better economic outcomes.

This study contributes to the broader literature on the Fiscal Theory of the Price Level by providing new insights into the optimal management of public debt portfolios. It underscores the importance of considering the maturity structure of debt as a strategic tool for economic stabilization.

The paper’s analytical framework and numerical results offer practical guidance for policymakers aiming to balance inflation and output through strategic debt management.

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1 Introduction

Rising fiscal strain and debt levels following the Covid crisis has brought to surface concerns that governments in OECD economies may not be able to raise enough surpluses to finance debt over the long-run.[1] At the same time, the world economy has entered a shock-prone era characterised by large supply (cost-push) shocks and a risk of heightened inflation volatility stemming from a steep trade-off between inflation and output stabilization. Yet, when public debt is not backed by (future) surpluses, cost-push shocks may not be the main source of inflation variability. Any shock (including demand shocks which are otherwise possible to confront with a well designed monetary policy) can potentially lead to a significant inflation-output trade-off since, in this situation, inflation adjusts to ensure the solvency of public debt. When the shocks that impact the 'debt constraint' are able to reach inflation, then inflation variability can be large (e.g. Leeper and Leith (2016)).

The Fiscal Theory of the Price Level is the workhorse model that macroeconomists use to investigate the driving forces of inflation in these types of circumstances. In the context of this framework, it is well known that Ricardian Equivalence breaks down, and the maturity structure of debt plays a key role in determining how economic shocks propagate through the economy and ultimately impact inflation and output dynamics. (Cochrane (2001); Leeper and Leith (2016)). Therefore, the composition of public debt portfolios can alter the inflation-output trade-off and becomes an important policy margin complementing monetary policy in its objective to stabilize inflation and output fluctuations.

This paper studies the interactions between monetary policy and public debt portfolios of short and long term bonds in the Fiscal Theory framework with both supply- and demand-side disturbances. We construct a New Keynesian model with price rigidities and a fiscal block, the consolidated budget constraint. Government debt is not backed by taxes and instead government deficits can be financed by inflation which reduces the real payout of debt and/or by output changes that result in fluctuations in the real interest rates, bond prices and the present value of government surpluses. We solve a Ramsey policy problem to determine the optimal structure of debt along with the paths of inflation and output. The objective of the planner is to minimize the volatility of inflation and of the output gap, as suggested by a second order approximation of the household welfare function in our model (e.g. Woodford (2003a), Chapter 6).

Using our framework we investigate the optimal maturity policies which enable to alleviate the inflation and output trade-off. We identify the key driving forces and the relative importance of demand and supply side disturbances in shaping these policies. Furthermore, we evaluate the relevance of the debt maturity margin in improving the inflation-output outcomes. Our findings are that the optimal public debt maturity structure is driven mainly by supply side shocks; the Ramsey policy targets a portfolio that alleviates the impact of these types of shocks on the government debt constraint and consequently on the inflation-output trade-off induced by the constraint. Moreover, we find that the effect of managing inflation and output fluctuations using public debt portfolios can be substantial; the optimal policy can accomplish a considerable reduction in the volatility of the targeted macroeconomic variables.

Our approach towards reaching these conclusions is incremental. We begin by studying theoretically how the maturity of debt shapes the inflation-output trade-off. In Section 2 we derive an analytical formula expressing the trade-off as a function of two terms: the shocks to the Phillips curve (the standard supply shock induced trade-off in the New Keynesian model) and the current and lagged growth rates of the Lagrange multiplier attached to the budget constraint. In Ramsey policy models the multiplier captures the impact of structural shocks on the government debt constraint. We provide an analytical expression showing this

[1] See e.g. IMF (2021, Chapter 2).

dependence explicitly.

Our formula identifies three pivotal channels via which debt maturity influences inflation and output, and which have been emphasized by the existing literature: Persistence, Discounting and Hedging. Persistence and Discounting are captured by the coefficients on the current and lagged values of the multiplier, whereas Hedging is measured by the variance of the Lagrange multiplier. Both objects are functions of the debt portfolio.

The Persistence channel states that, when debt is long term, any shock leading to an imbalance between the value of debt and the present value of surpluses does not have to be compensated by an abrupt temporary change in the price level; inflation can persistently change to adjust the real payout of debt which matures in the distant future. (e.g. Sims (2013); Leeper and Zhou (2021); Leeper and Leith (2016); Lustig et al. (2008) among others). This results in a smoother trajectory of output since, from the Phillips curve, output fluctuations are related to the expected growth of inflation.

The Discounting channel, on the other hand, emphasizes that when debt is (at least partly) short-term, the Ramsey planner will it find advantageous to distort output intertemporally in order to alter real interest rates and adjust favorably the present value of government surpluses that compensate for debt. This may also improve the inflation-output trade-off induced by the debt constraint (e.g. Leeper and Zhou (2021)). Using analytical examples we demonstrate the working of these forces and elucidate our inflation-output formula.

In Section 3 we turn towards the Hedging channel. We consider three types of structural shocks: to government spending and to the household discount rate (demand factors) and cost-push shocks (supply factor). The Hedging channel emphasizes that when these shocks hit the economy, they induce fluctuations in bond prices which well targeted portfolios can take advantage of in order to finance deficits and alleviate the need of using inflation to do so.

We characterize Hedging analytically as a function of three 'fundamental portfolios'. These are maturity structures which, in the presence of a single shock in the economy, can fully insulate the government budget. When such complete hedging is possible, inflation and output evolve as in the standard New Keynesian model. For demand shocks this occurs when government debt is long term. Long bond prices covary negatively with the government deficits following a demand shock and issuing long-term debt enables a drop in the market value of government liabilities in times of high deficits (see e.g. Angeletos (2002); Buera and Nicolini (2004)). For supply side shocks, the fundamental portfolio hinges on the relative weight attached to output/inflation in the planner's objective function. Depending on the relative weight, the risk for the government budget could be mainly deriving from inflation or mainly from output fluctuations. In the latter case, cost-push shocks affect the debt constraint through changes in real interest rates and these can be compensated by long term debt (similar to demand shocks). However, when inflation displays more volatility following a cost-push shock, the optimal portfolio needs to be tilted towards short bonds to insulate the government budget: The real value of short term debt is less sensitive to persistent inflation shocks, than is the analogous real value of long bonds.

In the full model, with the three shocks together, complete hedging is not feasible. We derive analytically the Lagrange multipliers in the Ramsey solution as a function of the three fundamental portfolios. Deviating from these portfolios yields an increase in the volatility of the Lagrange multipliers which is proportional to the variances of the shocks and parameters determining their significance as sources of risk for the budget constraint.

Our derivations thus show clearly the three factors (Discounting, Persistence, and Hedging) determining the optimal maturity composition of debt in our Ramsey model. In Section 4 of the paper we turn to a numerical analysis in order to investigate which of these three forces exerts a stronger influence on the optimal public debt portfolio. So as to not constrain our experiments to only one calibration of the model, we rely on a prior predictive analysis (see Leeper et al. (2017)) whereby we characterize the optimal debt structure over a plausible distribution of values for the parameters which we can identify as key for the optimal portfolio. The distributions that we use are standard in the context of DSGE models.

Our main conclusion from the numerical experiments is that while the Persistence and Discounting channels exert only a small impact on optimal portfolio decisions, the Hedging channel is the main driving force. Moreover, when we ask which of the three structural shocks accounts for the bulk of Hedging, we find that it is the cost push shock that has the most significant contribution. Supply-side shocks turn out to be a more important risk for the debt constraint than demand shocks. The Ramsey optimal policy thus targets a portfolio which nearly eliminates the trade-off between output and inflation induced by the debt constraint in the presence of supply shocks; what remains is the demand shock induced trade-off and the standard cost-push New Keynesian trade-off. Moreover, this outcome turns out to be quite close to the New Keynesian optimal policy without the debt constraint. Our experiments show that accounting for the optimal maturity structure as a complement for monetary policy is important when public debt is not backed by taxes.

Our work primarily builds upon the previously mentioned papers that examine the interactions between debt and monetary policies within the Ramsey context (Leeper and Zhou (2021); Schmitt-Grohé and Uribe (2004); Faraglia et al. (2013); Lustig et al. (2008); Sims (2013); Leeper and Leith (2016); Bouakez et al. (2018)) and which characterize the welfare-maximizing maturity structure when inflation bears some or all of the burden of ensuring government debt solvency.[2][3] Our contribution is twofold. First, we provide an analytical characterization of the inflation output trade-off, a formula which facilitates the transparent exploration of the three critical dimensions of optimal policy: Persistence, Discounting, and Hedging. Second, we explicitly assess the relative significance of these dimensions, alongside the significance of demand and supply disturbances.

In terms of analytical solutions, our results complement those of Leeper and Zhou (2021), who have made significant progress in solving their Ramsey problem. While we will later on highlight differences in terms of our modelling assumptions and which led us to derive distinct and thus complementary analytical results, we emphasize that our main contribution relative to Leeper and Zhou (2021) lies in explicitly solving for the Lagrange multipliers associated with the consolidated budget. Our solution, which expresses the multiplier as a function of fundamental portfolios, appears to be new to the literature and underscores the important concept of Hedging.

The second contribution of our paper, which involves disentangling the relative importance of the three policy margins and structural shocks, also brings novel insights to the existing literature. A well-established

[2] Much of this work considers optimal policies when the Ramsey planner can set taxes and inflation simultaneously. This may be a different setup than the one we consider here (we wish to focus on environments in which taxes cannot adjust to make debt solvent) however our results should also extend to models with optimal distortionary taxation. Intuitively, models with jointly optimal fiscal and monetary policies are essentially Fiscal theory models. This is so because, unless taxes fully absorb shocks to the debt constraint (not the solution to a Ramsey program) inflation will also react to these shocks. We thus expect that our insights are applicable to the broad literature, including papers studying joint policies.

That said, we acknowledge that there may also be significant differences, stemming (for example) from the fact that for optimal taxes, the Discounting and Hedging channels remain applicable, but not the Persistence channel. In models with joint policies, the debt maturity affects the optimal policy mix, the relative use of taxes vs. inflation to absorb shocks (e.g. Leeper and Zhou (2021)). The methodological approach that we follow in this paper to derive analytical solutions can be extended to consider optimal taxation and evaluate the differences.

[3] These papers assume full commitment policies. There is also a rich literature studying optimal inflation and debt under no-commitment. See Eggertsson (2006, 2008); Burgert and Schmidt (2014); Matveev (2021); Leeper et al. (2021); de Beauffort (2023, 2024); Bhattarai et al. (2023a).

and significant finding in previous studies is that when inflation is tasked with ensuring debt sustainability, the optimal maturity of debt tends to be long. This result is commonly attributed to the Persistence channel of inflation. However, our experiments reveal that Hedging plays a more crucial role as a policy margin. Moreover, our findings demonstrate the possibility that in Ramsey models the optimal debt may be short-term if the fundamental portfolios predominantly feature short-term debt. We discuss the conditions under which the optimal portfolios are reversed, favoring short-term debt.

Our paper also relates to a literature on optimal debt management using real macroeconomic models (Angeletos (2002) and Buera and Nicolini (2004); Faraglia et al. (2019, 2010); Nosbusch (2008); Debortoli et al. (2017); Aparisi de Lannoy et al. (2022); Greenwood et al. (2015) among others). In these models optimal debt portfolios serve the purpose of smoothing distortionary taxes across time. Though this is a different source of distortions than the inflation output trade-off we focus on in this paper, we draw important insights from this work. Thus, our result that long bonds are useful to hedge against demand shocks basically echoes the findings of Angeletos (2002), Buera and Nicolini (2004) and Debortoli et al. (2017) in the tax smoothing context.

Furthermore, an important difference (on the methodological side) between these papers and ours is that while for its most part the debt management literature studies optimal policies in non-linear business cycle models relying on numerical approximations, we work with a tractable log-linear model which allows us to derive analytical solutions expressing portfolios as functions of the shock processes and other measurable model parameters. This approach which is useful to reduce a complicated portfolio choice problem to simple formulae that can be estimated from the data, bears some resemblance to the recent work of Aparisi de Lannoy et al. (2022). This paper derives transparent closed form solutions for optimal government portfolios as functions of financial and macroeconomic data moments using second order/small noise Taylor expansions. In contrast, we constrain attention to constant (steady state) portfolios, employing first order accurate solutions of the model. Whereas the formulae of Aparisi de Lannoy et al. (2022) are richer and include second order (covariance) terms which are undoubtedly important in the asset pricing/ optimal portfolio context, our first order analytical approach can be easily integrated with a broad spectrum of empirically estimated DSGE models, a facet we briefly delve into in Section 4 of our paper.

Finally, our work relates to the vast literature on the Fiscal Theory of the Price Level (e.g. Sargent et al. (1981); Leeper (1991); Sims (1994); Woodford (1994, 1995, 2001); Cochrane (1998, 2001); Schmitt-Grohé and Uribe (2000); Bassetto (2002); Eggertsson (2008); Canzoneri et al. (2010); Del Negro and Sims (2015); Reis (2016); Jarociński and Maćkowiak (2018); Bhattarai et al. (2014, 2023b); Leeper and Leith (2016); Kumhof et al. (2010); Bi and Kumhof (2011); Benigno and Woodford (2007); Bianchi and Ilut (2017); Bianchi and Melosi (2017); Leeper et al. (2017); Cochrane (2018); Leeper and Zhou (2021); among many others).[4] From this literature closest to our study is the work of Cochrane (2001). Like us, Cochrane (2001) studies the optimal composition of public debt in the context of the Fiscal Theory. He assumes a simplistic Fisherian model with exogenous surplus shocks assuming also that the planner's objective is to stabilize inflation. In contrast to Cochrane (2001) we employ a fully fletched New Keynesian model with a microfounded dual objective for output and inflation stabilization, a New Keynesian Phillips curve, as well as consider more structural shocks to demand and supply as the sources of risk for the government's budget constraint. Our approach is thus more akin to the recent DSGE literature with New Keynesian frictions. Otherwise, our paper is complementary to Cochrane (2001).

[4]See in particular Leeper and Leith (2016) for a very comprehensive overview of this literature focusing on the interactions between monetary and fiscal policy.

2 The Model

We consider an optimal policy problem in which a Ramsey planner sets the path of inflation, output and debt subject to the dynamic equations that define the competitive equilibrium. Our framework is a standard New Keynesian model, featuring monopolistically competitive firms operating technologies which are linear in the labour input and setting prices subject to adjustment costs as in Rotemberg (1982). The model is augmented with the consolidated budget constraint. Since this is a standard setup we will describe the competitive equilibrium using the equations of the log-linear model. We leave it to the appendix to characterize the household and firm optimal behavior from the (background) non-linear model.

We use the standard notation $\hat{x}_t$ to denote the log deviation of variable x_t (in the nonlinear model) from its steady state value, x. The following equations define the competitive equilibrium:

$$\hat{\pi}_t = \kappa(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t \hat{\pi}_{t+1} + \hat{\mu}_t \tag{1}$$

where $\kappa \equiv -\frac{(1+\eta)Y}{\theta}(\varphi + \sigma \frac{Y}{C}) > 0$.

$$\hat{i}_t = \frac{Y}{C}\sigma E_t\Big((\hat{Y}_{t+1} - \hat{Y}_{t+1}^n) - (\hat{Y}_t - \hat{Y}_t^n)\Big) + E_t(\hat{\pi}_{t+1} + \hat{r}_t^n) \tag{2}$$

$$p_L b_L(\hat{b}_{t,L} + \hat{p}_{t,L}) + p_S b_S(\hat{b}_{t,S} + \hat{p}_{t,S}) = -S\hat{S}_t + (1+p_L)b_L(\hat{b}_{t-1,L} - \hat{\pi}_t) + p_L b_L \hat{p}_{t,L} + b_S(\hat{b}_{t-1,S} - \hat{\pi}_t) \tag{3}$$

$$\hat{p}_{L,t} = -\hat{i}_t + \beta\, E_t \hat{p}_{L,t+1} \tag{4}$$

$$\hat{p}_{S,t} = -\hat{i}_t \tag{5}$$

(1) is the Phillips curve at the heart of our model. $\hat{\pi}_t$ represents inflation and $\hat{Y}_t - \hat{Y}_t^n$ is the deviation of aggregate output from its natural level (the output gap). Parameters $\eta < 0$ and $\theta > 0$ govern the elasticity of substitution across the differentiated (monopolistically competitive) goods produced in the economy and the degree of price stickiness, respectively.[5] σ denotes the inverse of the intertemporal elasticity of substitution and φ is the inverse of the Frisch elasticity of labour supply. These objects influence the slope of the Phillips curve, κ, through their influence on the response of hours/output to changes in marginal costs (wages). $\hat{\mu}_t$ is a cost-push shock, a shifter of the inflation output trade-off defined by the Phillips curve.

(2) is the standard log-linear IS-Euler equation which prices a short term nominal asset. $\hat{\xi}$ is a preference shock which affects the relative valuation of current vs. future utility by the household. A drop in the value of $\hat{\xi}_t$ (relative to the expected value $E_t\hat{\xi}_{t+1}$) makes the household relatively patient, willing to substitute current for future consumption.

Equation (3) is the consolidated budget constraint. The left hand side (LHS) of this equation represents the value of debt issued in period t. The terms $\hat{b}_{t,L}$, and $\hat{b}_{t,S}$, denote the quantities of real net government bonds issued in t and held by the private sector. We assume that debt can be issued in two different debt instruments: A short term (S) bond that pays one unit of nominal income in one model period, and a long term bond (L) which is a consol that pays 1 unit of income in perpetuity. The prices of these assets are denoted $\hat{p}_{t,S}$ and $\hat{p}_{t,L}$ respectively.[6]

The term $S\hat{S}_t$ denotes the surplus of the government. We will focus on equilibria where the surplus is

[5] θ is the parameter that governs the magnitude of price adjustment costs in the standard quadratic cost function of Rotemberg (1982). When θ equals zero prices are fully flexible.

[6] Focusing on these types of assets, simplifies considerably our analytical formulae and moreover, it is a common modelling assumption in the debt management literature we referred to previously (see e.g. Debortoli et al (2018)). Our findings can, however, be easily generalized to alternative structures, for example when we make the also common assumption that long bonds are perpetuities paying decaying coupons. Assuming decaying coupons will not change our conclusions.

exogenous with respect to the debt level and so government debt is not backed by taxes. Specifically,

$$S\hat{S}_t = -(1+\omega_1)G\hat{G}_t - \omega_2(\hat{Y}_t - \hat{Y}_t^n)$$

where $\hat{G}_t$ denotes government spending in t. Parameters ω_1, ω_2 are not zero when the government uses a mixture of lump sum/transfers and distortionary taxes/subsidies which are kept constant through time. For example, when the government sets a subsidy s to eliminate distortions from monopolistic competition but otherwise its revenues derive from lump sum taxation, then $\omega_1 = \frac{sY}{Y+\frac{\phi}{\sigma}C} > 0$ and $\omega_2 = sY\left(1+\phi+\frac{\sigma Y}{C}\right) > 0$. Alternatively, if distortionary taxes are levied on (labour) income, or the government's transfers to the private sector depend on macroeconomic conditions (i.e. the output gap) then the parameters ω_1, ω_2 can be adjusted accordingly to reveal the dependence of revenue on output and spending levels. We could then have (for example) $\omega_2 < 0$ in which case a shock that induces a drop in the output gap leads to a fiscal deficit. Our experiments below will be conducted for different values of the ω_2 parameter.

Moreover, we define $\hat{Y}_t^n$ and $\hat{r}_t^n$, the natural output level and the natural interest rate respectively, as:[7]

$$\hat{Y}_t^n \equiv \frac{G}{Y+\frac{\phi}{\sigma}C}\hat{G}_t$$

$$\hat{r}_t^n \equiv -\frac{\phi G}{Y+\frac{\phi}{\sigma}C}(E_t\hat{G}_{t+1} - \hat{G}_t) - E_t(\hat{\xi}_{t+1} - \xi_t)$$

Finally, equations (4) and (5) give the bond pricing formulae for long and short term bonds. (5) sets the short term price equal to the negative of the short-term nominal rate of interest $\hat{i}_t$. (4) defines the recursive formula that determines the price of long-term debt in period t.

Iterating this equation forward and substituting the equilibrium prices in (3) and rearranging, it is possible to write the consolidated constraint as:

$$\begin{aligned}
&\beta b_S\left(\hat{b}_{S,t} - \sigma\frac{Y}{C}(E_t\widetilde{Y}_{t+1} - \widetilde{Y}_t) - E_t\hat{\pi}_{t+1} - \hat{r}_t^n\right) + \\
&b_L\frac{\beta}{1-\beta}\left(\hat{b}_{L,t} - \sigma\sum_{j\geq 1}\beta^{j-1}\frac{Y}{C}(E_t\widetilde{Y}_{t+j} - \widetilde{Y}_{t+j-1}) - \sum_{j\geq 1}\beta^{j-1}E_t\hat{\pi}_{t+j} - \sum_{j\geq 1}\beta^{j-1}\hat{r}^n_{t+j-1}\right) = \\
&(1+\omega_1)G\hat{G}_t + \omega_2\widetilde{Y}_t + b_S(\hat{b}_{S,t-1} - \hat{\pi}_t) + b_L\frac{1}{1-\beta}(\hat{b}_{L,t-1} - \hat{\pi}_t) \\
&-\frac{\beta}{1-\beta}b_L\left(\sigma\sum_{j\geq 1}\beta^{j-1}\frac{Y}{C}(E_t\widetilde{Y}_{t+j} - \widetilde{Y}_{t+j-1}) + \sum_{j\geq 1}\beta^{j-1}E_t\hat{\pi}_{t+j} + \sum_{j\geq 1}\beta^{j-1}\hat{r}^n_{t+j-1}\right)
\end{aligned} \tag{6}$$

where for convenience we use the notation $\widetilde{Y}_t = \hat{Y}_t - \hat{Y}_t^n$ to define the deviation of output from its natural level (the output gap).

2.1 Optimal Policy

Objective Function. Ramsey policy chooses the paths of the competitive equilibrium quantities and prices to maximize the following policy objective:

$$-\frac{1}{2}E_0\sum_{t=0}^{\infty}\left(\hat{\pi}_t^2 + \lambda_Y\widetilde{Y}_t^2\right) \tag{7}$$

[7] See Woodford (2003a) or Galí (2015).

In the appendix we derive (7) as a second order approximation of the household utility function (see e.g. Woodford (2003a, Chap. 6)). For this microfounded objective, $\lambda_Y = \frac{1}{\theta}\left(\sigma\frac{Y}{C} + \varphi\right)$ is the appropriate weight attached to the output gap. Our main results below concern the microfounded weight λ_Y. However, we will establish formulas applicable to any $\lambda_Y \geq 0$ as this broader approach enables us to highlight clearly how the relative weight attached to output influences the optimal policies. Additionally, solving explicitly for $\lambda_Y = 0$ (the optimal policy focuses on inflation stabilization only) and $\lambda_Y \to \infty$ (neglecting the relative weight on inflation) will offer analytical convenience in certain cases, when otherwise algebraic expressions are too complex to easily interpret. The insights that we can derive from these solutions will be valuable to understand the determinants shaping the optimal policy under the welfare based criterion.

Policy Program. We now describe the optimal policy problem. The following definition states formally the Ramsey program.

Definition (Ramsey Policy): *The optimal policy solves:*

$$\max_{(b_L,b_S)\ \{\hat{d}_t,\hat{\pi}_t\widetilde{Y}_t\}_{t\geq 0}} -\frac{1}{2}E_0\sum_{t=0}^{\infty}\left(\hat{\pi}_t^2 + \lambda_Y\widetilde{Y}_t^2\right)$$

subject to (1), (6),

$$d\hat{d}_t = b_S\hat{b}_{t,S} + \frac{1}{1-\beta}b_L\hat{b}_{t,L}$$

$$\text{and} \qquad b_S + \frac{b_L}{1-\beta} = \frac{S}{1-\beta}$$

A few comments are in order. Firstly, it's important to note that while the preceding paragraph derived the competitive equilibrium expressions for $\hat{i}_t, \hat{p}_{S,t}, \hat{p}_{L,t}$, these elements need not be included as constraints in the Ramsey program. By utilizing the paths of the variables determined by the Ramsey planner, one can employ the Euler and bond pricing equations to reconstruct the sequence $\hat{i}_t, \hat{p}_{S,t}, \hat{p}_{L,t}$. Consequently, we can omit (5), (4), and (2).

Secondly, considering that our log-linear model doesn't uniquely define the optimal portfolio sequence $\left\{\hat{b}_{S,t}, \hat{b}_{L,t}\right\}_{t\geq 0}$, we substitute the quantities of short and long bonds with the total debt face value variable, $\hat{d}_t$. The optimal policy will thus set $\hat{d}_t$ and this is equivalent to any combination of $\hat{b}_{S,t}$ and $\hat{b}_{L,t}$ that results in the same path of $\hat{d}_t$.

Moreover, although the specific composition of $\hat{b}_{S,t}, \hat{b}_{L,t}$ won't influence the optimal policy, as will become apparent later, the steady state quantities (b_L, b_S) will impact the solution. We assume that the planner optimally sets (b_L, b_S) alongside the remaining variables to maximize welfare, while adhering to $b_S + \frac{b_L}{1-\beta} = \frac{S}{1-\beta}$, the steady state intertemporal budget constraint.

The bulk of our analysis below explores the optimal composition of public debt in terms of (b_L, b_S). Notice that the inclusion of these variables in the Ramsey policy essentially makes ours a non linear-quadratic program. To solve for the optimal portfolio composition we therefore proceed in two steps: First, we fix (b_L, b_S) and solve a linear quadratic program to determine the optimal sequence $\left\{\hat{d}_t, \hat{\pi}_t, \widetilde{Y}_t\right\}_{t\geq 0}$ through solving the system of first order conditions. Then, we vary (b_L, b_S) to determine the portfolio by tracing the upper envelope defined by the optimality conditions in the first step. This iterative approach allows us to map out the range of feasible solutions, and characterize the optimal maturity composition of debt.

Finally, note that the analogue of our approach in a fully non-linear context would be to determine optimally the proportions of short and long bonds in the government's portfolio and hold the maturity composition constant letting the government vary total debt over time. It turns out that such a solution to a non-linear model aligns closely to the fully optimal portfolio composition, wherein the share of short-term debt can fluctuate over time. Ramsey models with fully optimal portfolios typically predict that the proportions of short and long bonds are constant or display little variation over time.[8] Therefore, even though our exercise restricts attention to cases where the maturity of debt is held constant, this assumption is not too restrictive for fully non-linear models.

2.2 Optimality

We solve for the first step optimal policies using a Lagrangian.[9] Attach a multiplier $\psi_{\pi,t}$ to the Phillips curve and $\psi_{gov,t}$ to the consolidated budget. The first order conditions of the Ramsey program can be written as:

$$-\hat{\pi}_t + \Delta\psi_{\pi,t} + b_S \Delta\psi_{gov,t} + \frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j} = 0 \tag{8}$$

$$-\lambda_Y \widetilde{Y}_t - \psi_{\pi,t}\kappa + b_S\sigma\frac{Y}{C}\Delta\psi_{gov,t} + b_L\sigma\frac{Y}{C}\sum_{j\geq 0}\Delta\psi_{gov,t-j} - \omega_2\psi_{gov,t} - \sigma\frac{Y}{C}S\psi_{gov,t} = 0 \tag{9}$$

$$\psi_{gov,t} - E_t\psi_{gov,t+1} = 0 \tag{10}$$

(see appendix). (8) is the first order condition for inflation, (9) is the derivative of the Lagrangean with respect to the output gap and finally, (10) is the optimality condition for debt.

According to the above conditions, under the optimal policy, inflation and output are functions of the current and lagged values of the Lagrange multipliers $\psi_{gov,t}, \psi_{\pi,t}$. Moreover, from (10) the multiplier on the government budget follows a random walk process.

To interpret these results consider first the case where $\psi_{gov,t} = 0$ for all t. Notice that this corresponds to a scenario in which the consolidated budget constraint does not influence the optimal solution. Under this assumption, and combining what is left from (8) and (9) into a single equation, we get:

$$\hat{\pi}_t + \frac{\lambda_Y}{\kappa}\Delta\widetilde{Y}_t = 0 \tag{11}$$

Equation (11) describes the standard inflation-output trade-off under optimal policy with commitment in the New Keynesian model (e.g. Giannoni and Woodford (2003)).

Now, consider bringing back the multipliers ψ_{gov}. We can write this trade-off equation as follows:

$$\begin{aligned}\hat{\pi}_t + \frac{\lambda_Y}{\kappa}\Delta\widetilde{Y}_t = {} & b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) + b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) + \\ & -\frac{\omega_2}{\kappa}\Delta\psi_{gov,t} - S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t} + b_S\Delta\psi_{gov,t} + \frac{b_L}{1-\beta\delta}\sum_{j\geq 0}\Delta\psi_{gov,t-j}\end{aligned} \tag{12}$$

[8] See e.g. Angeletos (2002); Buera and Nicolini (2004); Faraglia et al. (2019).

[9] Following numerous papers, we assume a *timeless perspective*. As is well known, solving for optimal policies under this assumption, requires to introduce additional constraints on the initial allocation (e.g Woodford (2003b)), or the program can be stated in terms of an objective function that accounts explicitly for the lagged Lagrange multipliers at the beginning of the planning horizon (e.g Faraglia et al. (2016)). To avoid introducing explicitly all these elements we do not state the Lagrangian here.

which reveals that $\hat{\pi}_t + \lambda_Y \Delta\widetilde{Y}_t$ is now a function of the current and lagged values of the multiplier attached to the consolidated budget.

What do these terms capture? Shocks that hit the economy will impact the value of debt. When the debt constraint affects the optimal policy solution, inflation and output will need to adjust in order to satisfy the consolidated budget constraint and thus to ensure the solvency of debt. The terms $\Delta\psi_{gov,t-j}$ essentially capture the influence of shocks when they are filtered through the consolidated budget constraint.

To clarify this further, let us state the *intertemporal consolidated budget* constraint which we can derive from (6) through forward substitution. This object reads:

$$E_t \sum_{j\geq 0} \beta^j \Big(-\omega_1 \hat{G}_{t+j} - \omega_2 \widetilde{Y}_{t+j} - \sigma \frac{Y}{C} S(\widetilde{Y}_{t+j} - \widetilde{Y}_t) + S(\overbrace{\frac{G\phi}{Y + \frac{\phi}{\sigma}C}(\hat{G}_{t+j} - \hat{G}_t) + \hat{\xi}_{t+j} - \hat{\xi}_t}^{-\hat{r}^n_t - \hat{r}^n_{t+1} - \ldots - \hat{r}^n_{t+j}}) \Big) =$$
$$b_S(\hat{b}_{S,t-1} - \hat{\pi}_t) + \frac{1}{1-\beta} b_L(\hat{b}_{L,t-1} - \hat{\pi}_t) + b_L \beta E_t \sum_{j\geq 1} \beta^{j-1} \Big[-\sigma \frac{Y}{C}(\widetilde{Y}_{t+j} - \widetilde{Y}_t) - \sum_{k=0}^{j} \hat{r}^n_{t+k} - \sum_{k=1}^{j} \hat{\pi}_{t+k} \Big] \tag{13}$$

and it links the present discounted value of the surplus (terms in the top row of (13)) to the real value of debt outstanding in t (terms in the bottom row). Consider a shock which lowers the intertemporal surplus relative to the value of debt. In response to such a shock, the constraint tightens, and the value of the multiplier $\psi_{gov,t}$ increases. To satisfy the constraint the planner needs to engineer a drop in the real payout of debt. This requires to increase current inflation-output and also possibly adjust these variables in the future (if debt is long term). Whereas the term $\Delta\psi_{gov,t}$ in (12) captures the effect of a shock that occurs in t on current inflation and output, the lagged terms capture the influence of past shocks on inflation and output in t.

Finally, note that the multiplier ψ_{gov} evolves as a random walk because the planner wants to spread the distortions associated with using inflation to satisfy the intertemporal constraint, evenly across periods. This is a standard property of optimal Ramsey policy (e.g. Aiyagari et al. (2002); Schmitt-Grohé and Uribe (2004); Lustig et al. (2008); Faraglia et al. (2013, 2016), among others).

2.3 Optimal Inflation-Output Trade-offs

We now present our main formula for the inflation output trade-off. Combining (12) with the Phillips curve to substitute out output growth, we derive the following difference equation for inflation:

$$E_t \hat{\pi}_{t+1} - \left(\frac{\kappa^2}{\lambda_Y \beta} + \frac{1}{\beta} + 1 \right) \hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} = -\frac{1}{\beta}\Delta\hat{\mu}_t - \zeta_t - \frac{\kappa^2}{\lambda_Y \beta} O(\Delta\psi_{gov}) \tag{14}$$

$O(\Delta\psi_{gov})$ picks up all the terms involving the Lagrange multiplier in (12) and ζ_t is a shock to the expectation of inflation (see appendix). The two roots of the characteristic polynomial are:

$$\lambda_{1,2} = \frac{1}{2}\left[(\frac{\kappa^2}{\lambda_Y \beta} + \frac{1}{\beta} + 1) \pm \sqrt{(\frac{\kappa^2}{\lambda_Y \beta} + \frac{1}{\beta} + 1)^2 - \frac{4}{\beta}} \right]$$

with one root being stable (say λ_1) and the other unstable ($\lambda_2 > 1$).

In the appendix we prove the following Proposition:

Proposition 1 (Inflation-Output Trade-off): *Optimal inflation and output in the Ramsey policy equi-*

librium are given by:

$$\hat{\pi}_t = \overbrace{\boxed{\lambda_1 \hat{\pi}_{t-1} + \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_t - \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_{t-1}}}^{\text{New Keynesian Trade-off}} + \overbrace{\boxed{\Big(\nu_1 - \nu_2 L + \nu_3 \sum_{j\geq 0} L^j\Big)\Delta\psi_{gov,t}}}^{\text{Debt Constraint Trade-off}} \tag{15}$$

$$\widetilde{Y}_t = \underbrace{\boxed{\lambda_1 \widetilde{Y}_{t-1} + \frac{1}{\kappa}\frac{1}{\beta\lambda_2}\frac{\Big(1+\beta-\beta(\lambda_1+\lambda_2)\Big)}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_t}}_{\text{New Keynesian Trade-off}} + \underbrace{\boxed{\frac{1}{\kappa}\Big(\nu_4 - \nu_2 L + \nu_3(1-\beta)\sum_{j\geq 0} L^j\Big)\Delta\psi_{gov,t}}}_{\text{Debt Constraint Trade-off}} \tag{16}$$

$$\begin{aligned}
\nu_1, \nu_4 :=& \quad \text{Impact} \\
\nu_2 L = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y \beta} b_S \frac{\sigma}{\kappa}\frac{Y}{C} L :=& \quad \text{Discounting} \\
\nu_3 \sum_j L^j := \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\Big(\frac{1}{1-\frac{1}{\lambda_2}}\Big)\sum_j L^j :=& \quad \text{Persistence} \\
\Delta\psi_{gov,t} = \nu_5 \begin{bmatrix} u_{G,t} \\ u_{\xi,t} \\ u_{\mu,t} \end{bmatrix} :=& \quad \text{Hedging}
\end{aligned}$$

and L is the lag operator.

Equations (15) and (16) (along with the definitions that follow) are our formula for the inflation-output trade-off under Ramsey policy. The leading terms (first box) represent the standard inflation-output trade-off in the New Keynesian model. The second box is the trade-off induced by the debt constraint and is a function of the current and lagged values of the multiplier ψ_{gov}.

We will devote the next few paragraphs to interpret the formula in Proposition 1. Consider first the leading terms in (15) and (16), the trade-off in the standard New Keynesian model. The property that inflation does not react to the spending and preference shocks in this model is easy to understand. Since the planner smooths deviations of output from its natural level, government spending shocks will not impact the Phillips curve and effectively will not be driving an inflation-output trade-off. The same holds for preference shocks, which only produce fluctuations in the natural rate of interest. Optimal policy can perfectly stabilize the target variables in response to these shocks. This is the well known divine coincidence property.

Cost-push shocks, however, induce a trade-off between output and inflation. According to (15) this is resolved by making inflation partially absorb the shock, depending on the weight attached to output stabilization in the policy objective function. If λ_Y is a very small number, then λ_2 approaches infinity and cost-push shocks do not exert any influence on inflation. Conversely, if λ_Y is infinite (equivalent to the planner only seeking to stabilize the output gap) then $\lambda_2 \to \frac{1}{\beta}$ and all of the effect of the cost-push shock is absorbed by inflation.

Consider now the persistence of inflation, λ_1. The higher is λ_Y the higher is this coefficient. Therefore, attaching a larger weight to output stabilization yields a more persistent inflation process. The intuition behind this property is simple: Since from the Phillips curve output variability is proportional to the variability of the changes in inflation, the planner makes inflation react persistently to shocks in order to smooth the output target. In the limit, when policy only cares about smoothing output fluctuations, then $\lambda_1 = 1$ and inflation displays a unit root.

These results are standard properties of optimal policy in the New Keynesian model. We now turn towards the terms in the second box in (15) and (16) to explain how the debt constraint influences policy. That these terms also lead to an inflation-output trade-off is already evident from the formula in Proposition 1: The 'shock' $\Delta\psi_{gov}$ enters into (15) and (16) in the same manner as the cost-push shocks, inducing volatility in inflation and the output gap under the optimal policy. Moreover, according to Proposition 1 $\Delta\psi_{gov}$ is itself a function of the 3 structural shocks of the model: $\left[u_{G,t}, u_{\xi,t}, u_{\mu,t}\right]$ denotes the vector of i.i.d innovations of the processes $\hat{G}_t, \hat{\xi}_t, \hat{\mu}_t,$. ν_5 is a 1×3 vector of appropriate coefficients. Thus, the debt constraint induces a trade-off for optimal policy which is driven by both demand supply side shocks. This trade-off depends on the debt portfolio b_S, b_L and on the parameter λ_Y, as well as on their interactions.

We will now turn to explaining these features of the model. To do so, in the next subsection, we will utilize analytical examples focusing on the terms labeled Persistence and Discounting in Proposition 1 (objects $\nu_2 L$ and $\nu_3 \sum_j L^j$) and we will also clarify the Impact terms (objects ν_1 ν_4).[10] In Section 3, where we will explain in detail the concept of Hedging we will complete our derivations by introducing a general analytical solution for the vector ν_5 and showing explicitly the dependence of its elements on the bond portfolio. We will then be in place to solve for the optimal debt maturity composition.

2.4 Persistence and Discounting

We begin by rewriting the intertemporal budget constraint (equation (13)) as

$$\text{Shock}_t = -\sigma E_t \sum_{j\geq 0} \beta^j \frac{Y}{C}\left(b_L - S\right)\left(\widetilde{Y}_{t+j} - \widetilde{Y}_t\right) - b_S\hat{\pi}_t - b_L E_t \sum_{j\geq 0}\beta^j \left(\sum_{k=0}^{j} \hat{\pi}_{t+k}\right) \tag{17}$$

On the LHS of (17), the term Shock_t represents a shock to the intertemporal debt constraint. Note that though this can be thought of as a term that groups together the structural shocks of the model, for our analytical examples in this subsection, we will treat this term simply as a one off shock to the intertemporal surplus.[11]

Our first analytical result characterizes the optimal inflation output trade-off in a Fisherian version of the model, setting $\sigma = 0$[12]. For tractability, we focus on the cases where debt is either only short-term or only long-term.

[10]Though Proposition 1 did not provide an analytical expression for the Impact coefficients, our derivations below show these coefficients in special cases of the model. See appendix for the general solution.

[11]From (17) we get:

$$\text{Shock}_t \equiv E_t \sum_{j\geq 0} \beta^j \Big(-\omega_1 \hat{G}_{t+j} + (S - b_L)\big(\frac{G\phi}{Y + \frac{\phi}{\sigma}C}(\hat{G}_{t+j} - \hat{G}_t) + \hat{\xi}_{t+j} - \hat{\xi}_t\big)\Big)$$

and so Shock_t can be thought of as a term which groups together the structural shocks. Notice that according to this expression Shock_t depends also on the portfolio composition (b_L). This dependence is at the core of the Hedging argument which we develop in Section 3. Thus, in our examples in this section, we think of Shock_t simply as a disturbance leading to an imbalance in the intertemporal debt constraint in order to abstract from Hedging. This approach will prove useful later on (in section 4) to separately study separately the Persistence and Discounting channels in our quantitative model.

[12]Assuming $\sigma = 0$ is a common modelling setup in the Fiscal theory (see for example Cochrane (2001)) . It amounts to assuming that real interest rates are exogenous and hence output fluctuations do not play any role in debt stabilization. This is a convenient setup to focus on the impact of inflation on the intertemporal solvency of debt.

Example 1 (Persistence): *Assume $\sigma = 0$. i) When debt is only short-term ($b_S > 0$, $b_L = 0$) the optimal inflation output trade-off is:*

$$\hat{\pi}_t = \lambda_1 \hat{\pi}_{t-1} \underbrace{- \frac{1}{b_S}\text{Shock}_t}_{\equiv \nu_1 \Delta\psi_{gov,t}} \tag{18}$$

$$\widetilde{Y}_t = \lambda_1 \widetilde{Y}_{t-1} \underbrace{- \frac{1}{\kappa}(1-\lambda_2^{-1})\frac{1}{b_S}\text{Shock}_t}_{\equiv \nu_4 \Delta\psi_{gov,t}} \tag{19}$$

ii) When debt is only long-term $\left(b_S = 0, b_L > 0\right)$ and moreover $\lambda_Y = 0$, optimal inflation and output are given by:

$$\hat{\pi}_t = \underbrace{- \frac{(1-\beta)^2}{b_L}\sum_{j=0}^{t}\text{Shock}_{t-j}}_{\equiv \nu_3 \sum_{j\geq 0} L^j \Delta\psi_{gov,t}} \qquad \widetilde{Y}_t = \underbrace{- \frac{1}{\kappa}\frac{(1-\beta)^3}{b_L}\sum_{j=0}^{t}\text{Shock}_{t-j}}_{\equiv (1-\beta)\nu_3 \sum_{j\geq 0} L^j \Delta\psi_{gov,t}} \tag{20}$$

Consider first part i) of Example 1. When debt is only short term, the intertemporal budget can be written as:

$$\text{Shock}_t = -b_S \hat{\pi}_t \tag{21}$$

Thus, a negative Shock_t (equivalent to a drop in the intertemporal surplus) will tighten the constraint. We then have $\Delta\psi_{gov,t} > 0$. From Proposition 1, $\nu_2 = \nu_3 = 0$ (since $b_L = \sigma = 0$) and therefore only the terms labeled 'Impact' ν_1, ν_4 will determine the solution for output and inflation. According to (18) the impact of the shock to inflation is $-\frac{1}{b_S}$ regardless of the policy objective λ_Y. Attaching a higher weight to output smoothing makes inflation more persistent, keeping the impact of the shock to inflation constant.

When debt is short term, a negative surplus shock can only be compensated by raising inflation in t. Higher persistence of inflation is thus wasteful from the point of view of fiscal solvency but it is desirable because of the standard New Keynesian trade-off considerations: a more persistent response of inflation to a shock leads to a smoother trajectory of output. This trade-off is evident in (19): $(1 - \lambda_2^{-1})$ is decreasing in λ_Y.

Next, consider part ii). With only long term debt, the intertemporal debt constraint is:

$$\text{Shock}_t = -b_L \sum_{j\geq 1} E_t \beta^j \left[\sum_{k=0}^{j} \hat{\pi}_{t+k}\right]$$

and a shock to the surplus can now be compensated with a rise in inflation in t and in any period after t, since debt is a perpetuity. The optimal policy is then to make inflation follow a random walk, as this enables to minimize the convex losses from inflation variability. In this case, the impact term (ν_1) equals zero and $\nu_3 \sum_j L^j$ determines the solution for inflation.

Part ii) of Example 1 assumes $\lambda_Y = 0$ for analytical convenience.[13] The simple result that we derived

[13] An analytical solution for the general case is possible, but the algebra is cumbersome and so we leave it for the appendix.

however is revealing of an important property concerning the interaction between the New Keynesian inflation output trade-off and the trade-off induced by the debt constraint. With $\lambda_Y = 0$, the New Keynesian trade-off would make inflation an i.i.d process, since output fluctuations do not matter for the planner's objective. However, the optimal policy with long term debt makes inflation a random walk, a result that would also be consistent with a very large weight on output stabilization. Besides being an important illustration of the interactions between the two trade-offs, this result also suggests that with long debt the objectives of smoothing inflation and output tend to align, diminishing the significance of λ_Y in determining the optimal policy. This property is well known to the literature (see e.g. Leeper and Zhou (2021) among others).

Our next example demonstrates a different angle via which the debt composition affects the inflation output trade-off.

Example 2 (Discounting): *Assume that debt is only short term, $\sigma > 0$ and $\lambda_Y = 0$. Optimal inflation is given by:*

$$\hat{\pi}_t = \underbrace{-\frac{1}{b_S}\frac{\text{Shock}_t}{1+\frac{\beta\widetilde{\omega}}{1-\beta}\left[1+\frac{\widetilde{\omega}}{1+\widetilde{\omega}}\right]}}_{\equiv \nu_1 \Delta\psi_{gov,t}} + \underbrace{\frac{1}{b_S}\frac{\widetilde{\omega}}{1+\widetilde{\omega}}\frac{\text{Shock}_{t-1}}{1+\frac{\beta\widetilde{\omega}}{1-\beta}\left[1+\frac{\widetilde{\omega}}{1+\widetilde{\omega}}\right]}}_{\equiv -\nu_2 \Delta\psi_{gov,t-1}} \tag{22}$$

where $\widetilde{\omega} \equiv \frac{\sigma}{\kappa}\frac{Y}{C}$.

Equation (22) shows how the inflation process changes with the assumptions we made in Example 2.[14] Notice the new element in the solution for inflation (relative to the solution in Example 1) is the Discounting term $\nu_2 L$. The presence of this term implies that the shock does not only exert an impact effect on inflation, but also the lagged value of the shock affects inflation. Thus, in response to a negative Shock in t, inflation will increase on impact and but after one period, it will turn negative. Subsequently, inflation will be zero.

What is going on? When the planner is not concerned about output stabilization, targeting a negative inflation rate one period after the shock hits, leads to a stronger reaction of output on impact. Thus, a higher output level is possible, and this is warranted to reduce the magnitude of the response of inflation to the shock. A simple inspection of the intertemporal budget is sufficient to clarify this.

$$\text{Shock}_t = \sigma S \sum_{j\geq 1} E_t \beta^j (\widetilde{Y}_{t+j} - \widetilde{Y}_t) - b_S \hat{\pi}_t$$

$$= \underbrace{\sigma S \frac{1}{\kappa}\beta E_t \hat{\pi}_{t+1}}_{\equiv \sigma S \sum_{j\geq 1} E_t \beta^j \widetilde{Y}_{t+j}} - \sigma S \frac{\beta}{1-\beta}\widetilde{Y}_t - b_S \hat{\pi}_t = \frac{\sigma S}{\kappa}\frac{\beta}{1-\beta}E_t \hat{\pi}_{t+1} - (b_S + \frac{\sigma S}{\kappa}\frac{\beta}{1-\beta})\hat{\pi}_t$$

Focus on the top equality. A negative Shock creates an imbalance between the LHS and the RHS of the equation and it can be compensated by either higher inflation or higher output in t. In particular, targeting a higher $\widetilde{Y}_t$ reduces the real interest rates rates of future (constant) surpluses, increasing the intertermporal surplus. This is the Discounting channel.

Note that assuming $\lambda_Y = 0$, implies that the planner will find advantageous to shift as much of the burden as possible to output. The bottom equation clarifies how inflation in $t + 1$ matters for this. We have used the Phillips curve, to substitute out output. The last equality thus decomposes the response of output in terms of the responses of date t and $t + 1$ inflation rates.

[14] The derivation for output can be found in the appendix.

According to this expression, a negative Shock can be financed by higher date t inflation or by lower expected inflation in $t+1$. It should not be surprising that making inflation mildly negative in $t+1$ is preferable to setting $E_t\hat{\pi}_{t+1} = 0$ and concentrating all of the adjustment on period t inflation, since the losses from inflation are convex.

The appendix extends Proposition 2 to the case where $\lambda_Y > 0$. In this case, inflation becomes persistent, and the positive impact effect dominates over discounting and therefore seeing a switch in the sign of inflation becomes less likely. Targeting a smoother output trajectory reduces the incentive of the planner to distort output intertemporally in order to ensure the satisfaction of the intertemporal budget constraint. Still, inflation continues responding to the lagged Shock, which implies that the Discounting channel remains significant.

Consider now the case where debt is only long term and $\sigma > 0$. Then, distorting output intertemporally is never optimal regardless of λ_Y. To see this, consider equation (17) noting that when $b_L = S$ the first term on the RHS drops. The solution for inflation in this model is quite similar to part ii) of Example 1.

2.4.1 Discussion. Examples 1 and 2 helped us elucidate a few noteworthy features of the formula in Proposition 1. The Persistence and Discounting mechanisms of optimal policy we studied are not new to the literature (see e.g. Leeper and Zhou (2021); Leeper and Leith (2016)). However, highlighting them has been an important groundwork for the characterization of the optimal debt policies in the model. Furthermore, the formulae that we derived in this subsection are complementary to the content of Leeper and Zhou (2021). Whereas, Leeper and Zhou (2021) setup a more complicated model in which the Ramsey planner sets fiscal and monetary variables simultaneously, we solve the standard New Keynesian model augmented with the consolidated budget and assuming constant taxes. This enabled us to augment the usual trade-off equations of that model with the additional elements induced by the debt constraint in (15) and (16).

Our next task is to solve explicitly for $\Delta\psi_{gov}$ which will link the trade-off induced by the debt constraint with the structural shocks of the model. We thus turn to the Hedging channel of optimal policy.

3 The Hedging Channel

3.1 Complete Hedging against spending, preference and cost-push shocks.

Our analytical solution derives $\Delta\psi_{gov}$ as a weighted sum of the structural disturbances to demand and supply. The weights can be conveniently expressed as functions of three 'fundamental portfolios' which we will derive in this subsection.

More specifically, we will consider model versions in which there is only one structural disturbance (at a time). Then, we will find a portfolio which will fully insulate the government budget constraint from the risk associated with this structural shock. If such a portfolio exists, it will fully eliminate the trade-off induced by the debt constraint, ensuring that $\Delta\psi_{gov,t} = 0$ for all t. Inflation and output will evolve as in the standard New Keynesian framework.

This approach to portfolio management essentially follows Angeletos (2002) and Buera and Nicolini (2004). These papers consider a debt management exercise in which an optimizing government sets the portfolio to ensure that the intertemporal budget constraint (given a desirable path of taxes) holds at all times and in all contingencies. Similarly, we consider here a policy exercise in which a desired path of inflation (the optimal path in the New Keynesian model) can be supported through portfolios that ensure satisfaction of the intertemporal budget in all states and in all periods.

We do this separately for each of the shocks, since finding a portfolio that can simultaneously absorb preference, spending and cost-push shocks is not possible.[15] However, the three fundamental portfolios that we derive in this paragraph, will be an important benchmark for the optimal debt maturity policy in full model (when all shocks can occur simultaneously). Our analytical solution for the term $\Delta\psi_{gov,t}$ in the next section will relate the sources of the volatility of $\Delta\psi_{gov,t}$ to the deviations of debt maturity from the fundamental portfolios.

For the sake of brevity we relegate all derivations to the appendix. We state the formulae for the three portfolios in the following Proposition:

Proposition 2 ('Fundamental Portfolios'): *Consider the solution (b_S, b_L) in the Ramsey Program with one source of risk for the government budget. Then, $\Delta\psi_{gov,t} = 0$ for all t and the optimal level of long term debt satisfies:*

i) $b_L^G = S + \frac{(1+\omega_1)}{1-\rho_G}\frac{Y+\frac{\phi}{\sigma}C}{\phi}\frac{1-\beta}{\beta}$ *when spending shocks are the source of risk for the government budget.*

ii) $b_L^\xi = S$ *when preference shocks are the risk for the government budget.*

iii)

$$b_L^\mu = \frac{\frac{\omega_2}{\kappa}(\frac{1}{1-\beta\rho_\mu} - \theta_1) + S\frac{\beta\widetilde{\omega}}{(1-\beta)}\Big[(\theta_1 - \frac{1}{1-\beta\rho_\mu})(1-\rho_\mu) + \theta_1(1-\lambda_1)\Big] + \frac{S}{1-\beta}\theta_1}{\frac{\beta\widetilde{\omega}}{(1-\beta)}\Big[\theta_1(1-\lambda_1) + (1-\rho_\mu)(\theta_1 - \frac{1}{1-\beta\rho_\mu})\Big] - \frac{\theta_1\beta}{(1-\beta)(1-\beta\lambda_1)}\Big[\lambda_1 + (\frac{\rho_\mu - 1}{(1-\beta\rho_\mu)})\Big]}$$

when cost-push shocks are the risk for the government budget. Moreover, $\theta_1 = \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}$. *Short term bonds are given by* $b_S^i = \frac{S-b_L}{1-\beta}$ for $i = G, \xi, \mu$

The claim that b_L^μ, b_L^ξ, b_L^G solve the Ramsey program should not be surprising. With one structural shock the best outcome that the optimal maturity policy can aim for is to eliminate the volatility of inflation and output attributed to the debt constraint.[16]

The result in i) states that in order to absorb fiscal shocks the optimal share of long debt needs to exceed 100 percent (short term bonds are negative). This result is easy to interpret in light of well known findings of Angeletos (2002) and Buera and Nicolini (2004)). When the government issues long term debt it benefits from the negative covariance between long bond prices and spending shocks. Thus, in times of high spending needs, the real value of debt drops, enabling to smooth distortions stemming from changes in inflation and

[15] Angeletos (2002); Buera and Nicolini (2004) focus on optimal debt management when spending shocks are the main source of risk for government budgets however, (in contrast to us) they use a non-linear model. When non-linearities are present, an optimal portfolio ensuring satisfaction of the intertemporal budget constraint at all times and in all contingencies can be found when the number of states is equal to the number of maturities of debt issued. Since we assumed continuous stochastic processes, a nonlinear version of our model would predict that the government should issue debt using an infinite number of different instruments in order to eliminate the volatility of inflation and output driven by debt sustainability. Under the linear model we use here however, such an complex debt policy is not necessary, as all of the (infinitely many) states of a given shock process line up and two bonds become sufficient.

[16] Beyond this, in the standard New Keynesian model, the maturity structure obviously has no effect on the inflation output trade-off.

the output gap.[17]

In part ii) of Proposition 2 we state another result which is known in the literature. Hedging against preference shocks requires a flat maturity structure of debt and at the optimum we have $b_L^{\xi} = S$ and $b_S^{\xi} = 0$. (see e.g. Debortoli et al. (2017)). Preference shocks influence the intertemporal surplus through the changes in real interest rates. However, when long bonds are consols, setting $b_L^{\xi} = S$ implies that the payment profiles of debt coincide with the stream of net revenues to the government. Then, a negative shock to preferences increases the present value of surpluses and it increases also the long bond prices by equal amounts.

Part iii) considers supply side shocks and for this case our result in Proposition 2 is new to the literature. We will thus devote a couple of paragraphs to explain it. Notice first that in contrast to the expressions for b_L^G, b_L^{ξ} which are independent of the weight attached to output stabilization, the expression for b_L^{μ} depends on parameters λ_1, λ_2 (and thus also on λ_Y). Under complete hedging, demand shocks do not lead to an inflation output trade-off. Supply shocks do.

Let us simplify by first assuming $\lambda_Y = 0$. We then have $\theta_1 = 0$ and $\lambda_1 = 0$ and

$$b_L^{\mu} = S - \frac{\omega_2}{(1-\rho_\mu)} \frac{(1-\beta)}{\beta} \frac{C}{Y\sigma} < S \tag{24}$$

Therefore, assuming that the planner only cares about inflation stabilization, we get that $b_L^{\mu} < S$ and $b_S^{\mu} > 0$ if parameter ω_2 is positive, and $b_L^{\mu} > S$, and $b_S^{\mu} < 0$ if ω_2 is negative.

To understand this finding, recall that under no output smoothing, inflation will be constant through time. The cost-push shock thus affects the intertemporal debt constraint through the changes in the path of output and therefore the Ramsey planner will set the optimal portfolio to exploit output fluctuations and stabilize debt intertemporally. The way that this can be accomplished is simple to investigate analytically.

[17] This principle can be illustrated using the intertemporal budget. When fiscal shocks are the only source of risk for the government budget, and $\Delta\psi_{gov,t} = 0$, the planner can smooth perfectly inflation and close the output gap. (13) can be written as:

$$E_t \sum_{j\geq 0} \beta^j \Big(-(1+\omega_1) G\hat{G}_{t+j} + S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}(\hat{G}_{t+j} - \hat{G}_t)\Big) = b_L \beta E_t \sum_{j\geq 1} \beta^{j-1} \Big(\frac{G\phi}{Y+\frac{\phi}{\sigma}C}(\hat{G}_{t+j} - \hat{G}_t)\Big) \tag{23}$$

The terms on the RHS capture the fluctuations in the real long bond price driven by a spending shock in t: An increase in $\hat{G}_t$ will lower the price of long debt. Then, what is on the LHS is the change in the intertemporal surplus due to the shock: $-(1+\omega_1)G\hat{G}_{t+j}$ is the direct effect (higher $\hat{G}_{t+j}$ results in a larger fiscal deficit) whereas $S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}(\hat{G}_{t+j} - \hat{G}_t)$ measures that effect of higher real rates on the present value of (constant) surpluses S.

Quite evidently, if the leading term on the LHS were equal to zero, then setting $b_L = S$ would be required to satisfy (23) at all t. As the payment profiles of the consol coincide with the constant stream of S government surpluses the sequence of real interest rates affects both in the same way and so (23) can be satisfied for any G when the maturity structure of debt is flat. However, since a rise in the spending level leads to a deficit (taxes are constant by assumption), the optimal debt issuance needs to be tilted more towards long term debt, in order to induce an even bigger drop in the real value of debt when spending levels rise. Thus, $b_L > S$ is the optimal policy.

As it is also evident from the formula for b_L^G in i), the optimal debt issuance depends on the parameters ρ_G (the first order autocorrelation coefficient of the spending processes), Y, C, φ and σ. It is interesting to explore how these parameters affect the optimal policy. First, note that assuming a higher autocorrelation coefficient implies a larger size of the long bond position is needed to absorb the fiscal shocks. This finding reflects how the yield curve will respond to a fiscal shock in the model, depending on coefficient ρ_G. More persistent shocks, lead to a flatter response of the yield curve and therefore smaller movements in long bond prices are induced by changes in $\hat{G}_t$. Then, a larger quantity of long bonds is needed to satisfy (23) and this position needs to be financed with more short term assets (see Angeletos (2002) and Buera and Nicolini (2004)).

Parameters σ, Y, C, φ exert an influence for two reasons: First, they influence the response of the bond prices to spending shocks, accounting for the curvature of the utility function of the household. Second, they enter in the definition of the target natural level of output in the model. Since the goal of policy is to stabilize output around this natural level, it is not surprising that these terms enter into the calculation of the optimal bond portfolio.

The intertemporal budget can be written as:

$$E_t \sum_{j \geq 0} \beta^j \left(-\omega_2 \widetilde{Y}_{t+j} - \sigma \frac{Y}{C} S(\widetilde{Y}_{t+j} - \widetilde{Y}_t) \right) = b_L \beta E_t \sum_{j \geq 1} \beta^{j-1} \left[-\sigma \frac{Y}{C} (\widetilde{Y}_{t+j} - \widetilde{Y}_t) \right] \tag{25}$$

and from the Phillips curve we have $\widetilde{Y}_t = -\frac{1}{\kappa}\hat{\mu}_t$.

Thus, a positive $\hat{\mu}_t$ shock will lower the value of debt on the RHS and the discounted surpluses on the LHS. If this was the only channel through which the cost-push shock affected the debt constraint, then $b_L = S$ would be optimal. However, the lower value of debt also needs to compensate for the term $-\omega_2 \widetilde{Y}_{t+j}$ which determines the response of the government's surplus to output following a cost push shock. If this elasticity is positive ($\omega_2 > 0$) then the LHS of (25) drops less than the RHS. The planner must then issue a positive amount of short term debt as the value of this debt is not affected by the shock. Opposite, when the surplus drops following a cost push shock, it becomes optimal to tilt the portfolio more towards long debt and short term bonds will be negative. We will discuss this implication of the model further below.

Now consider the case where the policy objective only targets output. We then have $\lambda_1 = 1$, $\lambda_2 = \frac{1}{\beta}$ and $\theta_1 = \frac{1}{1-\beta\rho_\mu}$. The formula in iii) simplifies to

$$b_L^\mu = -S \frac{(1 - \beta\rho_\mu)}{\beta\rho_\mu}$$

The long bond position is negative. When the cost-push shock is only absorbed by inflation it will not impact the real value of surpluses or the deficit of the government. The LHS of the intertemporal budget is thus constant and, at the same time, on the RHS, higher inflation reduces the real payout of government debt. The optimal portfolio is the one that neutralizes the impact of inflation. It is simple to find by solving the following equation:

$$-b_S \hat{\pi}_t - b_L \hat{\pi}_t - b_L \beta \sum_{j \geq 1} \beta^{j-1} \sum_{k=1}^{k} \hat{\pi}_{t+k} = 0$$

A persistent shock can therefore be absorbed if long term debt is negative enough to satisfy the above condition.[18]

The formula in part iii) of Proposition 2 states that the optimal portfolio is somewhere in between these two cases. A higher λ_Y makes the inflation risk more important for the government budget and leads to a more negative value of long term debt. In contrast, a low λ_Y means that output fluctuations are a more significant source of risk and depending on the sign and the magnitude of the ω_2 parameter, a more balanced portfolio could be optimal.

3.2 Hedging in the full model

Proposition 3 derives the solution for $\Delta\psi_{gov,t}$ in the full model with three structural shocks:

[18] Recall that $b_L = S - b_S(1 - \beta)$ is a smaller number than b_S. For a persistent shock having negative b_L is the only way to ensure satisfaction of the intertemporal budget. For a purely temporary cost-push shock, it is not possible to find a portfolio so that $\Delta\psi_{gov,t} = 0$.

Proposition 3: *The solution for $\Delta\psi_{gov,t}$ is*

$$\Delta\psi_{gov,t} = \frac{1}{\nu_6}\left[\left(b_L^G - b_L\right)\chi_G \quad \left(b_L^\xi - b_L\right)\chi_\xi \quad \left(b_L^\mu - b_L\right)\chi_\mu\right]\begin{bmatrix} u_{G,t} \\ u_{\xi,t} \\ u_{\mu,t}\end{bmatrix} \tag{26}$$

where

$$\chi_G \equiv \frac{G\phi}{Y + \frac{\phi}{\sigma}C}\frac{(1-\rho_G)}{(1-\beta\rho_G)} \approx \frac{G\phi}{Y + \frac{\phi}{\sigma}C}$$

$$\chi_\xi \equiv \frac{(1-\rho_\xi)}{(1-\beta\rho_\xi)} \approx 1$$

$$\chi_\mu \equiv \widetilde{\omega}\left[\theta_1(1-\lambda_1) + (1-\rho_\mu)(\theta_1 - \frac{1}{1-\beta\rho_\mu})\right] - \frac{\theta_1}{(1-\beta\lambda_1)}\left[\lambda_1 + (\frac{\rho_\mu - 1}{(1-\beta\rho_\mu)})\right]$$

The expression for coefficient ν_6 is provided in the appendix.

Equation (26) is our formula for Hedging. It expresses $\Delta\psi_{gov,t}$ as the weighted sum of the three structural shocks of the model where the weights are functions of the deviations of the long-term debt level b_L from the portfolios b_L^G, b_L^ξ, b_L^μ. Together with equations (15) and (16), (26) provides the complete characterization of the inflation output trade-off in our model.

The term $\left(b_L^i - b_L\right)\chi_i$ captures the relative importance of shock i in the fluctuations of $\Delta\psi_{gov,t}$. When $b_L^i = b_L$ then the shock i will not matter at all for the solution, since the planner has targeted a portfolio that fully hedges the budget constraint against this shock. Conversely, if b_L is far from b_L^i then shock i can contribute to the variability of $\Delta\psi_{gov,t}$. Parameter χ_i then determines the increase in the variance of $\Delta\psi_{gov,t}$ which obtains from the deviation $|b_L^i - b_L|$, and holding constant the variance of the shock i.

To further explore the result in Proposition 3, let us consider the debt management policy which maximizes Hedging, or equivalently, minimizes the variance of $\Delta\psi_{gov,t}$. Letting $\sigma_G^2, \sigma_\xi^2, \sigma_\mu^2$ be the variances of the structural shocks it is easy to show that

$$b_{L,\text{Hedging}} = \frac{1}{\sum_{i\in\{G,\xi,\mu\}}\chi_i^2\sigma_i^2}\left(\sum_{i\in\{G,\xi,\mu\}} b_L^i\chi_i^2\sigma_i^2\right) \tag{27}$$

where $b_{L,\text{Hedging}}$ is the minimum variance portfolio. Quite evidently, this portfolio will be tilted towards the b_L^i for which the product $\chi_i^2\sigma_i^2$ is highest.

The expressions in Proposition 3 give sense of the relative magnitudes of the parameters. In standard calibrations we would have $\chi_G < 1 \approx \chi_\xi$. Thus, for equal variances of preferences and spending shocks, the former are a more significant risk for the government budget. Furthermore, assuming for simplicity $\lambda_Y = 0$ we get $\chi_\mu \approx -\widetilde{\omega} = -\frac{\sigma}{\kappa}\frac{Y}{C}$. A plausible calibration is $\sigma = 1$ and the ratio $\frac{Y}{C}$ is strictly greater than (but close to) 1. Moreover, $\kappa \in [0.1, 0.3]$. Thus, $\chi_\mu^2 >> \chi_\xi^2$. Supply shocks then exert a bigger impact of the debt constraint. Optimal policy will prioritize hedging against these shocks.

This numerical example is indicative of the results that we will derive from solving the model (over a wide range of parameter values) in Section 4. One of our key findings will be that the optimal portfolio composition prioritizes hedging against supply shocks. The analytics of this section shed light on the mechanics behind this property.

3.3 A closed form solution for the loss function.

The portfolio that maximizes Hedging need not coincide with the optimal portfolio in the Ramsey model. To determine the fully optimal solution one has to first plug the formula (26) into equations (15) and (16), and given the processes of output and inflation, compute the welfare loss function.

The following proposition derives the loss function when $\lambda_Y = 0$.

Proposition 4: *Assume* $\lambda_Y = 0$. *Optimal debt maturity can be found by maximizing the following objective function:*

$$-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\sigma^2_{\hat{\pi},t} = -\frac{1}{2}\overbrace{f(b_L)}^{\text{Persistence and Discounting}}\underbrace{\sum_{i\in\{G,\xi,\mu\}}\left[b^i_L - b_L\right]^2\chi_i^2\sigma_i^2}_{\text{Hedging}}$$

The policy objective is the product of the function $f(b_L)$ measuring Persistence and Discounting and the Hedging term that governs the variance of $\Delta\psi_{gov,t}$.

In the appendix we derive $f(b_L)$ as the composition of b_L^2 and an inverse function of the structural parameters of the model. A key property of f is that it is minimized for either very negative or very positive long bond positions. This property merits a brief comment.

As we saw in Section 2, when debt is only long term, the variability of inflation is reduced, since the planner can spread distortionary inflation over many periods following a shock (the Persistence channel). At the same time however, with short term debt the planner could rely on output changes to satisfy the intertemporal budget and this also lowered inflation variability (Discounting). A portfolio with long term debt and short term savings, or long term savings and short term debt, effectively enables the planner to exploit both of these channels of reduction in the variance of inflation.[19] The ability to do so is maximized when b_L is either very large or very negative.

This force will be present in our experiments in the next section when we will characterize the optimal debt policy using the microfounded loss function. Unfortunately, deriving the welfare losses in this case is not as simple, and so we will rely on the numerical solution of the Ramsey problem, to inspect the properties of the optimum.[20] As in the case of $\lambda_Y = 0$ however, the optimal policy in the Ramsey model will balance the benefit of using Discounting and Persistence against the analogous benefit of targeting a portfolio that maximizes Hedging.

4 Optimal Portfolios: A DSGE analysis

We now solve numerically the full Ramsey program we laid out in Section 2 to investigate the optimal maturity structure of debt. The solution compiles all of the forces we analyzed in the previous sections via which debt maturity impacts the inflation output trade-off. Our task in this section is to characterize the optimal policy and to quantify the contribution of each force on optimal portfolio management. Furthermore,

[19] i.e. when $b_S < 0$ and $b_L > S$ a reduction in the value of debt needed to compensate for a shock, can occur both with positive long run inflation and with negative inflation when the shock hits.

[20] With $\lambda_Y > 0$, our analytical formulae are however useful to easily calculate numerically the welfare loss function, as they represent the impulse responses of inflation and output to the shocks. The welfare losses can then be calculated as the intertemporal sums of the variances of inflation and output induced by these responses.

This approach is considerably more efficient than calculating the welfare losses using Monte Carlo simulations, especially if a large number of alternative parameterizations of the model is being considered.

we are interested in evaluating quantitatively the extent to which optimal debt management can complement optimal monetary policy in achieving its price and output stability goals.

To solve the model numerically we need to assign values to the parameters. Instead of using a standard calibration procedure, which requires to constrain our solution to one set of values for the parameters, [21] we rely on a prior predictive analysis (e.g. Geweke (2010); Leeper et al. (2017)). We solve the model over the joint prior distribution of parameter values and summarize key moments regarding the optimal maturity structure of debt, based on the resulting distribution of portfolios.[22] We present results for various draws from the distribution to show the robustness of our findings to different parameterizations of the model.

In Table 1 we report the prior distributions that we assume, along with the values of the parameters that we calibrate directly from the data.[23] To motivate the choices of the priors, let us note that the ranges of values considered are inclusive of estimated values that we found in the empirical literature. For example, risk aversion coefficients within the range of $[1, 3]$ are compatible with numerous empirical studies and are also very typical in calibrated macroeconomic models. Furthermore, a sizable empirical literature has provided estimates of the Frisch elasticity of labour supply. Though most micro-studies suggest that the elasticity is below unity, in macroeconomic models it is typical to set $\varphi = 1$.[24] Our choice of the prior for this parameter is inclusive of both the micro and macro values. Finally, the empirical counterpart for parameters $\rho_\xi, \rho_G, \rho_\mu$ and the variances of the shocks can be found in the estimated DSGE models literature. For these parameters we chose standard prior distribution functions, our assumptions are quite common for DGSE models.

Besides making these choices, another important parameter whose value we need to determine is the elasticity of the government's surplus with respect to the output gap (when we hold the spending level constant), ω_2.[25] Recall that there are different ways of choosing the value of this parameter. In the stricter sense of our model (which assumes that the government subsidizes firms to eliminate distortions from monopolistic competition), we have $\omega_2 = sY\left(1 + \phi + \frac{\sigma Y}{C}\right) > 0$. However, if we also consider transfers contingent on the output gap we can fix the value of ω_2 to be zero, or even negative.

Our baseline results are for $\omega_2 = sY\left(1 + \phi + \frac{\sigma Y}{C}\right)$. We make this assumption because we want to have a model in which there is a steep trade-off between the hedging against preference and cost-push shocks. Recall our results in Section 3.1. We showed that $b_L^\xi = S$ and $b_L^\mu < S$ when $\omega_2 > 0$ and/or λ_Y is a large number. However, assuming the microfounded weight in this section, makes λ_Y a small number so that if we also set $\omega_2 = 0$ we would get $b_L^\mu \approx S$. Then, in our numerical exercises it would be difficult to distinguish between preference and cost-push shocks in terms of hedging and to evaluate which of the two shocks exerts a more significant influence on the optimal policy. We thus set $\omega_2 > 0$ and in the appendix consider the case

[21]The calibration approach would perhaps be too restrictive in certain cases. Though for some of the parameters of the model (e.g. C, Y, G, the average level of debt), the appropriate values can be easily recovered from the data, for other parameters (i.e. the shock processes, σ, φ, κ) one can find in the literature a wide range of assumed or estimated values. Our formulae in the previous paragraphs revealed the dependence of the Hedging portfolio on these parameters and it is worthwhile exploring the sensitivity of the optimal debt maturity to them.

[22]For further details on this methodology and its application in the context of DSGE models, we refer to reader to Leeper et al. (2017).

[23]A standard procedure of calibrating the value of G is to use the sample average for government consumption in the US data. We therefore assume $G = 6.2\%$ of aggregate output. Then, normalizing $Y = 1$ implies that $C = 1 - 0;062 = 0.938$ in the steady state with zero inflation. Note that these values are standard in the literature. In fact most papers that estimate DGSE models calibrate exogenously these parameters based on the US data.

[24]This can in turn be justified on the ground that small elasticities at the micro level, can add up to a large elasticity at the macro level. The literature has explained that this can be so in heterogeneous agents models when labour supply choices are at the extensive margin, when there are fixed costs of entry into the labour force etc. We will not summarize this work here, however, it is worth mentioning these findings to show that setting $\varphi = 1$ or lower, can be justified in our representative agent model.

[25]Clearly, a spending shock will induce an increase in $\widetilde{Y}_t$ and at the same time it will lead to a higher deficit. Therefore, spending driven deficits are procyclical.

Table 1: **Prior predictive analysis**

Priors					
Parameter	Description	Distribution	Mean	Std. Dev.	90% HPD interval
σ_g	Std. error G shock	IG	0.1	1	[0.033 ; 0.249]
σ_ξ	Std. error preference shock	IG	0.1	1	[0.033 ; 0.249]
σ_μ	Std. error cost-push shock	IG	0.1	1	[0.033 ; 0.249]
ρ_g	Persistence G shock	B	0.5	0.2	[0.172 ; 0.828]
ρ_ξ	Persistence preference shock	B	0.5	0.2	[0.172 ; 0.828]
ρ_μ	Persistence cost-push shock	B	0.5	0.2	[0.172 ; 0.828]
κ	Slope NKPC	G	0.25	0.1	[0.111 ; 0.434]
σ	Intertemporal elasticity of substitution	N	2	0.5	[1.178 ; 2.822]
φ	Inverse Frish elasticity	N	2	0.5	[1.178 ; 2.822]

Calibrated parameters			
Parameter	Description	Value	Target/Source
β	Discount factor	0.995	2% annual interest rate in SS
η	CES parameter	-6.88	Steady state markup
Y	Steady-state output	1	Normalisation
G	Steady-state government spending	0.062	US data average 1980-2019
$\frac{S}{1-\beta}$	Steady-state value of surpluses/debt	240%	60% annual debt-to-GDP ratio

Notes: The table reports the prior distributions of model parameters used in the prior predictive analysis. The third column indicates the assumed prior distribution (B: beta, G: gamma, IG: inverse gamma, N: normal). The fourth and fifth columns report the mean and standard deviation of the priors, and the sixth columns provides the 90% HPD interval. The bottom panel of the table reports the assumed values for the parameters that are calibrated. The last row explains the respective targets.

where $\omega_2 = 0$.

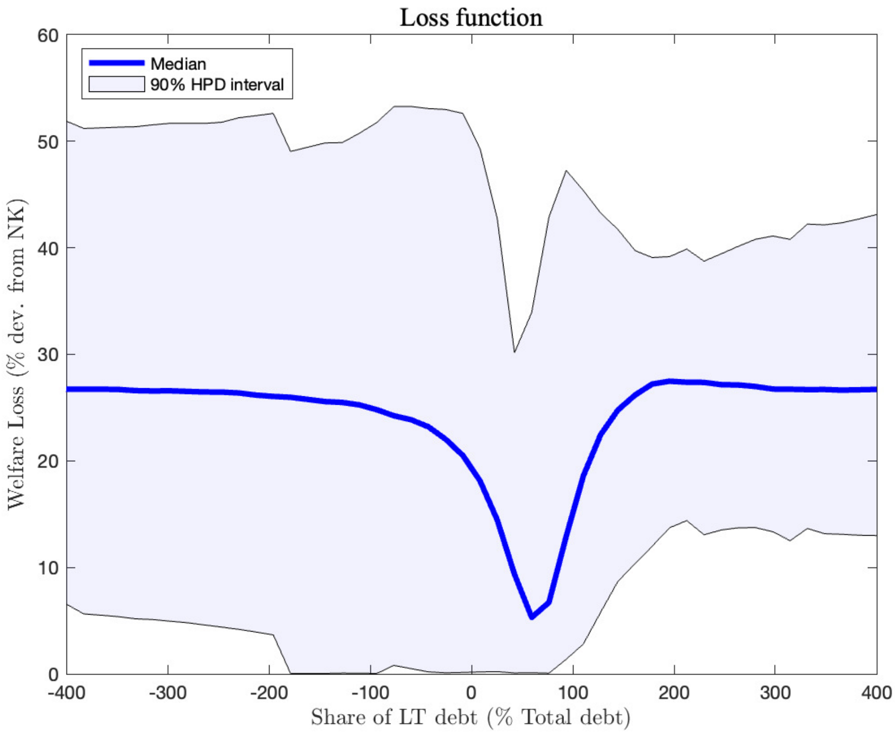
4.1 Baseline Results

Figure 1 summarizes the distribution of the welfare losses as a function of the share of long term debt over total government debt (x-axis). The solid line corresponds to the median loss function over the prior distribution and the light blue shaded areas are the 90% HPD intervals for the loss function. The losses are expressed as percentage deviations from the New Keynesian model (without the debt constraint). A deviation of say 20%, means that the ratio of loss function of the optimal policy, relative to the loss function of the NK model, is 1.2. If the percentage deviation is 0, then the model outcome coincides with the NK outcome.

Concentrate first on the solid line which depicts the median losses. Notice that the optimal policy is to issue some long term debt, the share of the long bonds in the optimal portfolio is roughly 60 percent. Therefore, the short-term bond quantity is also positive. The portfolio is 'balanced' in the sense that it features both positive long term and short term debt.

Considering the 90 percent intervals shown by the grey areas we can however see that this result may not hold more generally over the joint distribution of parameter values. The intervals are quite wide, and the loss function deviations frequently attain much lower values than the median, over a substantial range for the share of long term debt. In Figure 2 we plot the distribution of the optimal portfolios (left panel) along with the corresponding deviations of the loss functions evaluated at the optimal policies (on the right). As can be seen from the left panel, though in most of the cases the shares of long term debt are close to the median, for a non-negligible fraction of the draws from the distribution we obtain optimal shares that are close to 0 or negative; in some (though much fewer) cases the shares exceed 100 percent. Thus, there is indeed a

Figure 1: **Welfare losses across portfolios**



Notes: The figure displays the behaviour of welfare losses as a function of the share of long-term debt in total debt issuances. The results are obtained by simulating the model over a large sample of parameters drawn from the prior distributions described in Table 1. The solid blue line depicts the median value of the micro-founded loss function across all draws. The light blue area provides the 90% HPD interval.

large range of outcomes suggesting that the optimal portfolios are sensitive to some of the parameters of the model. We next decompose these results, characterizing the influence of the parameters and investigating how the Hedging vs Persistence/Discounting channels affect the optimal policies.

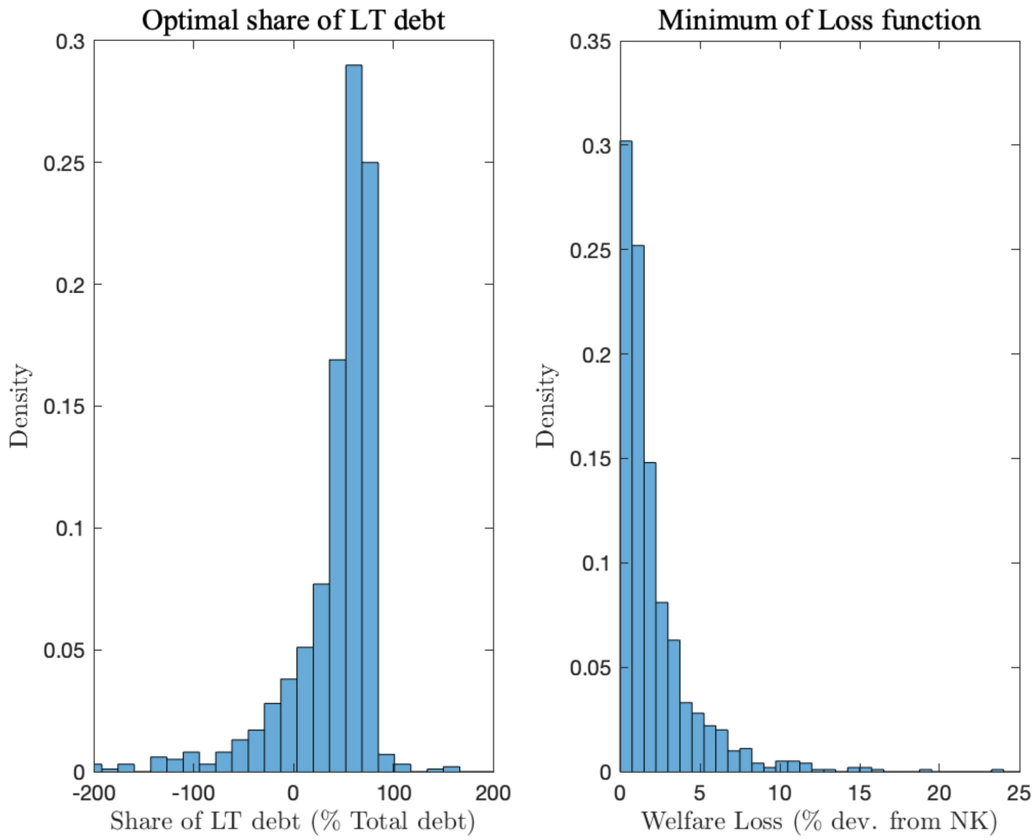
4.2 What is driving the results?

4.2.1 Hedging vs. Discounting and Persistence. To unpack the driving forces behind the results of the previous paragraph we first concentrate on the relative importance of the Hedging and the Persistence/Discounting channels of debt maturity. In Figure 3 we plot the median loss function (solid blue, left scale) alongside two additional functions of debt maturity (right scale). The black dotted graph shows the 'hedging variance' $\sum_{i\in\{G,\xi,\mu\}}\left[b_L^i - b_L\right]^2 \chi_i^2\sigma_i^2$ (see Proposition 4). The minimum value of this graph corresponds to the portfolio $b_{L,\text{Hedging}}$ in (27). Finally, the dashed red line shows the loss function when the only shock in the economy is a shock to the intertemporal surplus as in section 2.4. In this scenario, Hedging is completely absent, and the optimal debt structure is determined solely by the Discounting/Persistence arguments.[26] This is analogous to the $f(b_L)$ function derived in Proposition 4.

Consider first the red dashed graph which isolates the role of Discounting/Persistence. The optimal portfolios that yield the most favorable inflation-output trade-off feature either very long-term or very-short

[26] In constructing this plot, we assumed that the variance of the 'Shock' is such that the loss function coincides with the model's outcome when all debt is short-term. Therefore, the readers should only focus on the portfolios that minimize variances rather than on the specific magnitudes of these functions.

Figure 2: **Distribution of optimal portfolios**



Notes: The figure displays the distribution of the optimal portfolios across draws from the prior distribution of model parameters. The left panel shows the distribution of the share of long-term debt in total debt issuances that minimises the microfounded loss function. The right panel shows the associated distribution of loss function values. Loss values are expressed in deviations from the loss in the 'New-Keynesian' version of the model, in which the intertemporal government budget constraint is not a constraint for the planner.

term debt (shares = ±400 %). This finding should not be surprising given the previous discussion. Portfolios featuring very long term debt and short term savings (or very long assets and short debt) enable the planner to benefit from both Discounting and Persistence. However, as is evident from Figure 3, such portfolios are far from the Ramsey optimum, depicted in the solid blue line. We can thus conclude that Persistence and Discounting do not significantly affect the optimal policy.

Next, focus on the black dotted graph which traces the hedging variance. The minimal variance portfolio, $b_{L,\text{Hedging}}$, effectively coincides with the optimal Ramsey solution. It is thus evident that the main factor behind the optimal portfolio decisions in this version of the model is Hedging.

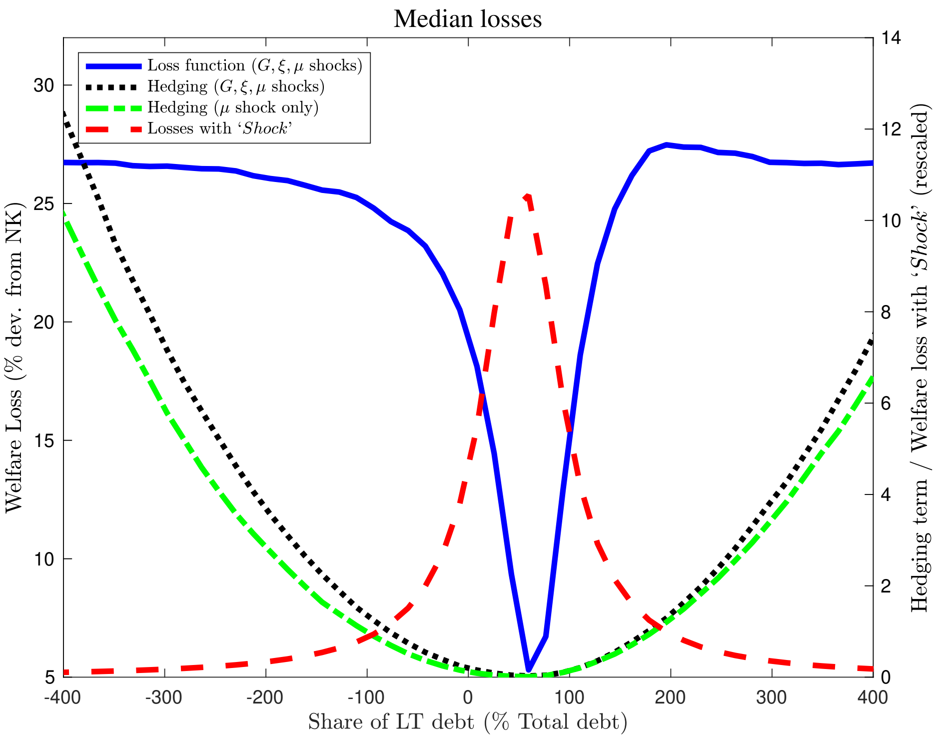
4.2.2 Hedging against which shock?

The green (dashed-dotted, right scale) line in Figure 3 shows the hedging variance when we shut down preference and spending shocks. At the minimum value of this function we obtain the fundamental portfolio for cost push shock, b_L^{μ}. Notice that this object basically coincides with $b_{L,\text{Hedging}}$ and with the optimal Ramsey solution. It is easy to see that the key driving force behind the optimal portfolio decision in our model is hedging against the cost push shock.

4.2.3 Hedging under different shocks and parameter values.

We now consider how alternative parameterizations of the model affect the optimal portfolio. We plot the median losses of the model when we

Figure 3: **Welfare losses and Hedging variance**



Notes: The figure displays the behaviour of welfare losses components as a function of the share of long-term debt in total debt issuances. The displayed results depict the median losses over results obtained by simulating the model over a large sample of parameters drawn from the prior distribution which is described in Table 1. The solid blue line displays the loss function using the three structural shocks we consider in the paper. The dotted black line shows the Hedging variance described in text, the dash-dotted green line displays the same variance assuming μ (supply) shocks only, and the dashed red line depicts the loss function in a model version with surplus shocks only, as in section 2.3.

fix one of the parameters values, assuming that the remaining parameters follow the distributions shown in Table 1. Thus, in the top left panel of Figure 4 we consider three different values for ρ_G, the autocorrelation coefficient of the spending shock, the value that corresponds to the bottom 5 percentile of the prior distribution (0.17) the median value (0.5) and the 95 percentile value (0.83). The parameters and the values that we consider in the remaining panels are reported in the graphs.

Consider first the parameters that pertain to the shock processes. According to the graphs varying the parameter values of σ_G^2, ρ_G does little to the optimal portfolios. We continue finding that the optimal share of long term is close to 60 percent. The same finding emerges when we consider ρ_ξ σ_ξ^2; changing the value of these parameters has very little bearing on the optimal debt structure. Finally, varying σ_μ^2 does seem to affect the optimal solution a bit. The effect is however not large.

The finding that the optimal portfolio decisions are not driven by parameters related to the spending process, is not surprising given the results of the previous sections. To understand why, recall that from (27), $b_{L,\text{Hedging}}$ will be tilted more towards b_L^G when the relative weight for the spending shock, χ_G is larger. However, in our numerical solutions χ_G is always at least one order of magnitude lower than χ_ξ and χ_μ. Therefore, increasing the variance of the spending shock is not enough to convince the planner to tilt the optimal portfolio towards b_L^G. Furthermore, the persistence of the shock does not drastically affect the coefficient χ_G. Higher persistence has two main impacts: On the one hand, the shock exerts a bigger effect on the intertemporal budget constraint; on the other hand, higher persistence flattens the yield curve's response to the shock and this makes it more difficult to hedge against it. The latter effect compensates for the former, and so the weight χ_G, is nearly independent of ρ_G.

For the same reason, ρ_ξ and σ_ξ^2 are unimportant for the optimal portfolio. What however may seem counter-intuitive is that the variance of the cost-push shock exerts a small influence on the optimal policy. Note however, that increasing the variance has little effect precisely because the optimal portfolio is already the one that maximizes hedging against the shock. Therefore, a change in the variance need not lead to a change in the bond positions.

The bottom left panel in Figure 4 shows that the parameter that impacts the optimal share of long term debt the most, is the first order autocorrelation coefficient of the cost-push shock, ρ_μ. Given the results and the discussion in Section 3, this should not be an odd observation. It reflects that b_L^μ is quite sensitive to the first order autocorrelation coefficient in line with the analytical expressions we showed in the previous section. The sensitivity of the optimal portfolio to ρ_μ also explains the flat part of the loss function in Figure 1.

Finally, the remaining parameters of the model σ, φ, κ exert a small influence on the optimal bond positions. Since the effects are not large, we will not offer a discussion of these results. The formulae that we provided reveal the dependence of the portfolios on the parameters and they can be used to assess qualitatively their effects.

To sum up, our analysis identifies Hedging as the most important component of the Ramsey optimal policy. Moreover, the cost-push shock is the key driving force behind $b_{L,\text{Hedging}}$. The Ramsey planner prioritizes hedging against this shock and designs the maturity structure accordingly.

4.3 Gains and Losses from Varying Debt Portfolios.

How important is targeting optimal portfolios? We can answer this question by investigating the relative losses under the optimal policy and in the case where debt maturity is not optimal. The right panel of Figure 2 plots the distribution of losses under the optimal policy drawing from the joint prior. As the graph indicates, for the larger part of the parameter space, the losses relative to the NK model are less that 10 percentage points and with the bulk being less than 5 percent.

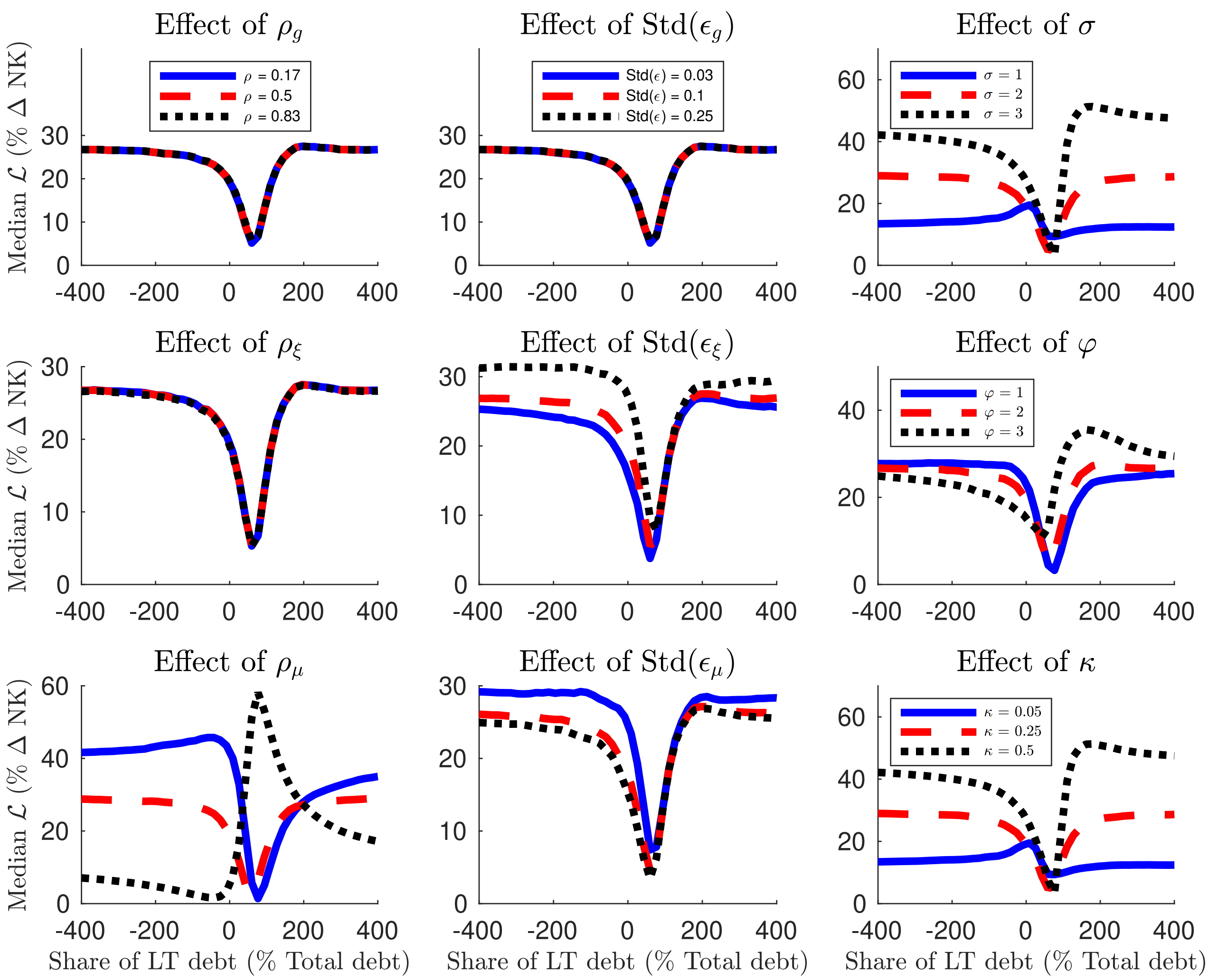
Figures 1 and 4 show how shifting the composition of debt away from the optimal portfolio impacts the loss function. As it is evident, shifting the composition away from the optimum, rapidly increases the losses. In Figure 1, for example, the median losses at the optimum are 5.31%; they become three times as large (15%) when debt is only long term, and increase up to nearly 30 percent when the share of long debt is further from the optimum. We get analogous numbers for most of the specifications we considered in Figure 4. The Ramsey optimal policy is a portfolio that maximizes hedging of the government budget against the cost-push shock. The excess welfare losses relative to the New Keynesian model at the optimum are driven solely by the demand shocks. Moving away from this optimal solution increases the losses because cost-push shocks impact the debt constraint.

These results prove that the debt maturity structure is an important policy margin to reduce the volatility of inflation and output induced by the debt constraint.

4.4 An approximate formula for the optimal debt maturity structure.

A significant finding of this section is that the key driving force behind optimal portfolios in the model is Hedging. The Ramsey planner will chose a portfolio close to $b_{L,\text{Hedging}}$ which minimizes the variance of $\Delta\psi_{gov,t}$. The expression in (27) is thus the approximate solution for the optimal debt maturity policy in the model.

Figure 4: **Welfare losses across parameter values**



Notes: The figure displays the effect of the parameters governing exogenous shock processes (the persistence of the AR(1) processes and the standard deviations of innovations) and the inter-temporal elasticity of substitution, Frisch labour elasticity and the slope of the NKPC, on welfare losses across values of the share of long-term debt. The results are obtained by simulating the model over a large sample of parameter drawn from the prior distributions described in Table 1, leaving the parameter of interest constant at the 5^{th} percentile (solid blue lines), the mean value (dashed red lines), and the 95^{th} percentile (dotted black lines) of its prior distribution, respectively. For each set of simulations, we plot the median values of the loss function. Top panels show the effects of the persistence and standard deviation of the government spending shock. Middle and bottom panels show the effects of the persistence and standard deviations of preference and cost-push shocks, respectively.

The analytical results that we have derived expressed this approximately optimal portfolio as a function of measurable model parameters. Though our approach in investigating the DSGE model has been somewhat agnostic about the values of the parameters, relying on the prior distributions, obtaining more precise values through an estimation of the model is feasible. However, our experiments also revealed that the optimal portfolio demonstrates robustness towards changes in the values of most of the parameters of the model. Notably, the parameters exerting significant influence primarily relate to the stochastic process of cost-push shocks. (We will also see in subsection 4.6 that the influence of ω_2 is analogous). Hence, using our approximate formula requires a precise measurement of only a few parameters.

4.5 Implications for the optimal debt structure in models with inflation.

Our findings have implications with regard to a well known and significant result in the literature, that when inflation is tasked with ensuring debt sustainability, the optimal maturity of debt tends to be long (e.g. Lustig et al. (2008); Faraglia et al. (2013); Leeper and Zhou (2021); Sims (2013)). This result is commonly attributed to the Persistence channel: Long term debt enables to spread inflation distortions over time.

The analytic results that we derived in this paper, enabled us to separate the the Persistence and the Hedging channels and to evaluate their relative importance in shaping optimal portfolio decisions. Our experiments revealed that Persistence did not matter much and instead Hedging played a more crucial role as a policy margin.

This result has significant implications for the optimality of long term bonds in models of debt driven inflation. If we can find cases in which hedging against shocks requires short term instead of long term debt, then the Ramsey policy can be reversed, and issuing short bonds may become optimal. Our numerical experiments in this section were revealing of the conditions under which this may occur.

4.6 Additional experiments in the appendix.

Our baseline exercise assumed a positive value for ω_2. As we discussed previously, our aim throughout this section has been to investigate the key driving forces behind optimal portfolio decisions in our simple model, and for this reason we set ω_2 equal to the value implied by the quadratic approximation of the household welfare objective function. In the appendix we extend our exercise to consider the case where $\omega_2 = 0$ assuming that government makes transfers to the private sector which fully compensate for the fluctuations in the subsidies it gives to firms. Our findings are as follows: With $\omega_2 = 0$ the median loss function is minimized when long term debt is approximately 100 percent of total debt. The optimal debt policy continues to be driven by Hedging and b_L^{μ} is now approximately equal to to b_L^{ξ}. Therefore, the Ramsey policy can kill two birds with one stone, minimizing the impact of both of these structural shocks on the intertemporal budget constraint. Under this optimal policy the welfare losses relative to the New Keynesian model are minuscule.

5 Conclusion

This paper explored the impact of the debt maturity structure on inflation and output within the Fiscal Theory of the Price Level. Leveraging the extensive literature on optimal Ramsey policy models, we relied on a mixture of analytical solutions and numerical simulations to characterize the the optimal composition of public debt in a New Keynesian framework, accounting for both demand and supply-side shocks.

Our findings yielded several intriguing conclusions: Firstly, the key factor behind the optimal maturity structure is to hedge the government budget against supply side shocks. Secondly, at the optimum the trade-off induced by the debt constraint is due to demand shocks only. Finally, targeting the optimal portfolio significantly improves the trade-off between inflation and output. Thus optimal debt maturity management is a useful policy margin to complement monetary policy in times where government debt is not backed by surpluses.

References

AIYAGARI, S. R., A. MARCET, T. J. SARGENT, AND J. SEPPÄLÄ (2002): "Optimal taxation without state-contingent debt," *Journal of Political Economy*, 110, 1220–1254.

ANGELETOS, G.-M. (2002): "Fiscal policy with noncontingent debt and the optimal maturity structure," *The Quarterly Journal of Economics*, 117, 1105–1131.

APARISI DE LANNOY, L. R., A. BHANDARI, D. EVANS, M. GOLOSOV, AND T. J. SARGENT (2022): "Managing Public Portfolios," Tech. Rep. w30501, National Bureau of Economic Research.

BASSETTO, M. (2002): "A game–theoretic view of the fiscal theory of the price level," *Econometrica*, 70, 2167–2195.

BENIGNO, P. AND M. WOODFORD (2007): "Optimal inflation targeting under alternative fiscal regimes," *Series on Central Banking, Analysis, and Economic Policies, no. 11*.

BHATTARAI, S., G. B. EGGERTSSON, AND B. GAFAROV (2023a): "Time consistency and duration of government debt: A model of quantitative easing," *The Review of Economic Studies*, 90, 1759–1799.

BHATTARAI, S., J. W. LEE, AND W. Y. PARK (2014): "Inflation dynamics: The role of public debt and policy regimes," *Journal of Monetary Economics*, 67, 93–108.

BHATTARAI, S., J. W. LEE, AND C. YANG (2023b): "Redistribution and the monetary-fiscal policy mix," *Quantitative Economics*, 14, 817–853.

BI, H. AND M. KUMHOF (2011): "Jointly optimal monetary and fiscal policy rules under liquidity constraints," *Journal of Macroeconomics*, 33, 373–389.

BIANCHI, F. AND C. ILUT (2017): "Monetary/fiscal policy mix and agents' beliefs," *Review of economic Dynamics*, 26, 113–139.

BIANCHI, F. AND L. MELOSI (2017): "Escaping the great recession," *American Economic Review*, 107, 1030–58.

BOUAKEZ, H., R. OIKONOMOU, AND R. PRIFTIS (2018): "Optimal debt management in a liquidity trap," *Journal of Economic Dynamics and Control*, 93, 5–21.

BUERA, F. AND J. P. NICOLINI (2004): "Optimal maturity of government debt without state contingent bonds," *Journal of Monetary Economics*, 51, 531–554.

BURGERT, M. AND S. SCHMIDT (2014): "Dealing with a liquidity trap when government debt matters: Optimal time-consistent monetary and fiscal policy," *Journal of Economic Dynamics and control*, 47, 282–299.

CANZONERI, M., R. CUMBY, AND B. DIBA (2010): "The interaction between monetary and fiscal policy," *Handbook of monetary economics*, 3, 935–999.

COCHRANE, J. H. (1998): "A frictionless view of US inflation," *NBER macroeconomics annual*, 13, 323–384.

——— (2001): "Long-term debt and optimal policy in the fiscal theory of the price level," *Econometrica*, 69, 69–116.

——— (2018): "Stepping on a rake: The fiscal theory of monetary policy," *European Economic Review*, 101, 354–375.

DE BEAUFFORT, C. (2023): "When is government debt accumulation optimal in a liquidity trap?" *Journal of Economic Dynamics and Control*, 147, 104591.

——— (2024): "Looking Beyond the Trap: Fiscal Legacy and Central Bank Independence," *Oxford Bulletin of Economics and Statistics*, 86, 385–416.

DEBORTOLI, D., R. NUNES, AND P. YARED (2017): "Optimal time-consistent government debt maturity," *The Quarterly Journal of Economics*, 132, 55–102.

DEL NEGRO, M. AND C. A. SIMS (2015): "When does a central bank's balance sheet require fiscal support?" *Journal of Monetary Economics*, 73, 1–19.

EGGERTSSON, G. B. (2006): "The deflation bias and committing to being irresponsible," *Journal of money, credit, and Banking*, 38, 283–321.

——— (2008): "Great Expectations and the End of the Depression," *American Economic Review*, 98, 1476–1516.

FARAGLIA, E., A. MARCET, R. OIKONOMOU, AND A. SCOTT (2013): "The impact of debt levels and debt maturity on inflation," *The Economic Journal*, 123, F164–F192.

——— (2016): "Long Term Government Bonds," Mimeo.

——— (2019): "Government debt management: The long and the short of it," *The Review of Economic Studies*, 86, 2554–2604.

FARAGLIA, E., A. MARCET, AND A. SCOTT (2010): "In search of a theory of debt management," *Journal of Monetary Economics*, 57, 821–836.

GALÍ, J. (2015): *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*, Princeton University Press.

GEWEKE, J. (2010): *Complete and incomplete econometric models*, Princeton University Press.

GIANNONI, M. P. AND M. WOODFORD (2003): "Optimal interest-rate rules: I. General theory," Tech. rep., National Bureau of Economic Research.

GREENWOOD, R., S. G. HANSON, AND J. C. STEIN (2015): "A Comparative-Advantage Approach to Government Debt Maturity," *The Journal of Finance*, 70, 1683–1722.

IMF (2021): "Fiscal Monitor, October 2021: Strengthening the Credibility of Public Finances," .

JAROCIŃSKI, M. AND B. MAĆKOWIAK (2018): "Monetary-fiscal interactions and the euro area's malaise," *Journal of International Economics*, 112, 251–266.

KUMHOF, M., R. NUNES, AND I. YAKADINA (2010): "Simple monetary rules under fiscal dominance," *Journal of Money, Credit and Banking*, 42, 63–92.

LEEPER, E. M. (1991): "Equilibria under 'active'and 'passive'monetary and fiscal policies," *Journal of monetary Economics*, 27, 129–147.

LEEPER, E. M. AND C. LEITH (2016): "Understanding inflation as a joint monetary–fiscal phenomenon," in *Handbook of Macroeconomics*, Elsevier, vol. 2, 2305–2415.

LEEPER, E. M., C. LEITH, AND D. LIU (2021): "Optimal time-consistent monetary, fiscal and debt maturity policy," *Journal of Monetary Economics*, 117, 600–617.

LEEPER, E. M., N. TRAUM, AND T. B. WALKER (2017): "Clearing up the fiscal multiplier morass," *American Economic Review*, 107, 2409–2454.

LEEPER, E. M. AND X. ZHOU (2021): "Inflation's role in optimal monetary-fiscal policy," *Journal of Monetary Economics*, 124, 1–18.

LUSTIG, H., C. SLEET, AND Ş. YELTEKIN (2008): "Fiscal hedging with nominal assets," *Journal of Monetary Economics*, 55, 710–727.

MATVEEV, D. (2021): "Time-Consistent Management of a Liquidity Trap with Government Debt," *Journal of Money, Credit and Banking*, 53, 2129–2165.

NOSBUSCH, Y. (2008): "Interest Costs and the Optimal Maturity Structure Of Government Debt," *The Economic Journal*, 118, 477–498.

REIS, R. (2016): "Funding quantitative easing to target inflation," .

ROTEMBERG, J. J. (1982): "Sticky prices in the United States," *Journal of Political Economy*, 90, 1187–1211.

SARGENT, T. J., N. WALLACE, ET AL. (1981): "Some unpleasant monetarist arithmetic," *Federal reserve bank of minneapolis quarterly review*, 5, 1–17.

SCHMITT-GROHÉ, S. AND M. URIBE (2000): "Price level determinacy and monetary policy under a balanced-budget requirement," *Journal of Monetary Economics*, 45, 211–246.

——— (2004): "Optimal fiscal and monetary policy under sticky prices," *Journal of economic Theory*, 114, 198–230.

SIMS, C. A. (1994): "A simple model for study of the determination of the price level and the interaction of monetary and fiscal policy," *Economic theory*, 4, 381–399.

——— (2013): "Paper money," *American Economic Review*, 103, 563–84.

WOODFORD, M. (1994): "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic theory*, 4, 345–380.

——— (1995): "Price-level determinacy without control of a monetary aggregate," in *Carnegie-Rochester conference series on public policy*, Elsevier, vol. 43, 1–46.

——— (2001): "Fiscal requirements for price stability," .

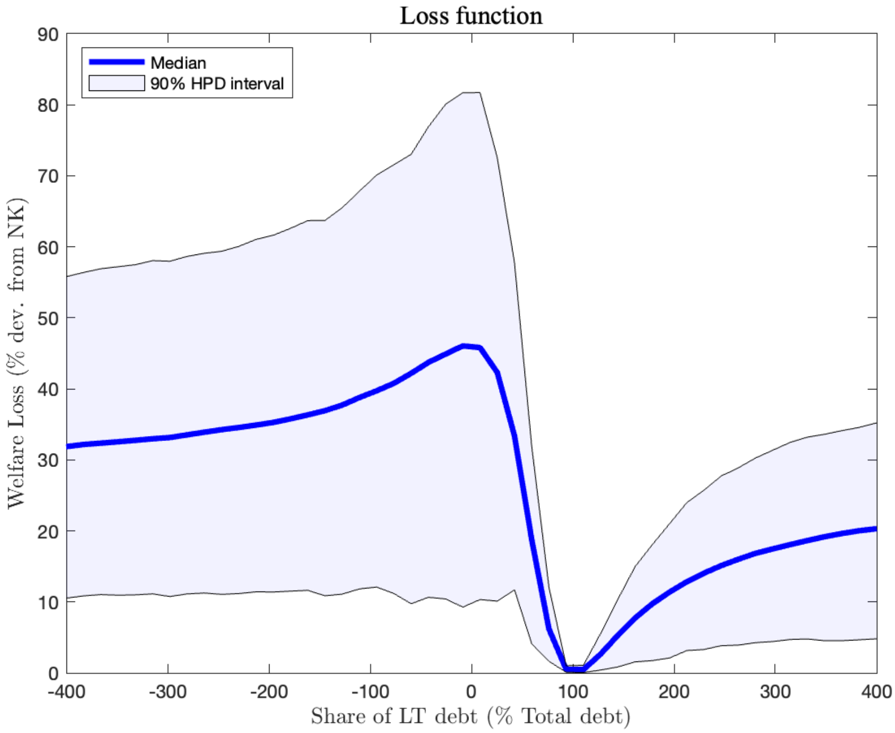
——— (2003a): *Interest and prices: Foundations of a theory of monetary policy.*, Princeton University Press.

——— (2003b): "Optimal interest-rate smoothing," *The Review of Economic Studies*, 70, 861–886.

Online Appendix (Not for Publication).

A Additional figures

Figure 5: **Welfare losses across portfolios with $\omega_2 = 0$**



Notes: The figure displays the behaviour of welfare losses as a function of the share of long-term debt in total debt issuances, assuming $\omega_2 = 0$. The results are obtained by simulating the model over a large sample of parameters drawn from the prior distributions described in Table 1. The solid blue line depicts the median value of the micro-founded loss function across all draws. The light blue area provides the 90% HPD interval.

B Derivations and Proofs

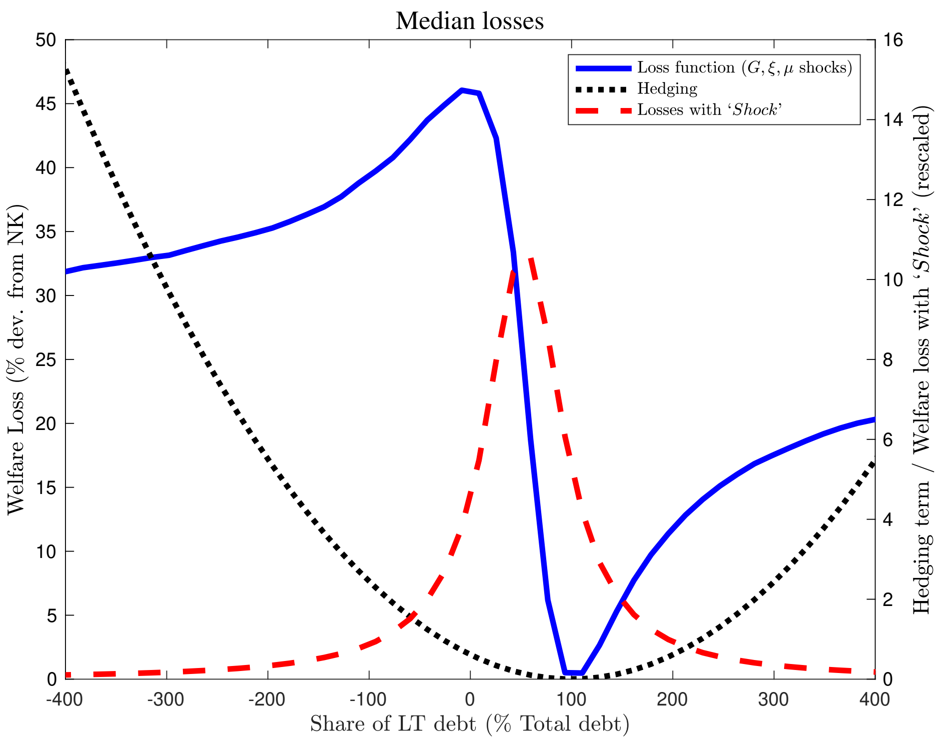
We now derive the first order conditions from the planner's program. To ease the exposition we repeat the constraint set of the planner:

$$\hat{i}_t = \sigma \frac{Y}{C}(E_t\widetilde{Y}_{t+1} - \widetilde{Y}_t) + E_t\hat{\pi}_{t+1} + \hat{r}_t^n$$
$$\hat{\pi}_t = \kappa\widetilde{Y}_t + \beta E_t\hat{\pi}_{t+1} + \hat{\mu}_t$$
$$\hat{p}_{L,t} = -\hat{i}_t + \beta E_t\hat{p}_{L,t+1}$$

together with the government budget constraint

$$b_S p_S(\hat{b}_{S,t} + \hat{p}_{S,t}) + b_L p_L(\hat{b}_{L,t} + \hat{p}_{L,t}) = -S\hat{S}_t + b_S(\hat{b}_{S,t-1} - \hat{\pi}_t) + b_L(1 + p_L)(\hat{b}_{L,t-1} - \hat{\pi}_t) + p_L b_L \hat{p}_{L,t}$$

Figure 6: **Welfare losses and Hedging variance with** $\omega_2 = 0$



Notes: The figure displays the behaviour of welfare losses components as a function of the share of long-term debt in total debt issuances, assuming $\omega_2 = 0$. The displayed results depict the median losses over results obtained by simulating the model over a large sample of parameters drawn from the prior distribution which is described in Table 1. The solid blue line displays the loss function using the three structural shocks we consider in the paper. The dotted black line shows the Hedging variance described in text, and the dashed red line depicts the loss function in a model version with surplus shocks only, as in section 2.3.

As discussed in text, we can simplify the Ramsey program noting that $\hat{i}_t$ and prices can be dropped. Then, the set of sufficient constraints is:

$$\hat{\pi}_t = \kappa\widetilde{Y}_t + \beta E_t\hat{\pi}_{t+1} + \hat{\mu}_t$$

$$b_S\beta\left(\hat{b}_{S,t} - \sigma\frac{Y}{C}(E_t\widetilde{Y}_{t+1} - \widetilde{Y}_t) - E_t\hat{\pi}_{t+1} - \hat{r}^n_t\right) +$$
$$b_L\frac{\beta}{1-\beta}\left(\hat{b}_{L,t} - \sigma\sum_{j\geq 1}\beta^{j-1}\frac{Y}{C}(E_t\widetilde{Y}_{t+j} - \widetilde{Y}_{t+j-1}) - \sum_{j\geq 1}(\beta)^{j-1}E_t\hat{\pi}_{t+j} - \sum_{j\geq 1}(\beta)^{j-1}\hat{r}^n_{t+j-1}\right) =$$
$$(1+\omega_1)G\hat{G}_t + \omega_2\widetilde{Y}_t + b_S\left(\hat{b}_{S,t-1} - \hat{\pi}_t\right) + b_L\frac{1}{1-\beta}\left(\hat{b}_{L,t-1} - \hat{\pi}_t\right)$$
$$-\frac{\beta}{1-\beta}b_L\left(\sigma\sum_{j\geq 1}\beta^{j-1}\frac{Y}{C}(E_t\widetilde{Y}_{t+j} - \widetilde{Y}_{t+j-1}) + \sum_{j\geq 1}\beta^{j-1}E_t\hat{\pi}_{t+j} + \sum_{j\geq 1}\beta^{j-1}\hat{r}^n_{t+j-1}\right)$$

To further simplify we can note that (with consols) the terms

$$b_L\frac{\beta}{1-\beta}\left(-\sigma\sum_{j\geq 1}\beta^{j-1}\frac{Y}{C}(E_t\widetilde{Y}_{t+j} - \widetilde{Y}_{t+j-1}) - \sum_{j\geq 1}(\beta)^{j-1}E_t\hat{\pi}_{t+j} - \sum_{j\geq 1}(\beta)^{j-1}\hat{r}^n_{t+j-1}\right)$$

cancel out from the LHS and the RHS of the constraint. We can therefore simplify and state the program as:

$$\max -\frac{1}{2}\sum_{t\geq 0}\beta^t E_t\left(\hat{\pi}_t^2+\lambda_Y\widetilde{Y}_t^2\right)$$

$$\text{subject to} \qquad \hat{\pi}_t=\kappa\widetilde{Y}_t+\beta E_t\hat{\pi}_{t+1}+\hat{\mu}_t$$

$$b_S\beta\left(\hat{b}_{S,t}-\sigma\frac{Y}{C}(E_t\widetilde{Y}_{t+1}-\widetilde{Y}_t)-E_t\hat{\pi}_{t+1}-\hat{r}_t^n\right)+$$

$$b_L\frac{\beta}{1-\beta}\hat{b}_{L,t}=(1+\omega_1)G\hat{G}_t+\omega_2\widetilde{Y}_t+b_S\left(\hat{b}_{S,t-1}-\hat{\pi}_t\right)+b_L\frac{1}{1-\beta}\left(\hat{b}_{L,t-1}-\hat{\pi}_t\right)$$

Given multipliers $\psi_{\pi,t}$ for the Phillips curve constraint and $\psi_{gov,t}$ for the budget constraint we can state the optimality condition for inflation as:

$$-\hat{\pi}_t+\Delta\psi_{\pi,t}+b_S\Delta\psi_{gov,t}+\frac{b_L}{1-\beta}\psi_{gov,t}=0$$

The first order condition for output is given by:

$$-\lambda_Y\widetilde{Y}_t-\psi_{\pi,t}\kappa+\beta b_S\sigma\frac{Y}{C}\psi_{gov,t}-b_S\sigma\frac{Y}{C}\psi_{gov,t-1}-\omega_2\psi_{gov,t}=0$$

To get the first equation in the format we show in text, add and subtract $\psi_{gov,t-1},\psi_{gov,t-2},\ldots$. Then, we have

$$-\hat{\pi}_t+\Delta\psi_{\pi,t}+b_S\Delta\psi_{gov,t}+\frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j}=0$$

The second equation can be rearranged as follows: First, notice that $(\beta-1)b_S=-S+b_L$. With this we have:

$$-\lambda_Y\widetilde{Y}_t-\psi_{\pi,t}\kappa+(b_L-S)\sigma\frac{Y}{C}\psi_{gov,t}+b_S\sigma\frac{Y}{C}\Delta\psi_{gov,t}-\omega_2\psi_{gov,t}=0$$

and so adding and subtracting $\psi_{gov,t-1},\psi_{gov,t-2},\ldots$ and rearranging we get:

$$-\lambda_Y\widetilde{Y}_t-\psi_{\pi,t}\kappa+b_L\sigma\frac{Y}{C}\sum_{j\geq 0}\Delta\psi_{gov,t-j}+b_S\sigma\frac{Y}{C}\Delta\psi_{gov,t}-\omega_2\psi_{gov,t}-S\sigma\frac{Y}{C}\psi_{gov,t}=0$$

which is the FONC for output shown in text.

We now derive the solutions for inflation and output we showed in Section 2.3. Combining the FONC for inflation and output we get:

$$-\hat{\pi}_t-\frac{\lambda_Y}{\kappa}\Delta\widetilde{Y}_t+b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\Delta\psi_{gov,t}-\Delta\psi_{gov,t-1})+b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{j\geq 0}(\Delta\psi_{gov,t-j}-\Delta\psi_{gov,t-j-1})-$$

$$-\frac{\omega_2}{\kappa}\Delta\psi_{gov,t}-S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t}+b_S\Delta\psi_{gov,t}+\frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j}=0$$

Then using:

$$\widetilde{Y}_t = \frac{1}{\kappa}(\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \hat{\mu}_t)$$

and

$$\widetilde{Y}_{t-1} = \frac{1}{\kappa}(\hat{\pi}_{t-1} - \beta E_{t-1} \hat{\pi}_t - \hat{\mu}_{t-1})$$

we can write:

$$\Delta\widetilde{Y}_t = \frac{1}{\kappa}(\hat{\pi}_t - \hat{\pi}_{t-1} - \beta E_t \hat{\pi}_{t+1} + \beta\hat{\pi}_t - \beta \underbrace{(\hat{\pi}_t - E_{t-1}\hat{\pi}_t)}_{\zeta_t} - \Delta\hat{\mu}_t)$$

where ζ_t is a shock to the expectation of inflation (to be pinned down later). With this addition we can write the trade-off equation as:

$$\frac{\lambda_Y}{\kappa^2}\beta E_t\hat{\pi}_{t+1} - (1 + \frac{\lambda_Y}{\kappa^2} + \beta\frac{\lambda_Y}{\kappa^2})\hat{\pi}_t + \frac{\lambda_Y}{\kappa^2}\hat{\pi}_{t-1} =$$
$$-\frac{\lambda_Y}{\kappa^2}\Delta\hat{\mu}_t - \frac{\lambda_Y}{\kappa^2}\beta\zeta_t - b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) - b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) +$$
$$+\frac{\omega_2}{\kappa}\Delta\psi_{gov,t} + S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t} - b_S\Delta\psi_{gov,t} - \frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j} = 0$$

or

$$E_t\hat{\pi}_{t+1} - (\frac{\kappa^2}{\lambda_Y\beta} + \frac{1}{\beta} + 1)\hat{\pi}_t + \frac{1}{\beta}\hat{\pi}_{t-1} =$$
$$-\frac{1}{\beta}\Delta\hat{\mu}_t - \zeta_t - \frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) - \frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) +$$
$$+\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_2}{\kappa}\Delta\psi_{gov,t} + \frac{\kappa^2}{\lambda_Y\beta}S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t} - b_S\frac{\kappa^2}{\lambda_Y\beta}\Delta\psi_{gov,t} - \frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j} = 0$$

which is a second order difference equation with forcing terms.

The two roots of the homogeneous equation λ_1, λ_2 were derived in text. Letting λ_2 denote the unstable root, we can factor the homogeneous equation into:

$$E_t(\hat{\pi}_{t+1} - \lambda_1\hat{\pi}_t) - \lambda_2(\hat{\pi}_t - \lambda_1\hat{\pi}_{t-1}) = 0$$

Define $\upsilon_t \equiv \hat{\pi}_t - \lambda_1\hat{\pi}_{t-1}$. We can solve the following difference equation forward:

$$\frac{1}{\lambda_2}E_t\upsilon_{t+1} + \frac{1}{\lambda_2}\frac{1}{\beta}\Delta\hat{\mu}_t + \frac{1}{\lambda_2}\zeta_t + \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\Delta\psi_{gov,t} - \Delta\psi_{gov,t-1}) +$$
$$+\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_2}{\kappa}\Delta\psi_{gov,t} - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t} +$$
$$+\frac{1}{\lambda_2}b_S\frac{\kappa^2}{\lambda_Y\beta}\Delta\psi_{gov,t} + \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{j\geq 0}\Delta\psi_{gov,t-j} = \upsilon_t$$

The solution to the above equation, when we iterate forward and impose the boundary condition $E_t \lim_{j\to} \frac{1}{\lambda_2^j} \upsilon_j = 0$ becomes:

$$\upsilon_t = \frac{1}{\beta\lambda_2}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\hat{\mu}_{t+l} + \frac{1}{\lambda_2}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\zeta_{t+l} + \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t(\Delta\psi_{gov,t+l} - \Delta\psi_{gov,t-1+l})$$

$$+\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{l\geq 0}\frac{1}{\lambda_2^l}\sum_{j\geq 0}E_t(\Delta\psi_{gov,t-j+l} - \Delta\psi_{gov,t-j-1+l}) - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_2}{\kappa}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t+l}+$$

$$-\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}S\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t+l} + \frac{1}{\lambda_2}b_S\frac{\kappa^2}{\lambda_Y\beta}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t+l} + \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{l\geq 0}\frac{1}{\lambda_2^l}\sum_{j\geq 0}E_t\Delta\psi_{gov,t-j+l}$$

We can solve each of the terms above analytically. We have:

$$\frac{1}{\beta\lambda_2}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\hat{\mu}_{t+l} = \frac{1}{\beta\lambda_2}(\hat{\mu}_t\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}(1-\frac{1}{\lambda_2}) - \hat{\mu}_{t-1})$$

$$\frac{1}{\lambda_2}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\zeta_{t+l} = \frac{1}{\lambda_2}\zeta_t$$

which follows from the definition of ζ_t and applying the law of iterated expectations.

$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_Y}{\kappa}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t+l} = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_Y}{\kappa}\Delta\psi_{gov,t}$$

which follows from the random walk property of the Lagrange multiplier.

$$\frac{1}{\lambda_2}b_S\frac{\kappa^2}{\lambda_Y\beta}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t+l} = \frac{1}{\lambda_2}b_S\frac{\kappa^2}{\lambda_Y\beta}\Delta\psi_{gov,t}$$

$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t(\Delta\psi_{gov,t+l} - \Delta\psi_{gov,t-1+l}) = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}(1-\frac{1}{\lambda_2})\Delta\psi_{gov,t} - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t-1}$$

$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{l\geq 0}\frac{1}{\lambda_2^l}\sum_{j\geq 0}E_t\Delta\psi_{gov,t-j+l} = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{j\geq 0}\sum_{l\geq 0}\frac{1}{\lambda_2^l}E_t\Delta\psi_{gov,t-j+l} \to$$

$$\to \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}\sum_{j\geq 0}\sum_{l=0}^{j}\frac{1}{\lambda_2^l}\Delta\psi_{gov,t-j+l} = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}(\frac{1}{1-\frac{1}{\lambda_2}})\sum_{j\geq 0}\Delta\psi_{gov,t-j}$$

and finally

$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}\sum_{l\geq 0}\frac{1}{\lambda_2^l}\sum_{j\geq 0}E_t(\Delta\psi_{gov,t-j+l} - \Delta\psi_{gov,t-j-1+l}) =$$

$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) - \frac{1}{\lambda_2^2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})\Delta\psi_{gov,t}$$

Bringing together the resulting expressions we obtain:

$$\upsilon_t = \frac{1}{\beta\lambda_2}(\hat{\mu}_t \frac{1}{1-\frac{\rho_\mu}{\lambda_2}}(1-\frac{1}{\lambda_2}) - \hat{\mu}_{t-1}) + \frac{1}{\lambda_2}\zeta_t - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{\omega_2}{\kappa}\Delta\psi_{gov,t} - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t} + \frac{1}{\lambda_2}b_S\frac{\kappa^2}{\lambda_Y\beta}\Delta\psi_{gov,t}+$$
$$+\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}(1-\frac{1}{\lambda_2})\Delta\psi_{gov,t} - \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}\Delta\psi_{gov,t-1} + \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}(\frac{1}{1-\frac{1}{\lambda_2}})\sum_{j\geq 0}\Delta\psi_{gov,t-j}+$$
$$\frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) - \frac{1}{\lambda_2^2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})\Delta\psi_{gov,t}$$

To simplify these expressions we can write:

$$\upsilon_t = \frac{1}{\beta\lambda_2}(\hat{\mu}_t \frac{1}{1-\frac{\rho_\mu}{\lambda_2}}(1-\frac{1}{\lambda_2}) - \hat{\mu}_{t-1}) + \frac{1}{\lambda_2}\zeta_t + \widetilde{a}_1\Delta\psi_{gov,t} - \widetilde{a}_2\Delta\psi_{gov,t-1} + \widetilde{a}_3\sum_{j\geq 0}\Delta\psi_{gov,t-j}+$$
$$+\widetilde{a}_4\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1})$$

for appropriately defined coefficients:

$$\widetilde{a}_1 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\left[-\frac{\omega_2}{\kappa} - S\frac{\sigma}{\kappa}\frac{Y}{C} + b_S + b_S\frac{\sigma}{\kappa}\frac{Y}{C}(1-\frac{1}{\lambda_2}) - \frac{1}{\lambda_2}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})\right]$$
$$\widetilde{a}_2 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_S\frac{\sigma}{\kappa}\frac{Y}{C}, \qquad \widetilde{a}_3 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\frac{b_L}{1-\beta}(\frac{1}{1-\frac{1}{\lambda_2}}) \qquad \widetilde{a}_4 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}b_L\frac{\sigma}{\kappa}\frac{Y}{C}(\frac{1}{1-\frac{1}{\lambda_2}})$$

Omitting the shocks for simplicity (we will introduce them later on) we can write the expected date t inflation as:

$$E_{t-1}\hat{\pi}_t = \lambda_1\hat{\pi}_{t-1} - \widetilde{a}_2\Delta\psi_{gov,t-1} + \widetilde{a}_3\sum_{j\geq 1}\delta^j\Delta\psi_{gov,t-j} + +\widetilde{a}_4\sum_{j\geq 1}\delta^j\Delta\psi_{gov,t-j} - \widetilde{a}_4\sum_{j\geq 0}\Delta\psi_{gov,t-j-1}$$

and so

$$\hat{\zeta}_t \equiv \hat{\pi}_t - E_{t-1}\hat{\pi}_t = \frac{1}{1-\lambda_2^{-1}}(\widetilde{a}_1 + \widetilde{a}_3 + \widetilde{a}_4)\Delta\psi_{gov,t}$$

Plugging this result, we obtain the following expression for inflation:

$$\hat{\pi}_t = \lambda_1\hat{\pi}_{t-1} + \frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}}(\widetilde{a}_1 + \widetilde{a}_3 + \widetilde{a}_4)\Delta\psi_{gov,t} + \widetilde{a}_1\Delta\psi_{gov,t} - \widetilde{a}_2\Delta\psi_{gov,t-1} + \widetilde{a}_3\sum_{j\geq 0}\Delta\psi_{gov,t-j}+$$
$$+\widetilde{a}_4\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) =$$
$$\lambda_1\hat{\pi}_{t-1} + a_1\Delta\psi_{gov,t} - a_2\Delta\psi_{gov,t-1} + a_3\sum_{j\geq 0}\Delta\psi_{gov,t-j} \tag{28}$$

[27] **Coefficients a_2 and a_3 are denoted in text ν_2 and ν_3 respectively.**

[27] Clearly, $a_4\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1}) = a_4\Delta\psi_{gov,t}$.

Moreover, using the above formulae and with a bit more algebra we can show that

$$\boxed{\nu_1 = \frac{\kappa^2}{\lambda_Y \beta}\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}}\left[-\frac{\omega_2}{\kappa} + b_S + b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\beta - \frac{1}{\lambda_2}) + \frac{1}{\lambda_2}\frac{b_L}{1-\beta}(\frac{1}{1-\frac{1}{\lambda_2}})\right]}$$

Now consider bringing back the cost-push shocks, assuming for simplicity that $\Delta\psi_{gov,t} = 0$ for all t. Then,

$$\upsilon_t = \frac{1}{\beta\lambda_2}(\hat{\mu}_t \frac{1}{1-\frac{\rho_\mu}{\lambda_2}}(1-\frac{1}{\lambda_2}) - \hat{\mu}_{t-1}) + \frac{1}{\lambda_2}\zeta_t$$

and from this, it becomes easy to derive:

$$\hat{\pi}_t = \lambda_1 \hat{\pi}_{t-1} + \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_t - \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_{t-1} = \lambda_1\hat{\pi}_{t-1} + \theta_1 \Delta\hat{\mu}_t \tag{29}$$

Combining the (28) and (29) we obtain the formula showed in text.

Given the analytical results for inflation it is not difficult to find the analogous expression for the output gap.

$$\kappa\widetilde{Y}_t = \hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} - \mu_t =$$

$$\lambda_1\kappa\widetilde{Y}_{t-1} - \beta\lambda_1\zeta_t - \mu_t + \lambda_1\mu_{t-1} + \theta_1\Delta\mu_t - \beta\theta_1(\rho_\mu - 1)\mu_t$$

$$\kappa\widetilde{Y}_t = \lambda_1\kappa\widetilde{Y}_{t-1} - \beta\lambda_1\zeta_t + \lambda_1\hat{\mu}_{t-1} + (\theta_1\Delta\hat{\mu}_t + O(\Delta\psi_{gov,t}, t)) - \beta E_t(\theta_1\Delta\hat{\mu}_{t+1} + O(\Delta\psi_{gov,t+1}, t+1)) - \hat{\mu}_t$$

$$\kappa\widetilde{Y}_t(1-\lambda_1 L) = (\theta_1\Delta\hat{\mu}_t + O(\Delta\psi_{gov,t}, t)) - \beta E_t(\theta_1\Delta\hat{\mu}_{t+1} + O(\Delta\psi_{gov,t+1}, t+1)) + \lambda_1\hat{\mu}_{t-1} - \hat{\mu}_t$$

$$-\beta\lambda_1\theta_1\hat{\mu}_t + \beta\lambda_1\rho_\mu\theta_1\hat{\mu}_{t-1} - \beta\lambda_1\frac{1}{1-\lambda_2^{-1}}(\widetilde{a}_1 + \widetilde{a}_3 + \widetilde{a}_4)\Delta\psi_{gov,t}$$

$$\rightarrow \widetilde{Y}_t = \lambda_1\widetilde{Y}_{t-1} + \frac{1}{\kappa}\left[\frac{1}{\beta\lambda_2}\frac{(1+\beta-\beta(\lambda_1+\lambda_2))}{1-\frac{\rho_\mu}{\lambda_2}}\hat{\mu}_t + (\widetilde{a}_1 + \beta a_2)\Delta\psi_{gov,t} - a_2\Delta\psi_{gov,t-1} + \right.$$

$$\left. + a_3(1-\beta)\sum_{j\geq 0}\Delta\psi_{gov,t-j} + a_4\sum_{j\geq 0}(\Delta\psi_{gov,t-j} - \Delta\psi_{gov,t-j-1})\right]$$

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Using the above expressions and rearranging we can obtain:

$$\boxed{\nu_4 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y\beta}\left[-\frac{\omega_2}{\kappa}(b_L - S)\frac{\sigma}{\kappa}\frac{Y}{C} + b_S + b_S\frac{\sigma}{\kappa}\frac{Y}{C}(\beta - \frac{1}{\lambda_2})b_S\frac{\sigma}{\kappa}\frac{Y}{C}\right]}$$

[28] The terms $\hat{\mu}_{t-1}$ drop since

$$-\theta_1 + \beta\lambda_1\rho_\mu\theta_1 + \lambda_1 = \frac{1}{\beta\lambda_2}\frac{1}{1-\rho_\mu/\lambda_2}(\beta\lambda_1\rho_\mu - 1) + \lambda_1 = \frac{1}{\beta\lambda_2}\frac{1}{1-\rho_\mu/\lambda_2}(\rho_\mu/\lambda_2 - 1) + \lambda_1 = \lambda_1 - \frac{1}{\beta\lambda_2} = 0$$

Then the terms $\hat{\mu}_t$ are

$$(-\beta\lambda_1\theta_1 - 1 + \theta_1 - \beta\theta_1(\rho_\mu - 1)) = \theta_1(1+\beta) - 1 - \beta\lambda_1\theta_1 - \beta\rho_\mu\theta_1 =$$

$$\theta_1(1+\beta) + \theta_1(-\beta\lambda_2 - \beta\lambda_1) = \theta_1(1+\beta) - \beta\theta_1(\lambda_2+\lambda_1) = \theta_1[1+\beta-\beta(\lambda_2+\lambda_1)]$$

B.1 Proof of Proposition 1.

The analytical derivations of this paragraph provided the proof. ■

B.2 Deriving the Optimal Portfolios.

Based on these expressions for output and inflation we can find the optimal government portfolios. To do so, we need to firstly use the intertemporal budget constraint and express $\Delta\psi_{gov,t}$ as a function of the shocks.

The intertemporal budget is:

$$E_t\sum_{j\geq 0}\beta^j\Bigg(-\omega_1\hat{G}_{t+j}-\omega_2\widetilde{Y}_{t+j}-\sigma\frac{Y}{C}S(\widetilde{Y}_{t+j}-\widetilde{Y}_t)+S(\overbrace{\frac{G\phi}{Y+\frac{\phi}{\sigma}C}(\hat{G}_{t+j}-\hat{G}_t)+\hat{\xi}_{t+j}-\hat{\xi}_t)}^{-\hat{r}^n_t-\hat{r}^n_{t+1}-\ldots-\hat{r}^n_{t+j}}\Bigg)=$$

$$b_S(\hat{b}_{S,t-1}-\hat{\pi}_t)+\frac{1}{1-\beta}b_L(\hat{b}_{L,t-1}-\hat{\pi}_t)$$

$$+b_L\beta E_t\sum_{j\geq 1}\beta^{j-1}\left[-\sigma\frac{Y}{C}(\widetilde{Y}_{t+j}-\widetilde{Y}_t)-\hat{r}^n_t-\hat{r}^n_{t+1}-\ldots-\hat{r}^n_{t+j}-\hat{\pi}_{t+1}-\ldots-\hat{\pi}_{t+j}\right]$$

To determine $\Delta\psi_{gov,t}$ we need to focus on period t innovations to this constraint. In other words terms that involve lags of state variables of the model (including terms such as $\Delta\psi_{gov,t-1}$, $\Delta\psi_{gov,t-2}$ etc) can be dropped from the calculations. Keeping this in mind we can calculate each of the components of the LHS and the RHS of the intertemporal constraint. We have:

$$-\omega_1 E_t\sum_{j\geq 0}\beta^j\hat{G}_{t+j}=-\omega_1\frac{1}{1-\beta\rho_G}\hat{G}_t=-\omega_1\frac{1}{1-\beta\rho_G}[\rho_G\hat{G}_{t-1}+u_{G,t}]=-\frac{1}{1-\beta\rho_G}u_{G,t}$$

when $\hat{G}_{t-1}$ has been accounted for.

$$E_t\sum_{j\geq 0}\beta^j\Bigg(-(\omega_2+S\sigma\frac{Y}{C})\widetilde{Y}_{t+j}+S\sigma\frac{Y}{C}\widetilde{Y}_t\Bigg)=$$

$$E_t\sum_{j\geq 0}\beta^j\Bigg(-(\omega_2+S\sigma\frac{Y}{C})\frac{1}{\kappa}[\hat{\pi}_{t+j}-\beta\hat{\pi}_{t+j+1}-\hat{\mu}_{t+j}]+S\sigma\frac{Y}{C}\widetilde{Y}_t\Bigg)=$$

$$-(\omega_2+S\sigma\frac{Y}{C})\frac{1}{\kappa}\hat{\pi}_t+(\omega_2+S\sigma\frac{Y}{C})\frac{1}{\kappa}\frac{\hat{\mu}_t}{1-\beta\rho_\mu}+S\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\hat{\pi}_t-\beta E_t\hat{\pi}_{t+1}-\hat{\mu}_t]=$$

$$-(\omega_2+S\sigma\frac{Y}{C})\frac{1}{\kappa}\hat{\pi}_t+S\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\hat{\pi}_t-\beta E_t\hat{\pi}_{t+1}]+(\omega_2+S\sigma\frac{Y}{C}\frac{\beta(\rho_\mu-1)}{(1-\beta)})\frac{1}{\kappa}\frac{u_{\mu,t}}{1-\beta\rho_\mu}$$

$$SE_t\sum_{j\geq 0}\beta^j\Bigg(\overbrace{\frac{G\phi}{Y+\frac{\phi}{\sigma}C}(\hat{G}_{t+j}-\hat{G}_t)+\hat{\xi}_{t+j}-\hat{\xi}_t}^{-\hat{r}^n_t-\hat{r}^n_{t+1}-\ldots-\hat{r}^n_{t+j}}\Bigg)=S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\hat{G}_t\frac{\beta(\rho_G-1)}{(1-\beta\rho_G)(1-\beta)}+$$

$$+S\frac{\beta(\rho_\xi-1)}{(1-\beta\rho_\xi)(1-\beta)}\hat{\xi}_t=S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\frac{\beta(\rho_G-1)}{(1-\beta\rho_G)(1-\beta)}u_{G,t}+S\frac{\beta(\rho_\xi-1)}{(1-\beta\rho_\xi)(1-\beta)}u_{\xi,t}$$

and the terms of the LHS (the present value of surpluses) of the intertemporal constraint.

On the RHS of the constraint we have:

$$-E_t \sum_{j\geq 1} \beta^{j-1}(\hat{\pi}_{t+1} + \hat{\pi}_{t+2} + ... + \hat{\pi}_{t+j}) = -E_t \sum_{j\geq 1} \beta^{j-1} \sum_{k=1}^{j} (\hat{\pi}_{t+k})$$

To simplify this term we need to figure out the expectation $E_t\hat{\pi}_{t+k}$ using the previous formulae. It is possible to write:

$$E_t\hat{\pi}_{t+k} = \lambda_1^k \hat{\pi}_t - a_2\lambda_1^{k-1}\Delta\psi_{gov,t} + a_3\frac{1-\lambda_1^k}{1-\lambda_1}\Delta\psi_{gov,t} + \theta_1(\rho_\mu - 1)\frac{\rho_\mu^k - \lambda_1^k}{\rho_\mu - \lambda_1}\hat{\mu}_t$$

where $\theta_1 = \frac{1}{\beta\lambda_2}\frac{1}{1-\frac{\rho_\mu}{\lambda_2}}$ Note that to arrive to this formula we only kept the terms $\Delta\psi_{gov,t}$ since these are the terms that the intertemporal constraint will pin down.

Then

$$-E_t \sum_{j\geq 1} \beta^{j-1} \sum_{k=1}^{j} (\hat{\pi}_{t+k}) = -\Bigg[\frac{\lambda_1}{(1-\beta)(1-\beta\lambda_1)}\hat{\pi}_t - \frac{a_2}{(1-\beta)(1-\beta\lambda_1)}\Delta\psi_{gov,t}$$
$$+a_3\frac{1}{(1-\lambda_1)(1-\beta)^2}\Delta\psi_{gov,t} + (\theta_1\frac{\rho_\mu - 1}{(\rho_\mu - \lambda_1)(1-\beta)})(\frac{\rho_\mu}{1-\beta\,\rho_\mu} - \frac{\lambda_1}{1-\beta\lambda_1})\hat{\mu}_t\Bigg]$$

Putting all of this together, we get the following expression for the budget constraint:

$$-\frac{\omega_1}{1-\beta\rho_G}u_{G,t} - (\omega_2 + S\sigma\frac{Y}{C})\frac{1}{\kappa}\hat{\pi}_t + S\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}] + (\omega_2 + S\sigma\frac{Y}{C}\frac{\beta(\rho_\mu - 1)}{(1-\beta)})\frac{1}{\kappa}\frac{u_{\mu,t}}{1-\beta\rho_\mu}$$
$$+S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\frac{\beta(\rho_G - 1)}{(1-\beta\rho_G)(1-\beta)}u_{G,t} + S\frac{\beta(\rho_\xi - 1)}{(1-\beta\rho_\xi)(1-\beta)}u_{\xi,t} =$$
$$= -b_S\hat{\pi}_t - \frac{1}{1-\beta}b_L\hat{\pi}_t + \beta b_L\Bigg[-\sigma\frac{Y}{C}\frac{1}{\kappa}E_t\hat{\pi}_{t+1} + \frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}] + \sigma\frac{Y}{C}\frac{(\rho_\mu - 1)}{(1-\beta)})\frac{1}{\kappa}\frac{u_{\mu,t}}{1-\beta\rho_\mu} +$$
$$+\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\frac{(\rho_G - 1)}{(1-\beta\rho_G)(1-\beta)}u_{G,t} + \frac{(\rho_\xi - 1)}{(1-\beta\rho_\xi)(1-\beta)}u_{\xi,t}\Bigg]$$
$$-\beta b_L\Bigg[\frac{\lambda_1}{(1-\beta)(1-\beta\lambda_1)}\hat{\pi}_t - \frac{a_2}{(1-\beta)(1-\beta\lambda_1)}\Delta\psi_{gov,t}$$
$$+a_3\frac{1}{(1-\beta)^2}\frac{1}{1-\beta\lambda_1}\Delta\psi_{gov,t} + (\theta_1\frac{\rho_\mu - 1}{(\rho_\mu - \lambda_1)(1-\beta)})(\frac{\rho_\mu}{1-\beta\rho_\mu} - \frac{\lambda_1}{1-\beta\lambda_1})u_{\mu,t}\Bigg] \quad (30)$$

B.3 Proof of Proposition 2

From (30) we can derive the optimal portfolios corresponding to each of the shocks. These are objects b_L^G, b_L^ξ, b_L^μ we showed in text. Recall that these portfolios can fully absorb the shocks and so we can set $\Delta\psi_{gov,t} = 0$.

Consider first the case of spending shocks only. It is not difficult to show that when $\Delta\psi_{gov,t} = 0$, inflation is 0 at all times and output is always equal to the natural level. From (30) we get:

$$-\frac{\omega_1}{1-\beta\rho_G}u_{G,t} + S\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\frac{\beta(\rho_G - 1)}{(1-\beta\rho_G)(1-\beta)}u_{G,t} = \beta b_L\frac{G\phi}{Y+\frac{\phi}{\sigma}C}\frac{(\rho_G - 1)}{(1-\beta\rho_G)(1-\beta)}u_{G,t} \rightarrow$$

$$b_L^G = S + \frac{\omega_1}{1-\rho_G}\frac{Y+\frac{\phi}{\sigma}C}{\phi}\frac{1-\beta}{\beta}$$

and so $b_S^G = \frac{S-b_L^G}{1-\beta} = -\frac{1}{1-\rho_G}\frac{Y+\frac{\phi}{\sigma}C}{\phi}\frac{1}{\beta} < 0$. The optimal portfolio features long term debt and short term savings.

Next consider b_L^{ξ}.

$$\beta S\frac{(\rho_\xi-1)}{(1-\beta\rho_\xi)(1-\beta)}u_{\xi,t} = \beta b_L\frac{(\rho_\xi-1)}{(1-\beta\rho_\xi)(1-\beta)}u_{\xi,t}$$

and so it is easy to see that $b_L^{\xi} = S$, $b_S^{\xi} = 0$ is the solution.

Next consider the optimal portfolio for cost-push shocks. When $\Delta\psi_{gov,t} = 0$ we can write for one off cost-push shocks:

$$\hat{\pi}_t = \theta_1 u_{\mu,t}$$
$$E\hat{\pi}_{t+1} = \theta_1\lambda_1 + \theta_1(\rho_\mu - 1)u_{\mu,t}$$
$$\hat{\pi}_t - \beta E\hat{\pi}_{t+1} = \theta_1(1-\beta\lambda_1-\beta(\rho_\mu-1))u_{\mu,t}$$

The terms involving the shock cancel out in the intertemporal budget constraint when:

$$\begin{aligned}
&-(\omega_2+S\sigma\frac{Y}{C})\frac{1}{\kappa}\theta_1 + S\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\theta_1(1-\beta\lambda_1-\beta(\rho_\mu-1))] + (\omega_2+S\sigma\frac{Y}{C}\frac{\beta(\rho_\mu-1)}{(1-\beta)})\frac{1}{\kappa}\frac{1}{1-\beta\rho_\mu}\\
&= -\frac{S}{1-\beta}\theta_1 + \beta b_L\Big[-\sigma\frac{Y}{C}\frac{1}{\kappa}(\theta_1\lambda_1+\theta_1(\rho_\mu-1)) + \frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}[\theta_1(1-\beta\lambda_1-\beta(\rho_\mu-1))]\\
&+\sigma\frac{Y}{C}\frac{(\rho_\mu-1)}{(1-\beta)})\frac{1}{\kappa}\frac{1}{1-\beta\rho_\mu}\Big] - \beta b_L\Big[\frac{\lambda_1}{(1-\beta)(1-\beta\lambda_1)}\theta_1 + (\theta_1\frac{\rho_\mu-1}{(\rho_\mu-\lambda_1)(1-\beta)})(\frac{\rho_\mu}{1-\beta\rho_\mu}-\frac{\lambda_1}{1-\beta\lambda_1})\Big]
\end{aligned}$$

and rearranging, we obtain

$$\begin{aligned}
&\frac{\omega_2}{\kappa}(\frac{1}{1-\beta\rho_\mu}-\theta_1) + S\frac{\beta\sigma}{\kappa(1-\beta)}\frac{Y}{C}\Big[(\theta_1-\frac{1}{1-\beta\rho_\mu})(1-\rho_\mu)+\theta_1(1-\lambda_1)\Big] + \frac{S}{1-\beta}\theta_1\\
&= \sigma\frac{Y}{C}\frac{\beta}{\kappa(1-\beta)}b_L^{\mu}\Big[\theta_1(1-\lambda_1)+(1-\rho_\mu)(\theta_1-\frac{1}{1-\beta\rho_\mu})\Big] - \theta_1\beta\frac{b_L^{\mu}}{(1-\beta)(1-\beta\lambda_1)}\Big[\lambda_1+(\frac{\rho_\mu-1}{(1-\beta\rho_\mu)})\Big]
\end{aligned}$$

the solution to which gives us the expression for b_L^{μ} shown in text.

Now consider the limiting cases $\lambda_Y = 0$ and $\lambda_Y = \infty$. In the first case $\lambda_1 = \theta_1 = 0$. We therefore have:

$$b_L^{\mu} = S - \frac{\omega_2}{(1-\rho_\mu)}\frac{(1-\beta)}{\beta}\frac{C}{Y\sigma}$$

Second, in the case of only output smoothing we have $\theta_1 = \frac{1}{1-\beta\rho_\mu}$ and $\lambda_1 = 1$ and so

$$b_L^{\mu} = -S\frac{(1-\beta\rho_\mu)}{\beta\rho_\mu}$$

B.4 Proof of Proposition 3

We now derive analytically the term $\Delta\psi_{gov,t}$ as a function of the shocks. To do so we utilize the impulse response function: We make use of the property that $\Delta\psi_{gov,t}$ is just a function of the date t innovations to the economy and therefore all lagged state variables of the model (lags of shocks, of $\Delta\psi_{gov,t}$ and of the debt levels) will not matter for this solution.

Equation (30) expresses debt solvency as a function of the shocks and of current and future inflation. The solution for $\Delta\psi_{gov,t}$ can be found from this equation. Writing current and future inflation as functions of $\Delta\psi_{gov,t}$ (omitting variables which are predetermined in t) we get:

$$\begin{aligned}
\hat{\pi}_t &= \overbrace{(a_1 + a_3 + a_4)}^{\equiv a_5} \Delta\psi_{gov,t} + \theta_1 u_{\mu,t} \\
E\hat{\pi}_{t+1} &= \lambda_1 \hat{\pi}_t + (a_3 - a_2)\Delta\psi_{gov,t} + \theta_1(\rho_\mu - 1)u_{\mu,t} = \\
&= \Big[a_5\lambda_1 + (a_3 - a_2)\Big]\Delta\psi_{gov,t} + \theta_1\Big[\lambda_1 + (\rho_\mu - 1)\Big]u_{\mu,t} \\
\hat{\pi}_t - \beta E\hat{\pi}_{t+1} &= \Big[(1-\beta\lambda_1)a_5 + \beta(a_2 - a_3)\Big]\Delta\psi_{gov,t} + \theta_1\Big[1 - \beta\lambda_1 + \beta(1-\rho_\mu)\Big]u_{\mu,t}
\end{aligned}$$

Next, we substitute the above expressions in the intertemporal budget (30) to solve for $\Delta\psi_{gov,t}$. Using the results of the previous paragraph to simplify the budget constraint we write it as:

$$\begin{aligned}
&\Big[b_L - b_L^G\Big]\chi_G u_{G,t} + \Big[b_L - b_L^\xi\Big]\chi_\xi u_{\xi,t} + \Big[b_L - b_L^\mu\Big]\chi_\mu u_{\mu,t} \\
&-(\omega_2 + S\sigma\frac{Y}{C})\frac{1}{\kappa}a_5\Delta\psi_{gov,t} + S\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\Big[(1-\beta\lambda_1)a_5 + \beta(a_2 - a_3)\Big] = -\frac{S}{1-\beta}a_5\Delta\psi_{gov,t} \\
&+\beta b_L\Big[-\sigma\frac{Y}{C}\frac{1}{\kappa}\Big(a_5\lambda_1 + (a_3 - a_2)\Big)\Delta\psi_{gov,t} + \frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\Big((1-\beta\lambda_1)a_5 + \beta(a_2 - a_3)\Big)\Big] \\
&-\beta b_L\Big[\frac{\lambda_1}{(1-\beta)(1-\beta\lambda_1)}a_5\Delta\psi_{gov,t} - \frac{a_2}{(1-\beta)(1-\beta\lambda_1)}\Delta\psi_{gov,t} + a_3\frac{1}{(1-\beta)^2}\frac{1}{1-\beta\lambda_1}\Delta\psi_{gov,t}\Big]
\end{aligned}$$

where

$$\begin{aligned}
\chi_G &\equiv \frac{G\phi}{Y + \frac{\phi}{\sigma}C}\frac{\beta(1-\rho_G)}{(1-\beta\rho_G)(1-\beta)} \\
\chi_\xi &\equiv \frac{\beta(1-\rho_\xi)}{(1-\beta\rho_\xi)(1-\beta)} \\
\chi_\mu &\equiv \sigma\frac{Y}{C}\frac{\beta}{\kappa(1-\beta)}\Big[\theta_1(1-\lambda_1) + (1-\rho_\mu)(\theta_1 - \frac{1}{1-\beta\rho_\mu})\Big] - \theta_1\beta\frac{1}{(1-\beta)(1-\beta\lambda_1)}\Big[\lambda_1 + (\frac{\rho_\mu - 1}{(1-\beta\rho_\mu)})\Big]
\end{aligned}$$

The above can be rearranged as follows

$$\begin{aligned}
&\Big[b_L - b_L^G\Big]\chi_G u_{G,t} + \Big[b_L - b_L^\xi\Big]\chi_\xi u_{\xi,t} + \Big[b_L - b_L^\mu\Big]\chi_\mu u_{\mu,t} \\
&= (\frac{\omega_2}{\kappa} - \frac{S}{1-\beta})a_5\Delta\psi_{gov,t} + \beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\Big[(1-\lambda_1)a_5 + (a_2 - a_3)\Big] \\
&\quad - \frac{\beta b_L}{(1-\beta)(1-\beta\lambda_1)}\Big[\lambda_1 a_5\Delta\psi_{gov,t} - (a_2 - \frac{a_3}{1-\beta})\Delta\psi_{gov,t}\Big] \qquad (31)
\end{aligned}$$

(31) can be solved to obtain $\Delta\psi_{gov,t}$ as a function of the shocks.

B.5 Proof of Proposition 4

We now simplify assuming. $\lambda_Y = 0$. Under this assumption we can show that that $\lambda_1 = 0$, $a_1 = -\frac{\omega_2}{\kappa} - S\frac{\sigma}{\kappa}\frac{Y}{C} + b_S + b_S\frac{\sigma}{\kappa}\frac{Y}{C}$, $a_2 = b_S\frac{\sigma Y}{\kappa C}$, $a_3 = \frac{b_L}{1-\beta}$ and $a_4 = b_L\frac{\sigma Y}{\kappa C}$ and $a_5 = a_1 + a_3 + a_4 = -\frac{\omega_2}{\kappa} - \beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C} + \frac{S}{1-\beta}$

Using these results equation (31) can be written as:

$$\left[b_L - b_L^G\right]\chi_G u_{G,t} + \left[b_L - b_L^\xi\right]\chi_\xi u_{\xi,t} + \left[b_L - b_L^\mu\right]\chi_\mu u_{\mu,t}$$

$$\underbrace{\left(\frac{\omega_2}{\kappa} - \frac{S}{1-\beta} + \beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\right)}_{-a_5} a_5\Delta\psi_{gov,t}$$

$$+\underbrace{\left(\beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C} + \frac{\beta b_L}{(1-\beta)}\right)}_{-\beta(a_2-a_3)}(a_2 - a_3)\Delta\psi_{gov,t} - a_3\frac{\beta^2 b_L}{(1-\beta)^2}$$

$$\rightarrow \Delta\psi_{gov,t} = \frac{\left[b_L^G - b_L\right]\chi_G u_{G,t} + \left[b_L^\xi - b_L\right]\chi_\xi u_{\xi,t} + \left[b_L^\mu - b_L\right]\chi_\mu u_{\mu,t}}{\left(\frac{\omega_2}{\kappa} - \frac{S}{1-\beta} + \beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\right)^2 + \beta\left((b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C} + \frac{b_L}{(1-\beta)}\right)^2 + \frac{\beta^2 b_L^2}{(1-\beta)^3}}$$

gives us a closed form expression for the change in the multiplier as a function of the shocks. Moreover, assuming $\lambda_Y = 0$ means that inflation follows:

$$\hat{\pi}_t = a_5\Delta\psi_{gov,t} + (a_3 - a_2)\Delta\psi_{gov,t-1} + a_3\sum_{j=2}^{t}\Delta\psi_{gov,t-j}$$

The assumption we made to derive this solution is that the economy does not get hit by any shock until period 0. Thus, we set $\Delta\psi_{gov,-1} = \Delta\psi_{gov,-2} = ... = 0$. This is useful to derive the objective function.

Using this formula, the variance of inflation in t can be written as:

$$\sigma^2_{\hat{\pi},t} = a_5^2\sigma^2_{\Delta\psi_{gov}}\mathcal{I}_{t\geq 0} + (a_2 - a_3)^2\sigma^2_{\Delta\psi_{gov}}\mathcal{I}_{t\geq 1} + a_3^2\sigma^2_{\Delta\psi_{gov}}(t-1)\mathcal{I}_{t\geq 2}$$

where the function $\mathcal{I}$ indicates when each of the terms on the RHS becomes relevant.

The welfare function is:

$$-\sum_{t=0}^{\infty}\beta^t\sigma^2_{\hat{\pi},t} = -\frac{1}{1-\beta}\left(a_5^2 + \beta(a_2 - a_3)^2\right)\sigma^2_{\Delta\psi_{gov}} - a_3^2\frac{\beta^2}{(1-\beta)^2}\sigma^2_{\Delta\psi_{gov}}$$

where

$$\sigma^2_{\Delta\psi_{gov}} = \frac{\left[b_L^G - b_L\right]^2\chi_G^2\sigma_G^2 + \left[b_L^\xi - b_L\right]^2\chi_\xi^2\sigma_\xi^2 + \left[b_L^\mu - b_L\right]^2\chi_\mu^2\sigma_\mu^2}{\left(a_5^2 + \beta(a_2 - a_3)^2 + \frac{\beta^2}{1-\beta}a_3^2\right)^2}$$

We can simplify this expression to:

$$-\sum_{t=0}^{\infty}\beta^t\sigma^2_{\hat{\pi},t} = -\frac{1}{1-\beta}\frac{\left[b_L^G - b_L\right]^2\chi_G^2\sigma_G^2 + \left[b_L^\xi - b_L\right]^2\chi_\xi^2\sigma_\xi^2 + \left[b_L^\mu - b_L\right]^2\chi_\mu^2\sigma_\mu^2}{a_5^2 + \beta(a_2-a_3)^2 + \frac{\beta^2}{1-\beta}a_3^2} =$$

$$-\frac{1}{1-\beta}\frac{\left[b_L^G - b_L\right]^2\chi_G^2\sigma_G^2 + \left[b_L^\xi - b_L\right]^2\chi_\xi^2\sigma_\xi^2 + \left[b_L^\mu - b_L\right]^2\chi_\mu^2\sigma_\mu^2}{\left(\frac{\omega_2}{\kappa} - \frac{S}{1-\beta} + \beta(b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C}\right)^2 + \beta\left((b_L - S)\frac{\sigma}{\kappa(1-\beta)}\frac{Y}{C} + \frac{b_L}{(1-\beta)}\right)^2 + \frac{\beta^2 b_L^2}{(1-\beta)^3}}$$

For the notation used in Proposition 4 we clearly have:

$$f(b_L) \equiv \left[\left(\frac{\omega_2}{\kappa} - \frac{S}{1-\beta} + (b_L - S)\frac{\beta\widetilde{\omega}}{(1-\beta)}\right)^2 + \beta\left((b_L - S)\frac{\widetilde{\omega}}{(1-\beta)} + \frac{b_L}{(1-\beta)}\right)^2 + \frac{\beta^2 b_L^2}{(1-\beta)^3}\right]^{-1}$$

B.6 Analytics of Section 2.4

We now derive the solutions for optimal inflation and output shown in Section 2.3 of the main text. Consider first the simple case where all debt is short term debt and $\sigma = 0$. We then have:

$$\hat{\pi}_t = \lambda_1\hat{\pi}_{t-1} + \left(\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}} + 1\right)\widetilde{a}_1\Delta\psi_{gov,t}$$

where $\alpha_1 = \frac{\kappa^2}{\lambda_2\lambda_Y\beta}b_S$. Moreover, it is easy to check that $\zeta_t = \frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1\Delta\psi_{gov,t}$ With this we can derive the process of the output gap as:

$$\begin{aligned}\widetilde{Y}_t &= \frac{1}{\kappa}(\hat{\pi}_t - \beta E_t\hat{\pi}_{t+1}) = \frac{1}{\kappa}\left(\lambda_1(\hat{\pi}_{t-1} - E_{t-1}\beta\hat{\pi}_t) + \left(\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}} + 1\right)\widetilde{a}_1\Delta\psi_{gov,t} - \beta\lambda_1\zeta_t\right)\\ &= \lambda_1\widetilde{Y}_{t-1} + \frac{1}{\kappa}\left(\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}} + 1\right)\widetilde{a}_1\Delta\psi_{gov,t} - \frac{\beta\lambda_1}{\kappa}\frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1\Delta\psi_{gov,t}\\ &= \lambda_1\widetilde{Y}_{t-1} + \frac{1}{\kappa}\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}}(\lambda_2 - 1)\widetilde{a}_1\Delta\psi_{gov,t}\end{aligned}$$

With this we can characterize the variance of output and the variance of inflation as functions of λ_Y given an analytical expression for $\Delta\psi_{gov,t}$. Assume an i.i.d shock to the present value of the government's surplus. In this simple model we have:

$$\text{Shock}_t = -b_S\hat{\pi}_t = -b_S\left(\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}} + 1\right)\widetilde{a}_1\Delta\psi_{gov,t} \rightarrow \Delta\psi_{gov,t} = -\text{Shock}_t\frac{1}{b_S\left(\frac{1}{\lambda_2}\frac{1}{1-\lambda_2^{-1}} + 1\right)\widetilde{a}_1}$$

Therefore we get:

$$\hat{\pi}_t = \lambda_1\hat{\pi}_{t-1} - \frac{1}{b_S}\text{Shock}_t$$

$$\widetilde{Y}_t = \lambda_1\widetilde{Y}_{t-1} - \frac{1}{\kappa}\text{Shock}_t(1-\lambda_2^{-1})\frac{1}{b_S}$$

which is the expression shown in text.

Now consider the case $\sigma > 0$ but keeping debt short-term. We get:

$$\widetilde{a}_1 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y \beta}\left[b_S + b_S \frac{\sigma}{\kappa}\frac{Y}{C}(1 - \frac{1}{\lambda_2})\right] \qquad \widetilde{a}_2 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y \beta} b_S \frac{\sigma}{\kappa}\frac{Y}{C},$$

and so inflation is:

$$\hat{\pi}_t = \lambda_1 \hat{\pi}_{t-1} + \frac{1}{1 - \lambda_2^{-1}}\widetilde{a}_1 \Delta\psi_{gov,t} - \widetilde{a}_2 \Delta\psi_{gov,t-1}$$

Using this result aggregate output becomes:

$$\begin{aligned}
\widetilde{Y}_t &= \frac{1}{\kappa}(\lambda_1(\hat{\pi}_{t-1} - E_{t-1}\beta\hat{\pi}_t) + (\frac{1}{\lambda_2}\frac{1}{1 - \lambda_2^{-1}} + 1)\widetilde{a}_1 \Delta\psi_{gov,t} - \widetilde{a}_2 \Delta\psi_{gov,t-1} + \beta \widetilde{a}_2 \Delta\psi_{gov,t} - \beta\lambda_1 \zeta_t) \\
&= \lambda_1 \widetilde{Y}_{t-1} + \frac{1}{\kappa}(\widetilde{a}_1 + \beta\widetilde{a}_2)\Delta\psi_{gov,t} - \frac{\widetilde{a}_2}{\kappa}\Delta\psi_{gov,t-1}
\end{aligned}$$

We can use the intertemporal budget to recover the multiplier. We have:

$$\begin{aligned}
\text{Shock}_t - \sum_{j\geq 0}\beta^j E_t S \frac{Y}{C}\sigma(\widetilde{Y}_{t+j} - \widetilde{Y}_t) &= -b_S \hat{\pi}_t \rightarrow \\
\text{Shock}_t - S\frac{Y}{C}\sigma\frac{1}{\kappa}(\frac{1}{1 - \lambda_2^{-1}}\widetilde{a}_1 \Delta\psi_{gov,t} - \frac{\frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1 \Delta\psi_{gov,t} + (\beta\widetilde{a}_2 - \beta\lambda_1 \frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1)\Delta\psi_{gov,t}}{1 - \beta}) & \\
&= -b_S \frac{1}{1 - \lambda_2^{-1}}\widetilde{a}_1 \Delta\psi_{gov,t} \rightarrow \\
\Delta\psi_{gov,t} = -\frac{\text{Shock}_t}{\left[b_S \frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1 + S\frac{Y}{C}\sigma\frac{\beta}{\kappa(1-\beta)}(\frac{1}{1-\lambda_2^{-1}}\widetilde{a}_1(1 - \lambda_1) + \widetilde{a}_2)\right]} &
\end{aligned}$$

Substituting the coefficients a_i and combining the above expressions it is possible to derive the solution for inflation and output shown in text.

Lastly, consider the model with $\sigma = 0$ but all debt is long term. We have:

$$\widetilde{a}_3 = \frac{1}{\lambda_2}\frac{\kappa^2}{\lambda_Y \beta}\frac{b_L}{1 - \beta}(\frac{1}{1 - \frac{1}{\lambda_2}})$$

and

$$\hat{\pi}_t = \lambda_1 \hat{\pi}_{t-1} + \frac{1}{\lambda_2 - 1}\widetilde{a}_3 \Delta\psi_{gov,t} + \widetilde{a}_3 \sum_{j\geq 0}\Delta\psi_{gov,t-j}$$

$$\begin{aligned}
\widetilde{Y}_t &= \frac{1}{\kappa}(\lambda_1(\hat{\pi}_{t-1} - E_{t-1}\beta\hat{\pi}_t) + \frac{1}{\lambda_2}\frac{1}{1 - \lambda_2^{-1}}\widetilde{a}_3 \Delta\psi_{gov,t} + \widetilde{a}_3(1 - \beta)\sum_{j\geq 0}\Delta\psi_{gov,t-j} - \beta\lambda_1 \zeta_t) \\
&= \lambda_1 \widetilde{Y}_{t-1} + \frac{1}{\kappa}(\frac{1}{\lambda_2}\frac{1 - \beta\lambda_1}{1 - \lambda_2^{-1}}\widetilde{a}_3 \Delta\psi_{gov,t} - \beta\lambda_1 \widetilde{a}_3 \Delta\psi_{gov,t} + \widetilde{a}_3(1 - \beta)\sum_{j\geq 0}\Delta\psi_{gov,t-j})
\end{aligned}$$

The intertemporal constraint can be written as:

$$\text{Shock}_t = -b_L \sum_{j\geq 0} \beta^j \sum_{k=0}^{j} \hat{\pi}_{t+k}$$

where

$$\hat{\pi}_{t+k} = \lambda_1^{k+1}\hat{\pi}_{t-1} + \frac{\lambda_1^k}{\lambda_2}\frac{1}{1-\lambda_2^{-1}}\widetilde{a}_3\Delta\psi_{gov,t} + \widetilde{a}_3\frac{1-\lambda_1^{k+1}}{1-\lambda_1}\Delta\psi_{gov,t}$$

We thus have:

$$\sum_{k=0}^{j}\hat{\pi}_{t+k} = \sum_{k=0}^{j}\frac{\lambda_1^k}{\lambda_2}\frac{1}{1-\lambda_2^{-1}}\widetilde{a}_3\Delta\psi_{gov,t} + \sum_{k=0}^{j}\widetilde{a}_3\frac{1-\lambda_1^{k+1}}{1-\lambda_1}\Delta\psi_{gov,t} =$$
$$\frac{1-\lambda_1^{j+1}}{(1-\lambda_1)}\widetilde{a}_3\Delta\psi_{gov,t}[\frac{1}{\lambda_2-1} - \frac{\lambda_1}{(1-\lambda_1)}] + \widetilde{a}_3\frac{(j+1)}{(1-\lambda_1)}\Delta\psi_{gov,t}$$

and so

$$-b_L\sum_{j\geq 0}\beta^j\sum_{k=0}^{j}\hat{\pi}_{t+k} = -b_L\Delta\psi_{gov,t}\widetilde{a}_3\sum_{j\geq 0}\beta^j\left(\frac{1-\lambda_1^{j+1}}{(1-\lambda_1)}[\frac{1}{\lambda_2-1}-\frac{\lambda_1}{(1-\lambda_1)}] + \frac{(j+1)}{(1-\lambda_1)}\right)$$
$$= -b_L\Delta\psi_{gov,t}\widetilde{a}_3\left([\frac{1}{\lambda_2-1}-\frac{\lambda_1}{(1-\lambda_1)}][\frac{1}{(1-\beta)(1-\beta\lambda_1)}] + \frac{1}{(1-\lambda_1)(1-\beta)^2}\right)$$
$$= -b_L\Delta\psi_{gov,t}\widetilde{a}_3\left([\frac{1-\lambda_1\lambda_2}{(1-\lambda_1)(\lambda_2-1)}][\frac{1}{(1-\beta)(1-\beta\lambda_1)}] + \frac{1}{(1-\lambda_1)(1-\beta)^2}\right)$$

To simplify focus on the case where $\lambda_Y = 0$. Then

$$\text{Shock}_t = -\frac{b_L^2}{(1-\beta)^3}\Delta\psi_{gov,t} \rightarrow \Delta\psi_{gov,t} = -\frac{(1-\beta)^3}{b_L^2}\text{Shock}_t$$

We thus have:

$$\hat{\pi}_t = -\frac{(1-\beta)^2}{b_L}\sum_j \text{Shock}_{t-j}$$

and so

$$Y_t = -\frac{1}{\kappa}\frac{(1-\beta)^3}{b_L}\text{Shock}_t$$

$$-b_L\sum_{j\geq 0}\beta^j\sum_{k=0}^{j}\hat{\pi}_{t+k} = -b_L\frac{1}{1-\beta}\frac{1}{1-\lambda_1}[\frac{\beta\lambda_1}{(1-\beta\lambda_1)^2} + \frac{1}{1-\beta}]\Delta\psi_{gov,t}$$
$$\rightarrow \Delta\psi_{gov,t} = -\frac{1}{b_L}\frac{(1-\beta)(1-\lambda_1)}{[\frac{\beta\lambda_1}{(1-\beta\lambda_1)^2}+\frac{1}{1-\beta}]}\text{Shock}_t$$

$$\hat{\pi}_t = \lambda_1 \hat{\pi}_{t-1} + \frac{1}{\lambda_2 - 1}\widetilde{a}_3 \Delta\psi_{gov,t} + \widetilde{a}_3 \sum_{j\geq 0} \Delta\psi_{gov,t-j}$$

■

C The non-linear model/ Deriving the welfare loss function.

C.1 Households, Firms and Government

C.1.1 Households Households in our model maximize the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{h_t^{1+\phi}}{1+\phi} \Big)$$

subject to a standard budget constraint which equates the consumption and income and the values of short term and long term bonds purchased by households, to the total net income (from salaries, bonds, and dividends). This program is standard and for brevity we simply state here the optimality conditions for bonds and consumption (the Euler equations):

$$R_t^{-1} = \beta \mathbb{E}_t d_{t+1,t} \left[\frac{C_t^{\sigma}}{\pi_{t+1} C_{t+1}^{\sigma}} \right]$$

$$P_{L,t} = \beta \mathbb{E}_t \left[\frac{C_t^{\sigma}(1 + P_{L,t+1})}{\pi_{t+1} C_{t+1}^{\sigma}} \right]$$

where $d_{t+1,t} \equiv \xi_{t+1}/\xi_t$

C.1.2 Firms As discussed in text, final output in the model is produced by a continuum of monopolistically competitive producer setting prices subject to adjustments costs as in Rotemberg (1982). The profit maximization problem of the generic firm j is given by :

$$\max_{P_t(j)} \quad E_t \sum_{s=0}^{\infty} Q_{t,t+s} \Big(\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - exp(\mu_t) \frac{MC_{t+s}(j)(1-s)}{P_{t+s}} Y_{t+s}(j) - AC_{t+s}(j) \Big)$$

$$\text{s.t.} \quad Y_{t+s}(j) = \Big(\frac{P_{t+s}(j)}{P_{t+s}} \Big)^{\eta} Y_{t+s}$$

$$AC_{t+s}(j) = \frac{\theta}{2} \Big(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - \overline{\pi} \Big)^2 Y_{t+s}$$

where s is a steady state employment subsidy. In a symmetric equilibrium (all firms charge the same price) the (non-linear) New-Keynesian Phillips curve is:

$$(\pi_t - \overline{\pi})\pi_t = \frac{\eta}{\theta} \Big(\frac{1+\eta}{\eta} - e^{\mu_t} w_t (1-s) \Big) + \beta E_t d_{t+1;t} \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - \overline{\pi})\pi_{t+1}.$$

C.1.3 Government and resource constraint The government issues debt to finance a random stream of expenditures denoted by G_t. Moreover, it levies constant lump sum taxes (T) and subsidizes monopolistic producers (s denotes the subsidy rate). The non-linear government budget constraint is:

$$\frac{b_{t-1,S}}{\pi_t} + \left(\frac{1+p_{L,t}}{\pi_t}\right) b_{L,t-1} = \frac{b_{S,t}}{R_t} + p_{L,t} b_{L,t} + T_t - G_t - s w_t Y_t$$

where b_S, b_L denote the real values of the short and long debt respectively.

Adding the budget constraint of the government and the analogous object for households we can obtain the economy wide resource constraint as:

$$C_t + G_t = Y_t\Big(1 - \frac{\theta}{2}\Big(\pi_t - \overline{\pi}\Big)^2\Big) \tag{32}$$

C.1.4 Efficient steady state The first best allocation is given by solving

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t \Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{h_t^{1+\phi}}{1+\phi}\Big)$$

such that (32) So, at the first best, we have

$$C_t^{-\sigma} = \chi Y_t^{\phi}$$

At steady state, we have

$$w = \frac{1+\eta}{\eta(1-s)} \qquad s = -\frac{1}{\eta} \qquad R = \beta^{-1} \qquad p_L = \frac{\beta}{1-\beta}$$
$$T = \Big(b_S + \frac{b_L}{1-\beta}\Big)(1-\beta) + G + swY$$

C.2 The micro-founded function

Second order approximation:

$$U \approx c^{-\sigma}\Big[C\hat{C}_t + \frac{1}{2}(1-\sigma)C\hat{C}_t^2 - Y\hat{Y}_t - \frac{1}{2}(1+\phi)Y\hat{Y}_t^2\Big] + tip \tag{33}$$

$$RC \approx C\hat{C}_t + \frac{1}{2}C\hat{C}_t^2 - Y\hat{Y}_t - \frac{1}{2}Y\hat{Y}_t^2 + \frac{1}{2}\theta Y\hat{\pi}_t^2 + tip \tag{34}$$

Combining the previous equations we get:

$$U \approx -\frac{1}{2}c^{-\sigma}\Big[-\sigma C\hat{C}_t^2 + \phi Y\hat{Y}_t^2 + \theta Y\hat{\pi}_t^2\Big] + tip \tag{35}$$

Using $\hat{C}_t^2 \approx \Big(\frac{Y}{C}\Big)^2 \hat{Y}_t^2 - 2\frac{GY}{C^2}\hat{Y}_t\hat{G}_t + tip = -\frac{Y}{C}\frac{\phi}{\sigma}\hat{Y}_t^2 + \Big(\frac{Y}{C}\Big)^2(1+\frac{\phi C}{\sigma Y})(\hat{Y}_t - \hat{Y}_t^n)^2 + tip$, we get

$$U \approx -\frac{1}{2}c^{-\sigma}Y\Big[\Big(\sigma\frac{Y}{C} + \phi\Big)(\hat{Y}_t - \hat{Y}_t^n)^2 + \theta\hat{\pi}_t^2\Big] + tip \tag{36}$$

which, after normalizing gives

$$-\frac{1}{2}(\lambda_Y(\hat{Y}_t - \hat{Y}_t^n)^2 + \hat{\pi}_t^2) \tag{37}$$

with $\lambda_Y \equiv \Big(\sigma\frac{Y}{C} + \phi\Big)\theta^{-1}$

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Brussels RLE – Company's number : 0203.201.340
Registered office : 14 Boulevard de Berlaimont – BE-1000 Brussels
www.nbb.be

Editor

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Layout : Analysis and Research Group
Cover : NBB CM – Prepress & Image

Published in July 2024