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Aggregate and distributional effects of a carbon tax

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Editorial

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Aggregate and distributional effects of a carbon tax

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Abstract

To identify the households most affected by a carbon tax I set up a multi-sector model with putty-clay technology. A $100-per-ton carbon tax cuts emissions by 25% after 5 years, but reduces output by 3% in the short run and 4% in the long run. Initially, the tax is progressive despite poorer households spending more on carbon-intensive goods, the prices of which rise. The complementarity of capital and energy causes a sharp decline in capital income, affecting top earners the most, and leads to job cuts in capital goods-producing industries that employ high-income earners. Over time the tax incidence flattens.

Keywords: carbon tax, putty-clay, input-output linkages, income distribution

JEL Codes: D57, E62, Q52

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1 Introduction

The impending climate crisis has pushed climate change to the forefront of global policy. Economists generally agree that carbon pricing would spur the adoption of new technologies and help mitigate climate change. But ambitious carbon pricing reforms are rare, mostly because policy makers are concerned about their impact on inequality and poverty (Metcalf, 2009; Carattini et al., 2019). Whereas policy analyses often focus on how a carbon tax would raise the cost of energy, little is known about the short-run general equilibrium effects of such a policy, including how higher energy prices trickle down into higher prices across the entire spectrum of consumer goods through input-output linkages, how wages and capital income respond, which sectors experience the sharpest declines in output and employment, and how these unequal consequences unfold over time.

This paper attempts to answer these questions by implementing a multi-sector model with household heterogeneity. The multi-sector framework is particularly appropriate in this context since differences in energy shares across sectors imply that sectors' exposure to a carbon tax varies greatly. The model makes rich predictions about the effects of a carbon tax on prices, output, labor market prospects etc. across sectors. Households, in turn, differ in their exposure to the various sectors due to differences in the goods they consume and their sector of employment, and they differ in the share of their income that is derived from capital vs. labor.

I extend the workhorse multi-sector model to include a sensible demand function for energy, the main carbon-intensive input in the economy. The extent to which a carbon tax reduces emissions and affects the economy critically depends on how sensitive firms' and households' demand responds to an increase in energy prices. As documented in Kilian (2008) and Labandeira et al. (2017), empirically, demand for energy is inelastic in the short run, but becomes more price sensitive at longer horizons. I capture this feature in a putty-clay model, where firms produce goods from a set of machines with fixed energy intensity. Energy demand is relatively insensitive in the short run when the energy intensity of the capital stock is pre-determined. Facing a higher cost of energy, firms invest, however, in more energy-efficient machines (Aghion et al., 2016; Calel and Dechezleprêtre, 2016; Hawkins-Pierrot and Wagner, 2022), which makes energy demand more elastic in the long run.[1] But the process of replacing the stock of old machines takes time, and, consequently, so does the transition to a less energy-reliant economy.

[1]Using plant-level data from the U.S. manufacturing census Hawkins-Pierrot and Wagner (2022) find that firms build less energy-efficient plants in times of higher energy prices and that these plants consume more energy throughout their lifetime, regardless of current electricity prices.

I use my model to quantify the aggregate economic and distributional income effects of a permanent carbon tax. The model's initial steady state is calibrated to the U.S. economy's 404 sectors and their input-output structure. Key input is a novel dataset on carbon emissions by economic sector published by the U.S. Environmental Protection Agency (EPA). I use this dataset to calculate the tax rates that each industry faces. I then simulate a permanent carbon tax of $100 per ton of carbon and solve for prices and quantities along the transition path to the new steady state for all sectors in the model.

The carbon tax reduces carbon emissions by 25% after 5 years and about 50% in the long run. Upon impact, energy consumption falls by 8% and GDP drops by about 3%. Even though energy accounts for only a small fraction of GDP, the fall in energy consumption markedly slows economic activity because it is achieved through a partial shutdown of machines. The drop in GDP goes along with a sharp decline in investment of more than 10%, but a modest, temporary increase in consumption. Over time, as firms invest into more energy efficient machines, energy consumption keeps on falling without causing a strong decline in GDP. Still, long-run GDP falls by 4% (ignoring any positive effects from reducing carbon emissions).

The strong short-run complementarity between energy and capital inherent to the putty-clay model has three implications for the distributional effects of a carbon tax. First, the carbon tax leads to a sharp decline in capital income. In the first year, net capital income falls by 15%, while labor income only falls by 3%. The tax is levied on capital owners who use energy to operate their machines. To what extent the tax is actually paid by capital owners or passed on to final consumers depends on firms' elasticity of energy demand. Since, consistent with the data, this elasticity is relatively low in the putty-clay model, most of the burden falls on capital owners. This result is an example of the inelastic side of the market bearing the tax incidence (see e.g. Kotlikoff and Summers, 1987).[2] The carbon tax reduces the value of the existing stock of capital by lowering expected dividends. Energy-intensive machines become "stranded assets", to use a term coined in an influential speech by the governor of the Bank of England, Mark Carney (Carney, 2015).[3] In support of these predictions, both Carattini and Sen (2019) and Känzig (2021) report that higher carbon prices empirically lead to a fall of stock prices.

Second, the complementarity between energy and capital affects the distribution of labor in-

[2]The fact that putty-clay models can generate substantial stock market volatility is also emphasized by Gourio (2011) who considers a model where capital and labor are combined in a putty-clay fashion.

[3]Carney (2015) was one of the first to warn against the financial risks of climate policies as an increase in stringency could lead investors to reevaluate the economy's productive capital stock, which could destabilize the financial system. I abstract from imperfections in financial markets that could potentially amplify the negative effects on capital income and GDP.

come across industries. With the net return on capital falling, the carbon tax leads to a fall in aggregate investment (and a change in the composition of investment towards high energy-efficient machines). This has direct implications for output and employment patterns across sectors. Economic activity markedly slows down in sectors producing capital goods and it is in these sectors that labor income falls the most. This echoes the empirical result in Goolsbee (1998) who finds that much of the benefit of investment tax breaks go to capital goods producers through higher prices rather than to investing firms. It is also consistent with empirical studies reported by Kilian (2008) and the results in Känzig (2021) who find that energy-intensive industries suffer less from an increase in energy prices than demand-sensitive industries such as industries producing capital goods.

Finally, the complementarity of energy and capital also affects the carbon tax's pass-through into output prices. Because part of the tax burden is borne by capital owners in the short run, output prices respond less than one-for-one with the tax. In the first year, the pass-through is less than 60%. It is only over time, as capital stocks fall and rental rates rise that the pass-through becomes one. This small pass through in the short run is consistent with Ganapati et al. (2020): Exploiting differences in the energy mix across industries and U.S. states they show that only 70% of energy price-driven changes in input costs get passed through to consumers over the first couple of years.

These findings — the fall of capital income, the distribution of labor income across sectors, and the low pass-through into output prices — are at the core of understanding the distributional consequences of the carbon tax. Households in the model differ by their sector of employment and their initial income level, which also affects their preferred basket of consumption goods due to non-homothetic preferences. I discipline the model's household heterogeneity using data from the consumer expenditure survey (CEX), the current population survey (CPS) and the distributional national accounts (DINA), splitting households in income percentiles.

Calculating the real consumption change for each income percentile, I find that the carbon tax is progressive in the short run. Consumption rises by almost 2% in the first year for the bottom half of the income distribution, but declines by 0.5% for the top 5%.[4] While poor households consume more carbon-intensive goods, the low short-run pass-through of the carbon tax into output price mutes this effect. Low-income households are less likely to work in capital-goods-

[4]I assume that the carbon tax is rebated to households through a fall in the consumption tax. Aggregate consumption initially rises because consumption is relatively cheap in the short run when the capital stock is higher than in the long run and households are willing to substitute consumption intertemporally.

producing industries and are more likely to work in service-oriented consumer sectors, such as restaurants and accommodation. As a result, poorer households are less vulnerable to the drop in demand for capital goods that follows the implementation of the carbon tax.[5] Most importantly, poorer households earn most of their income through labor rather than capital. It is the top earners that rely on capital income, which makes them particularly vulnerable to the carbon tax. In the long run, however, the carbon tax starts hurting low-income households more because the pass-through into consumer prices goes up. Still, a gap between capital and labor income remains such that even over time, high-income earners remain the ones most affected by the tax. This leads to an inverse u shape of consumption losses across the income distribution with households pertaining to the middle class loosing the least.

1.1 Comparison to the literature and contribution

Several papers have studied the effects of carbon taxes across the income distribution (see Wang et al., 2016; Shang, 2021, for surveys). Most studies focus on the effect of a carbon tax on consumer prices. Estimating the share of energy consumption in total consumption across income groups using household budget survey data, a common conclusion is that carbon taxes are strongly regressive, hurting the poor more than the average household (Hassett et al., 2009; Grainger and Kolstad, 2010; Mathur and Morris, 2014; Fremstad and Paul, 2017; Feindt et al., 2021). These studies typically assume a direct and perfect pass-through into consumer prices and ignore the distributional consequences of income changes. In an effort to overcome these challenges, several papers present general equilibrium models to predict the effect on prices and households' income. A common finding is that the consumption effects are progressive, but this conclusion is fully driven by redistribution policies rather than a progressive response of market income. For instance, Rausch et al. (2011) and Williams et al. (2015) assume that the carbon tax revenue is rebated in a lump-sum fashion, which benefits the poor much more than the rich. Fullerton et al. (2011) , Cronin et al. (2019) and Goulder et al. (2019) assume that social transfers are indexed to inflation, resulting in larger transfers in response to the inflation-inducing carbon tax. Goulder et al. (2019) state that "the progressive source-side impacts [i.e. the income effects] (...) are strongly driven by (...) in-

[5]While my model is consistent with Känzig (2021)'s finding that labor income across sectors is mostly driven by sectors' different exposure to fluctuations in final demand, household-level data for the United States suggest that the poor work in demand-sensitive sectors, whereas Känzig (2021) contends that, in the United Kingdom, the poor are more likely to work in demand-sensitive sectors. Whether, more generally, the poor are more exposed to business cycles in the United States is still subject to debate. See e.g. Parker and Vissing-Jorgensen (2009) for evidence that the rich have more volatile income (mostly due to the high volatility of capital income) and consumption, and Hoynes et al. (2012) and Patterson (2019) for evidence that the poor experience larger income drops during recessions.

creases in nominal transfer income." In a counterfactual where they keep nominal transfers fixed, the income effects become regressive.

I contribute to this literature in three ways: First, whereas papers in this literature base their predictions on models with a representative household, my model features the same dimensions of household heterogeneity as those used for the distributional analysis. This makes the analysis more consistent by accounting for the fact that household heterogeneity can affect aggregate outcomes. Second, I allow households to work in different sectors, which matters for how their income is affected by a carbon tax that hits some sectors more than others. Third, the model accounts for the observed strong complementarity of energy and capital in the short run. This generates the novel result that carbon taxes have a regressive effect on consumption.

At least since the energy crises in the 1970s, both a theoretical and empirical literature connecting macroeconomics and energy economics have emerged to study the dynamic effects of energy price shocks and, more recently, the effects of carbon pricing.

The empirical literature has found surprisingly large effects of energy price increases on economic activity (Kilian, 2008; Hamilton, 2008). In his literature review, Hamilton (2008) concludes that a 10% increase in oil prices reduces GDP by 1.4%. Känzig (2021) finds that an increase in carbon prices normalized to raise energy prices by 10% leads to a reduction in GDP by even more than 5%. These effects are particularly large given the modest share of energy in output (Hulten, 1978).

The theoretical literature has developed two main channels through which higher energy prices can reduce economic activity: Supply-side models appeal to strong complementarities between energy and capital goods in the production function (Atkeson and Kehoe, 1999; Finn, 2000), whereas demand-side models emphasize either direct, discretionary income effects (Baumeister and Kilian, 2016)—with households spending less on non-energy goods—or indirect, general equilibrium effects triggered by higher energy prices hitting households with particularly high marginal propensities to consume (MPCs) (Chan et al., 2024; Känzig, 2021).

My model combines supply-side channels through a combination of putty-clay technology and a utilization margin, and demand-side channels through household heterogeneity in MPCs and financial constraints on the household side.[6] This yields two main insights: First, the demand-side channels are strongly muted because I consider a permanent change in the carbon tax. Even in heterogenous agent models, MPCs out of permanent income changes are constant across the

[6] Since I consider a carbon tax with revenue being rebated through a consumption tax decrease, there are no direct, discretionary income effects in my model.

income distribution and equal to one. Hence, shifts in income across the income distribution neither amplify nor dampen aggregate demand. Even in the presence of temporary energy price shocks, the putty-clay technology would weaken the amplification through the demand side that is emphasized in Chan et al. (2024) and Känzig (2021). An important part explaining amplification in the class of heterogenous agent models is the positive correlation in real income volatility and high MPC (see e.g. Bilbiie, 2020). If high-MPC households' income is particularly volatile, then this tends to amplify business cycles. This is the case in Känzig (2021) who assumes that the carbon tax revenue is fully rebated to low-MPC households. Similarly, Chan et al. (2024) assume that labor and energy are strong complements, making (high-MPC) workers more vulnerable to higher energy prices than (low-MPC) firm owners. The putty-clay technology in my model, instead, implies a strong complementarity between capital and energy, resulting in capital income falling more than labor income. Hence, even in response to a temporary energy price increase, the model would generate little amplification through the demand side. Second, I show that, even absent any demand-side amplification, the supply-side channels can be strong enough to generate output elasticities to energy price shocks that are of the same magnitude as those found in the data, with GDP falling by about 3% for a 35% increase in energy prices.

The following two sections introduce the multi-sector model with putty-clay technology and discuss its calibration. Sections 4 and 5 examine the aggregate and distributional effects of instituting a permanent carbon tax. The final section concludes.

2 A Multi-Sector Model with Putty-Clay Technology

2.1 Overview

This section develops a New Keynesian model with multiple sectors and putty-clay technology. The model's main features are as follows: First, production takes place in J sectors that each produces a distinct good. Sectors are linked through a production network and firms are therefore exposed to the carbon tax not only through their own emissions, but also indirectly through their use of intermediate goods. Both capital markets and labor markets are segmented in the short run such that factor returns are sector specific.

Second, the production of capital follows the putty-clay approach in Atkeson and Kehoe (1999). Machines differ in their energy intensity and hence their carbon emissions. In response to a carbon tax, firms invest in more carbon-efficient machines, but since a machine's energy intensity is chosen once and for all at the time of investment, the carbon efficiency of the stock of operating machines takes some time to improve, especially for goods with a low depreciation rate. This means that in the short run, the price elasticity of demand for energy is very low and that short-run adjustments to the carbon tax differ from its long-run effects.

Third, the economy is populated by a continuum of households that potentially differ in their labor productivity, their ownership share of a national capital fund, and their preferences across consumption goods. Household heterogeneity provides a rationale for the distributional analysis in the later part of the paper, but also feeds back into the dynamics of the model through its effect on the demand for goods and the stochastic discount factor, as emphasized by the literature on heterogenous agent New Keynesian models (Kaplan et al., 2018).

Finally, I introduce a tax on carbon emissions. The tax takes two forms depending on how carbon is emitted. Most carbon is emitted through combustion of fuels, which is used to run the capital stock, and part of the carbon tax acts as a tax on fuel combustion. A second part of the carbon tax is a tax on output for certain products whose production process emits carbon (e.g. production of cement emits carbon through calcination).

I first describe the putty-clay approach to the production of capital services before discussing households' consumption and labor supply decisions. Households choose in which sector to work and labor unions decide how much each household works in a given sector. Section 2.4 presents the production and input-output structure of the intermediate and final goods and Section 2.5 specifies government policies, including how the carbon tax is set and rebated. The model is written in real terms with all prices being deflated by the aggregate consumer price index.

2.2 Capital

Each sector j has its own capital stock managed by capital funds. Capital funds buy energy to run the machines in each sector and then rent out the resulting capital services to firms. When purchasing energy, capital funds are taxed according to the carbon emitted during the combustion of energy.

Capital capacity The production of capital capacity follows the putty-clay approach in Atkeson and Kehoe (1999). The capital stock consists of a continuum of machines that require energy to run. For a machine in sector i, capital capacity is given by $k = a_i z^{\chi_i} e^{1-\chi_i}$, where a_i is a constant sector-specific productivity shifter, z is the size of the machine, e its energy requirement and

χ_i is a sector-specific factor that affects a sector's energy intensity. Both its size z and its energy requirement e are decided when the machine is constructed and remain constant thereafter. Hence (and since production features constant returns to scale), one can normalize $e = 1$ and write $k = a_i z^{\chi_i}$, where z denotes both the size and energy efficiency of the machine.

Let $x_{i,t}$ denote the number of new machines in sector i at time t and $z_{i,t}$ their energy efficiency (which is the same across all new machines). Investment is then given by $I_{i,t} = x_{i,t} z_{i,t}$. Installing new machines requires an adjustment cost $f\left(\frac{I_{i,t}}{I_{i,t-1}}\right) \equiv f_{i,t}$ with $f(1) = f'(1) = 0$ and $f''(1) \geq 0$. That is, when purchasing $x_{i,t}$ machines, only $x_{i,t}(1 - f_{i,t})$ get installed.[7] Machines have a probability $\delta_{i,t}$ of breaking down each period.[8]

The equilibrium of this economy features a cross-sectional distribution of machines. Denoting by $G_{i,t}(z)$ the measure of machines with energy intensity less than z, its law of motion reflects the number of machines that survived from the previous period plus the new machines built with efficiency $z_{i,t}$ (if $z_{i,t} \leq z$), net of adjustment costs:

$$G_{i,t+1}(z) = (1 - \delta_{i,t}) G_{i,t}(z) + x_t (1 - f_{i,t}) \mathbb{1}_{z_{i,t} \leq z}.$$

The number of machines in sector i is $X_{i,t} = \int_0^\infty dG_{i,t}(z)$ and capital capacity is $K_{i,t} = \int_0^\infty a_i z_{i,t}^{\chi_i} dG_{i,t}(z)$. Given the normalization of $e = 1$, the number of machines in a sector, $X_{i,t}$, is equal to the energy requirement of that sector's stock of machines.

A sector's energy requirement, $X_{i,t}$, and total capital capacity, $K_{i,t}$, are pre-determined because they are chosen in the previous period. They can be expressed as functions of their own lags, and some control variables. Using the law of motion for $G_{i,t}(z)$, the state variables evolve as follows:

$$X_{i,t+1} = (1 - \delta_{i,t}) X_{i,t} + x_{i,t} (1 - f_{i,t}), \tag{2.1}$$

and

$$K_{i,t+1} = (1 - \delta_{i,t}) K_{i,t} + x_{i,t} a_i z_{i,t}^{\chi_i} (1 - f_{i,t}). \tag{2.2}$$

Capital fund Capital funds manage the machines on behalf of the households. Following Finn (2000), capital funds decide for how long to run their machines and hence, how much energy the

[7] Investment adjustment costs are a common feature of DSGE model (Christiano et al., 2005) and are often preferred to capital adjustment costs because they can generate hump-shaped impulse responses and help match the empirically observed acyclical behavior of real interest rates.

[8] The model assumes that investment is irreversible so that machines will not be scrapped.

machines consume.[9] Recall that machines have been normalized such that they consume one unit of energy if run at normal capacity. Denoting the machines' running time (utilization) by $u_{i,t}$ actual energy consumption is the number of machines times their running time,

$$E_{i,t} = u_{i,t} X_{i,t}. \tag{2.3}$$

Raising the utilization of capital raises the depreciation rate: $\delta_{i,t} = \delta_i(u_{i,t})$, with $\delta_i'(1) > 0$ and $\delta_i''(1) > 0$. To operate the machines capital funds purchase energy $E_{i,t}$ at price $p_{E_i,t} + \tau_{E_i,t}$, where $p_{E_i,t}$ is the price of the energy bundle $E_{i,t}$ and $\tau_{E_i,t}$ is the corresponding energy tax. They rent out the resulting capital services to firms and receive, in return, capital income, $\sum_i u_{i,t} r_{i,t} K_{i,t}$, where $r_{i,t}$ is the rental price of the running machines. Part of the receipts are used to purchase $x_{i,t}$ machines of size $z_{i,t}$ at price $p_{I_i,t}$. Besides their income from renting out capital to firms, capital funds also receive income from bonds purchased in the previous period, $B_{t-1}\frac{1+i_{t-1}}{\pi_t}$, where i_{t-1} is the nominal interest rate and π_t is the CPI inflation rate.

Dividends are then composed of the return on machines, net of energy expenditures and investment, and the return on bonds:

$$div_t = \sum_{i=1}^{J} \left\{ u_{i,t} r_{i,t} K_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t}\right) u_{i,t} X_{i,t} - p_{I_i,t} x_{i,t} z_{i,t} \right\} + B_t - B_{t-1}\frac{1+i_{t-1}}{\pi_t}. \tag{2.4}$$

Capital funds then choose $K_{i,t+1}$, $X_{i,t+1}$, $x_{i,t}$, $z_{i,t}$, $u_{i,t}$ and B_t to maximize the expected discounted sum of their dividends, $\mathbb{E}_t\left(\sum_{s=0}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t} div_{t+s}\right)$, subject to the law of motions for the number of machines (2.1) and capital capacity (2.2). The stochastic discount factor $\beta^s \frac{\Phi_{t+s}}{\Phi_t}$ is defined in the next section. The relevant first-order conditions are discussed together with the quantitative results in Section 4 and are also in Appendix Section A.2.

2.3 Households

The economy is populated by a unit mass of households. I introduce three dimensions of household heterogeneity that make households differentially exposed to the carbon tax. First, households work in different sectors and their labor income is therefore tied to their sectors' exposure to the carbon tax. Second, households differ in their labor productivity and their level of capital

[9] I assume that all machines are run at equal capacity. This simplifies the solution relative to the setup in Gilchrist and Williams (2000). My assumption is natural in a setup where, once the machines are put in place, their capital services are perfect complements to each other, making each machine essential for production.

income. That is, households differ in the share of income that is derived from capital income and are therefore differentially exposed to a carbon tax that drives a wedge between the returns on labor and capital. Third, households have non-homothetic preferences over consumption goods, implying that households with different income levels consume different baskets of goods. Since the carbon tax has a differential impact on consumption prices, some households will experience a stronger increase in inflation than others.

The household side of the model features two ingredients that make the response and exposure to the carbon tax dynamic and allow the economy to rebalance in the long run: First, I assume that nominal wages only adjust sluggishly (Erceg et al., 2000). The carbon tax therefore translates into a fall in labor (rather than a fall in wages), which amplifies the recession. Over time, as wages adjust, labor returns to its initial level.[10] Second, I assume that the distribution of workers across sectors needs time to respond to wage differentials across sectors. Specifically, households have a constant probability ψ of death and each period, a cohort of size ψ is born (Blanchard, 1985). When a household is born they choose their sector of employment where they stay for their entire life. Since newcomers will choose to work in more thriving sectors, workers' income prospects across sectors will rebalance over time, even if the carbon tax hits some sectors more than others in the short run.[11]

Life-cycle and sector choice Each period, a cohort of size ψ is born. Newborns choose their sector of employment based on the expected lifetime utility from consumption and a household-specific preference shock for each sector i, denoted by $\varepsilon_{i,t}$. For each sector there is a one-time 'training' cost κ_i, measured in terms of utility, that needs to be paid. For households born in t the maximization problem is

$$\max_i \left\{ \left(\sum_{s=0}^{\infty} \left[\beta(1-\psi)\right]^s \mathbb{E}_t \left(\mathcal{U}_{i,t+s}\right) \right) + \frac{1}{\gamma}\varepsilon_{i,t} - \kappa_i \right\}.$$

Here, $\mathcal{U}_{i,t}$ is the utility a household in sector i receives, and the discount factor β is augmented by the probability of survival, $1-\psi$. The standard deviation of the idiosyncratic shocks is given by $\frac{1}{\gamma}$. Assuming that idiosyncratic shocks are distributed according to a Type-I extreme value

[10] This nominal rigidity also gives a lever for monetary policy to affect the economy's response and makes output partially demand-determined in the short run.

[11] Alternatively, I could assume that workers directly relocate across sectors (subject to some friction), as in Caliendo et al. (2019). These approaches are very similar in that they generate larger wage dispersion across sectors in the short run, but smaller wage dispersion in the long run.

distribution, one can show that the share of households that choose sector i is

$$\mu_{i,t} = \frac{\exp\left\{\left(\sum_{s=0}^{\infty}\left[\beta(1-\psi)\right]^s \mathbb{E}_t\left(\mathcal{U}_{i,t+s}\right)\right) - \kappa_i\right\}^{\gamma}}{\sum_k \exp\left\{\left(\sum_{s=0}^{\infty}\left[\beta(1-\psi)\right]^s \mathbb{E}_t\left(\mathcal{U}_{k,t+s}\right)\right) - \kappa_k\right\}^{\gamma}}, \tag{2.5}$$

with $\sum_i \mu_{i,t} = 1$. If labor market prospects rise in sector j relative to i, then $\mathcal{U}_{j,t}$ rises relative to $\mathcal{U}_{i,t}$, and households are more likely to choose to work in sector j rather than i. The inverse of the standard deviation of the idiosyncratic shocks, γ, disciplines the elasticity of sector choices to utility differentials. The number of households employed in sector i at time t, $n_{i,t}$, is then composed of those households employed last period and that survive, $(1-\psi)n_{i,t-1}$, and the new households that choose sector i, $\psi\mu_{i,t}$:

$$n_{i,t} = (1-\psi)n_{i,t-1} + \psi\mu_{i,t}. \tag{2.6}$$

Labor supply Once households choose their sector, they are (randomly) assigned a job $\iota \in [0,1]$ that pays a nominal wage $W_{i,t}(\iota)$. This wage is potentially sticky in the short run such that changes in labor demand translate into changes in equilibrium labor rather than being absorbed through wage changes (Erceg et al., 2000). This allows the carbon tax to have an effect on labor.

More precisely, households supply a fixed amount of labor, l^S, in their sector. For each job ι and each sector i, there is a labor union with market power that acts in the interest of its members in setting a wage rate. A set of competitive labor-aggregating firms hire households and sell effective labor to goods producers. Effective labor $l_{i,t}$ is produced from the following combination of jobs:

$$l_{i,t} = \zeta + \left(\int_0^1 \left(\omega_i^l(\iota)\right)^{\frac{1}{\psi_w}} a_l(\iota)\left(l_{i,t}(\iota) - \zeta\right)^{\frac{\psi_w - 1}{\psi_w}} d\iota\right)^{\frac{\psi_w}{\psi_w - 1}},$$

where $\psi_w > 1$ is the elasticity of substitution across jobs. This aggregator differs from the standard one commonly assumed in the New Keyensian literature: First, the assumption of $\zeta > 0$ ensures a positive equilibrium wage in the presence of inelastic labor supply (House et al., 2018). Second, and more important for this paper, the aggregator allows for different labor productivities across jobs, $a_l(\iota)$, to account for differences in income levels across households. The preference weights $\omega_i^l(\iota)$ acknowledge that sectors differ in their demand for jobs, with some sectors having more high-productive jobs than other sectors. In the non-stochastic steady, $\omega_i^l(\iota)$ is the density of type-ι jobs in sector i, with $\int_0^1 \omega_i^l(\iota)d\iota = 1$. I denote by $\omega_{i,t}^l(\iota)$ the actual density in period t. For future reference, I denote by $n_{i,t}(\iota) = n_{i,t}\omega_{i,t}^l(\iota)$ the number of households working job ι in sector i at time t.

Unions set wages to maximize payments to their workforce taking labor demand as given. Wages are set according to a Calvo mechanism with a wage reset probability of $1-\theta_w$. The unions' maximization problem then yields the following New Keynesian wage Phillips curves for each sector i:[12]

$$\tilde{\pi}^w_{i,t} = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w}\tilde{l}_{i,t} + \beta\mathbb{E}_t\left[\tilde{\pi}^w_{i,t+1}\right], \tag{2.7}$$

where a tilde denotes log deviations from the non-stochastic steady state and $\pi^w_{i,t}$ is wage inflation in sector i at time t. Notice that if wages were fully flexible ($\theta_w \to 0$), then hours would be constant and the carbon tax would, by assumption, have no effect on equilibrium labor.

Budget constraint A household working job ι in sector i receives labor income $w_{i,t}(\iota)l_{i,t}$, where $w_{i,t}(\iota) \equiv \frac{W_{i,t}(\iota)}{P_{c,t}}$ is the real wage. In addition, households also hold a share $a_{k,t}(\iota)$ of the capital fund that pays out dividends div_t. To keep the model tractable, I assume that the ownership shares in the capital fund are non-tradable.[13]

The household uses their net income to purchase a consumption bundle $c_{i,t}(\iota)$ at price $p_{c_i,t}(\iota)$. As discussed below, the composition of the consumption bundle is specific to each household and its price therefore depends on a household's sector of employment i and job ι. I assume that consumption is taxed at rate τ^C_t. The tax rate is time-varying because, as becomes clear later on, I assume that the carbon tax revenues are rebated to households through a reduction in the consumption tax rate. This ensures that the rebate does not affect the distribution of consumption and does not drive the distributional consequences of the carbon tax.

Taken together, the budget constraint for a household working job ι in sector i is

$$(1+\tau^C_t)p_{c_i,t}(\iota)c_{i,t}(\iota) = w_{i,t}(\iota)l_{i,t}(\iota) + a_{k,t}(\iota)div_t. \tag{2.8}$$

[12] See Appendix Section A.1 for more details. Notice that wage inflation differs across jobs ι because wages can be adjusted for some jobs, but not for others. Since the probability of having a stuck wage is i.i.d. across jobs and newborn workers choose sectors before knowing their job, this heterogeneity does not affect any dynamics and does not affect the distributional analysis that is done from an ex-ante point of view.

[13] Capital shares change slightly over time because sectors differ in their job profiles and grow / shrink at different rates over time. In practice, I assume $a_{k,t}(\iota) = \frac{a_k(\iota)}{\sum_i \int_0^1 n_{i,t}(\iota)a_k(\iota)}$ such that $\frac{a_{k,t}(\iota)}{a_{k,t-1}(\iota)} = \frac{a_{k,t}(\iota')}{a_{k,t-1}(\iota')}$.

Consumption A household employed in job ι of sector i consumes a bundle of goods

$$c_{i,t}(\iota) = \left(\sum_{j=1}^{J} \left(\omega_c^j(c_{i,t}(\iota)) \right)^{\frac{1}{\sigma}} \left(y_{c_i,t}^j(\iota) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.9}$$

where $\omega_c^j(c_{i,t}(\iota))$ is the preference weight that a household with consumption level $c_{i,t}(\iota)$ assigns to good j (with $\sum_j \omega_c^j(c_{i,t}(\iota)) = 1$), $y_{c_i,t}^j(\iota)$ is consumption of good j by household ι working in sector i, and σ is the elasticity of substitution between sector goods. The bundle of consumption goods includes housing services and services from motor vehicles. Allowing the preference weights to be a function of households' consumption level introduces non-homotheticicities in a very flexible manner (Faber and Fally, 2017). Denoting by $p_{j,t}$ the real price of good j, and by $p_{c_i,t}(\iota)$ the real price index associated with (2.9),

$$p_{c_i,t}(\iota) = \left(\sum_{j=1}^{J} \omega_c^j(c_{i,t}(\iota)) p_{j,t}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

demand for good j by a household working job ι in sector i is

$$y_{c_i,t}^j(\iota) = \omega_c^j(c_{i,t}(\iota)) c_{i,t}(\iota) \left(\frac{p_{c_i,t}(\iota)}{p_{j,t}} \right)^{\sigma}. \tag{2.10}$$

Utility and stochastic discount factor Utility is defined as $\mathcal{U}_{i,t}(\iota) = \ln c_{i,t}(\iota)$. The stochastic discount factor $\beta^s \frac{\Phi_{t+s}}{\Phi_t}$ reflects the ownership structure of the capital fund, with

$$\Phi_t = \frac{1+\tau^C}{1+\tau_t^C} \int_0^1 \sum_i a_{k,t}(\iota) \frac{c_i(\iota)}{p_{c_i,t}(\iota) c_{i,t}(\iota)} n_{i,t}(\iota) d\iota \tag{2.11}$$

denoting the price-adjusted average marginal utility of the owners of the capital fund, with weights corresponding to households' ownership shares $a_{k,t}(\iota)$.[14]

2.4 Production

Production takes place in a two-stage process. In a first stage, sector goods are produced from capital, labor and intermediate goods. At this stage, firms are taxed according to their direct emissions caused by the production process (beyond the combustion of energy). In a second stage,

[14] Note that marginal utilities are normalized by their steady-state value, $\frac{1}{c_i(\iota)}$, to ensure that the capital fund puts equal weight on each capital share, $a_k(\iota)$.

the various sector goods are bundled to produce final goods specific to each demand component.

First stage In the first stage, perfectly competitive firms operate in either of J sectors to produce sector goods using capital services, $u_{i,t}K_{i,t}$, labor, $L_{i,t}$, and a bundle of intermediates, $M_{i,t}$:

$$Y_{i,t} = \left\{ (1-\phi_i)^{\frac{1}{\xi}} \left[A_i \left(u_{i,t} K_{i,t} \right)^{\alpha_i} L_{i,t}^{1-\alpha_i} \right]^{\frac{\xi-1}{\xi}} + \phi_i^{\frac{1}{\xi}} M_{i,t}^{\frac{\xi-1}{\xi}} \right\}^{\frac{\xi}{\xi-1}}, \tag{2.12}$$

where ξ is the elasticity of substitution between intermediates and the capital-labor-aggregate, $1-\alpha_i$ is the labor income share in sector i, ϕ_i is a weight on intermediates in sector i's production function and A_i is a constant productivity shifter in sector i. Note that I abstract for simplicity from a feedback loop of carbon emissions on productivity through climate damages, as used e.g. in Golosov et al. (2014), because the positive effects from reducing carbon emissions are negligible in the short run, which is the focus of this paper.

Firms pay a tax $\tau_{Y_{i,t}}$ proportional to production $Y_{i,t}$. Denoting by $p_{i,t}$ the price of their output, their maximization problem is

$$\max_{L_{i,t},K_{i,t},M_{i,t}} \left\{ \left(p_{i,t} - \tau_{Y_{i,t}} \right) Y_{i,t} - w_{i,t} L_{i,t} - r_{i,t} u_{i,t} K_{i,t} - p_{M_i,t} M_{i,t} \right\} \tag{2.13}$$

subject to the production function (2.12). Here, p_{M_i} is the price of the bundle of intermediates used in sector i.

Second stage In a second stage, another set of perfectly competitive firms combine the sector goods from the first stage to produce J investment goods, I_j, J energy bundles E_j, J intermediate good bundles, and a government good G according to

$$y_{s,t} = \left(\sum_{i=1}^{J} \left(\omega_s^i \right)^{\frac{1}{\sigma}} \left(y_{s,t}^i \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2.14}$$

for $s = I_j, E_j, M_j, G$. Here, ω_s^i are preference weights that are specific to each composite s and satisfy $\sum_{i=1}^{J} \omega_s^i = 1$. For instance, energy bundles are composed of different sector goods than investment goods. Facing prices $p_{i,t}$ for each input i, the firms' maximization problem is analagous to the one for households choosing their optimal consumption bundle, yielding demand curves analagous to (2.10).

2.5 Government

The government purchases a constant amount of goods, G, at price $p_{G,t}$. The government imposes a tax on carbon emissions. Let Ψ_{E_i} denote emissions due to combustion per unit of energy, $E_{i,t}$ and Ψ_{Y_i} denote emissions due to the production process per unit of output, $Y_{i,t}$. Then, the tax rates on energy, $\tau_{E_i,t}$ and production, $\tau_{Y_i,t}$ are set proportional to emissions: $\tau_{E_i,t} = \tau_t^{carb}\Psi_{E_i}$, and $\tau_{Y_i,t} = \tau_t^{carb}\Psi_{Y_i}$, with τ_t^{carb} denoting the carbon tax rate per unit of emissions. Total revenue from the carbon tax is $\tau_t^{carb}carb_t$, with

$$carb_t = \sum_{i=1}^{J} \left(\Psi_{E_i} E_{i,t} + \Psi_{Y_i} Y_{i,t}\right)$$

denoting economy-wide emissions. The government adjusts the consumption tax rate τ_t^C to balance its budget at all times:

$$\tau_t^C \sum_{i=1}^{J} \left[\int_0^1 n_{i,t}(\iota) p_{c_i,t}(\iota) c_{i,t}(\iota) d\iota\right] = p_{G,t}G - \tau_t^{carb} carb_t.$$

Finally, the central bank sets the nominal interest rate according to a Taylor rule that targets consumer price inflation:

$$i_t = \varphi i_{t-1} + (1-\varphi)\left(\bar{i} + \varphi_\pi \pi_t\right). \tag{2.15}$$

2.6 Market clearing and aggregation

Labor market clearing requires $L_{i,t} = n_{i,t} l_{i,t}$. Bond market clearing requires $B_t = 0$. The market for sector good i clears whenever

$$Y_{i,t} = y^i_{G,t} + \sum_{j=1}^{J} \left(\int_0^1 n_{j,t}(\iota) y^i_{c_j,t}(\iota) d\iota + y^i_{I_j,t} + y^i_{M_j,t} + y^i_{E_j,t}\right).$$

To construct aggregate consumption in line with the national accounts, services from motor vehicles (sector J), $p_{J,t} y^J_{c_i,t}$, need to be removed from the set of consumption goods. Instead, purchases of motor vehicles, $p_{I_J,t} I_{J,t}$, and consumption of gasoline, $p_{E_J,t} E_{J,t}$, are added to consumption.

Hence, aggregate consumption is

$$C_t = \sum_{i=1}^{J-1} \int_0^1 n_{i,t}(\iota) \left[p_{c_i,t}(\iota) c_{i,t}(\iota) - p_{J,t} y^J_{c_i,t}(\iota) \right] d\iota + p_{I_J,t} I_{J,t} + p_{E_J,t} E_{J,t}. \tag{2.16}$$

Purchases of motor vehicles are removed from investment such that aggregate investment is $p_I I_t = \sum_{i=1}^{J-1} p_{I_i} I_{i,t}$. Real GDP consists of consumption C_t, aggregate investment, $p_I I_t$, and government purchases, $p_G G_t$:

$$GDP_t = C_t + p_I I_t + p_G G_t$$

Finally, I define aggregate labor income and capital income as

$$Y^L_t = \sum_{i=1}^{J} w_{i,t} L_{i,t} \qquad \text{and} \qquad Y^K_t = \sum_{i=1}^{J} \left(u_{i,t} r_{i,t} K_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t} \right) u_{i,t} X_{i,t} \right). \tag{2.17}$$

3 Solution Method and Calibration

3.1 Solution method

I solve the model's response by log-linearizing the equilibirum conditions around the initial steady state.

Household heterogeneity The underlying heterogeneity across households has the potential to affect aggregate and distributional outcomes through two channels: First, asymmetric movements in income across households affect households' demand for consumption goods due to non-homothetic preferences. For instance, if labor-intensive goods are primarily consumed by those households that suffer most from the carbon tax, then this would amplify the aggregate output response.[15]

Second, as shown in equation (2.11), the stochastic discount factor reflects the marginal utility of those households that receive dividends rather than the stochastic discount factor of a representative household. Hence, fluctuations in capital income play a more prominent role because they have a larger impact on consumption of those households that hold the capital stock. If house-

[15] Interactions of non-homotheticity and sectoral heterogeneity can either amplify or dampen shocks, through mechanisms similar to those pointed out by the literature on government spending multipliers. Non-homotheticity amplifies the recessionary impact of the carbon tax if spending shifts away from labor-intensive industries (Hall, 2009), industries that employ high MPC households (Flynn et al., 2022) or industries with particularly sticky prices (Cox et al., 2020).

holds were fully divided into workers and capital owners, the model would generate the same Euler equation as in the TANK literature, where the stochastic discount factor is only influenced by movements in capital income (see e.g. Cantore and Freund, 2020).[16]

3.2 Calibration

I calibrate the model at a quarterly frequency. Key input to calibrate the production structure are the BEA input-output tables from 2012 that distinguish between 404 sectors (U.S. Bureau of Economic Analysis, 2021). I add a 405$^{\text{th}}$ sector that combines motor vehicles and gasoline to provide motor vehicle services to households.

In the non-stochastic steady, household heterogeneity is fully captured by households' job, ι, that determines their income and consumption.[17] When calibrating and solving the model, I discretize the continuum of jobs into income percentiles. I rely on micro data from the BLS' Consumer Expenditure Survey (CEX, U.S. Department of Labor, 2021), the distributional national accounts (DINA, Piketty et al., 2018) and the current population survey (CPS, Flood et al., 2021) to estimate consumption patterns, income levels and income sources, as well as sectors of employment across the income distribution. In each survey, I restrict the sample to the active working-age population and assign respondents to income percentiles.[18]

Carbon intensity by product To calculate the carbon tax for each of the 405 sectors I rely on information from the environmental accounts published by the U.S. Envrionment Protection Agency (Yang et al., 2020; Ingwersen et al., 2022). These accounts include information on each industry's direct carbon emissions.[19]

A sector's tax burden depends on its carbon intensity measured as kilogram of carbon emis-

[16]The TANK literature emphasizes differences in marginal propensities to consume across households to generate this result. While technically households in my model all have an MPC of one, what matters for the aggregate dynamics is the "MPC" at the level of the capital fund who has access to investment and bonds to smooth out fluctuations in capital income. Since in my model, households receive both labor income and capital income but in different proportions, the model generates an Euler equation that lies somewhere in between the extreme cases of a representative agent model and a TANK model.

[17]In the non-stochastic steady state, job ι pays the same wage across all sectors and hence, the sector of employment is irrelevant for household heterogeneity. But the sector of employment does matter for the distributional consequences of the carbon tax because the response of labor income is tied to the sector of employment.

[18]As in Heathcote et al. (2017), I restrict the sample to the active working-age population (households that earn at least $15'000 for a two-adult household, which corresponds to one person working full-time at minimum wage, and $11'250 for a single-adult household, i.e. 30 hours per week at minimum wage). More details on how I use the micro data for the model calibration are provided in Appendix Section C.8.

[19]The environmental accounts list various greenhouse gases (carbon dioxide, methane, carbon monoxide,...). Since most public debate is concerned with emissions from CO_2, I restrict my focus on CO_2.

sions per \$1 value of production, $\frac{carb_{i,t}}{Y_{i,t}}$. The environmental accounts let me distinguish carbon emitted during the production process through chemical or physical transformation of goods from carbon emitted through combustion of fuels. Dividing the former by a sector's output gives me an estimate of Ψ_{Y_i}, whereas the latter corresponds to the carbon intensity of the energy bundle, Ψ_{E_i}, times the expenditure on energy, E_i. Together with data from the input-output tables on the revenue share spent on energy, this informaton allows me to back out Ψ_{E_i} for each sector.

I classify sectors producing coal, natural gas, electricity and petroleum as energy sectors.[20] Spending on petroleum by households is assumed to power motor vehicles, whereas spending on natural gas, coal and electricity is assumed to be used for housing and is assigned to the sectors 'Owner-occupied housing' and 'Tenant-occupied housing', together with the associated carbon emissions.[21] According to CEX data, renters spend about three times more on energy for a given amount of housing services than house owners. With about one quarter of housing services being provided by 'Tenant-occupied housing' according to the input-output tables, I assign about half of electricity and natural gas consumed by households to each housing sector.

Table 1 displays the total carbon intensity and the breakdown by source for the top 15 carbon-intensive sectors (ranked by total carbon intensity) as well as sectors that account for at least 1% of U.S. carbon emissions. The most carbon-intensive sectors are in electricity production, manufacturing of certain goods such as cement, lime, copper, fertilizers and chemical products, and, to a lesser extent transport (truck, pipeline, air and motor vehicle services). While most emissions are due to fuel combustion (they account for more than 95% of all emissions), for a few sectors, such as cement manufacturing or lime production, a substantial amount of emissions occur during the production process. The last column multiplies the total carbon intensity by a sector's output and expresses the resulting number as a percent of total carbon emissions of U.S. production. Just two sectors account for more than half of all emissions in the United States: electricity production by either state-owned or private utility companies (38%) and private use of motor vehicles (17%).

Production The production parameters α_i, χ_i and ϕ_i are set to match the labor income shares and the expenditure shares on energy and intermediate goods in each sector in the 2012 I-O ta-

[20]This concerns coal mining (BEA industry code 212100), oil and gas extraction (211000), natural gas distribution (221200), electric power generation, transmission, and distribution (221100), federal electric utilities (S00101), state and local government electric utilities (S00202), and petroleum refineries (324110).

[21]The environmental accounts do not assign any carbon emissions to 'Owner-occupied housing' and 'Tenant-occupied housing', but instead count the consumption of energy towards private consumption expenditure. In line with the model, I assign spending on residential energy to the two housing sectors and therefore also count the associated carbon emissions to those sectors.

bles.[22] Preference weights for investment, government consumption, private consumption and the various intermediate good bundles and energy bundles, ω_s^i, are calibrated to match the share of good i in total expenditure for demand component s as observed in the input-output tables. The elasticity of substitution between intermediates and the capital-labor-aggregate is set to $\xi = 0.1$ in line with the low substitutability at the industry level observed by Boehm et al. (2019). The elasticity of substitution across sector goods is set to $\sigma = 2$ as estimated by Hobijn and Nechio (2019).

Labor The wage rigidity parameter is set to $\theta_w = 0.85$ to match the 6-quarter average duration of wage contracts found in administrative data (Grigsby et al., 2021). I set the share of workers leaving / entering the workforce to $\psi = 0.025$ to match a work life of 40 years. The inverse of the standard deviation of the idiosyncratic preference shocks across sectors, γ, determines how much workers' sector choice responds to labor income differentials. I set $\gamma = 0.2$ in line with estimates by Artuç et al. (2010) and Caliendo et al. (2019) who find a fairly low responsiveness.

In the non-stochastic steady, $\omega_i^l(\iota)$ equals the share of jobs ι in all jobs of sector i. Equating 'jobs' with 'income percentiles' I estimate these shares using data from the CPS social and economic supplement. I sort workers into income bins based on their labor income, calculate their distribution across industries for each income bin and average across 2003 - 2019. I concord CPS industries to BEA IO industries using crosswalks provided by the BEA and CPS.

Capital I assume three different depreciation rates across capital stocks. The annualized depreciaton rates are set to 3% for residential capital[23] and 7% for non-residential capital to match the share of residential and non-residential investment in GDP over 2000 - 2019 (4.1% and 17.1%). Annualized depreciation rates for motor vehicles are set to 16% in line with those used by the BEA (Fraumeni, 1997). The value for the investment adjustment cost, $\zeta = 2.5$, lies within the range of industry-level estimates of Tobin Q-elasticities reported by House and Shapiro (2008).

The utilization adjustment cost parameter $\delta''(u)$ determines the short-run elasticity of energy to a change in energy prices. If these costs go to infinity, energy consumption is pre-determined and does not respond to changes in the energy prices within a quarter. Empirical estimates of the

[22] The sector 'motor vehicle services' employs no labor and requires no intermediates. Sales of the following goods and services to private households (as observed in the BEA tables) make up investment in its capital stock: Transportation Equipment Manufacturing (336), Motor Vehicle and Motor Vehicle Parts and Supplies Merchant Wholesalers (4231) and Motor Vehicle and Parts Dealers (441).

[23] Sectors relying on residential capital are 'Owner-occupied housing', 'Tenant-occupied housing' and 'Other real estate'.

price elasticity of electricity demand find very low short-run elasticities around 0.15-0.2 within a year (see e.g Kilian, 2008; Labandeira et al., 2017). Based on this evidence, I set $\delta''(u) = \frac{1}{30}$, which produces a within-year elasticity of 0.175.[24]

Consumption preferences Steady-state preference weights, $\omega_c^j(c(\iota))$, across sector goods for each income percentile are estimated from CEX data on expenditure shares. The BLS collects data on households' spending patterns for around 650 categories through quarterly interviews for infrequent purchases and weekly diaries for more frequent purchases. Within each survey, I assign households to an income percentile based on their total household income adjusted for the number of household members and then calculate expenditure shares averaged across 2004 - 2019. Using concordance tables provided by the BLS and the BEA, I concord the spending categories from the CEX to the 405 sectors present in the input-output tables. I adjust the resulting matrix of expenditure shares, $\omega_c^j(\iota)$, to ensure that, once summed up across all income percentiles, the expenditure shares correspond to those reported by the BEA I-O tables.

Up to a first-order accurate solution, I also need values for the elasticity of the preference weights to consumption, $\frac{\partial \ln \omega_c^j(c(\iota))}{\partial \ln c(\iota)}$, for each good j. I set them to the estimates of β_j from the following set of regressions that I run on the CEX data:[25]

$$\ln \omega_c^j(\iota) = \alpha_j + \beta_j \ln c(\iota) + \varepsilon_j(\iota). \tag{3.1}$$

Table A1 in the Appendix shows substantial variation with richer households spending larger fractions on philantropy, travelling and entertainment, whereas poorer households spending more on rent and food.

Income In the model, the distribution of labor productivity, $a_l(\iota)$, and capital fund shares, $a_k(\iota)$, pins down the income distribution and the income shares of labor and capital income for each income percentile. I estimate these objects from the DINA. For the DINA, Piketty et al. (2018) combine tax, survey and national accounts data to estimate the distribution of national income in the United States. I access their micro-files that contain information on income and its components for a synthetic set of individuals. By construction, adding up the income across individuals in these micro-files adds up to national income, while the distributions are consistent with those seen in

[24] See Appendix Section C.5 for details.

[25] In theory, these elasticities should "average out" to zero: $\sum_j \beta_j y_c^j = 0$. While for, simplicity, I run the regressions separately without imposing this restriction, I shift the estimates to ensure that this condition holds.

tax and survey data. Total pre-tax household income is composed of labor income, which maps to $a_l(\iota)wl$ in the model, and net capital income, which is capital income net of depreciation and maps to $a_k(\iota)div$. I assign households to an income percentile based on their total adjusted household income and then back out, for each income percentile, values for $a_l(\iota)$ and $a_k(\iota)$, averaged over the years 2002 - 2019.

Fiscal and monetary policy The share of government consumption in GDP is set to its average level between 2000 - 2019 (15%). The Taylor rule coefficients are set to standard values of $\varphi = 0.75$ and $\varphi_\pi = 1.5$.

Size of carbon tax I assume that the government unexpectedly implements a permanent carbon tax of $100 per ton (in 2012 prices) of carbon emissions. This value lies at the upper range of discussed policy options (Barron et al., 2018; Congressional Budget Office, 2018).[26] The direct, partial-equilibrium cost burden as a share of output of such a tax corresponds to a sector's total carbon intensity reported in Table 1 times 0.1. For the economy as a whole, the tax represents about 3.23% of GDP, to which the tax on gasoline for motor vehicles contributes about 0.54 percentage points.[27] But the tax is highly skewed towards a small set of sectors: For the median sector, the tax represents only 0.20% of total gross output. The top 10 sectors pay on average 30% of their gross output on carbon taxes, and the top sector (cement manufacturing) even more than 80%.

4 Aggregate effects

Figure 1 displays the transition dynamics triggered by the carbon tax for the first 10 years, expressed in log deviations from the initial steady state, as well as the long-run deviations. Panel (a) shows the response of aggregate economic variables, while panel (b) focuses on the response of variables related to energy and carbon emissions.

[26] Some proposals of carbon pricing also suggest a gradual increase of the carbon tax. I consider a constant price of carbon for two reasons: (i) Golosov et al. (2014) show that the optimal price of carbon is a constant fraction of GDP and (ii) such a policy experiment makes it easier to study the adjustment process and transition path generated by the model. The Paris agreement targets a reduction of 50% of carbon emissions by 2030 compared to 2005. This implies a 44% reduction compared to 2012, the baseyear of my calibration. As will be seen, my model suggests that a carbon tax of $100 will reach this target only in the very long run.

[27] From the last row in Table 1, it is observed that the average sector (weighted by output) emits 0.178 kg of carbon per $ of output. Given a tax of $100 per ton, or 0.1$/kg, the average tax burden for firms is $0.178\text{kg/\$} \times 0.1\text{kg/\$} = 1.78\%$ of output, or: 3.23% of value added.

Table 1: CARBON EMISSIONS

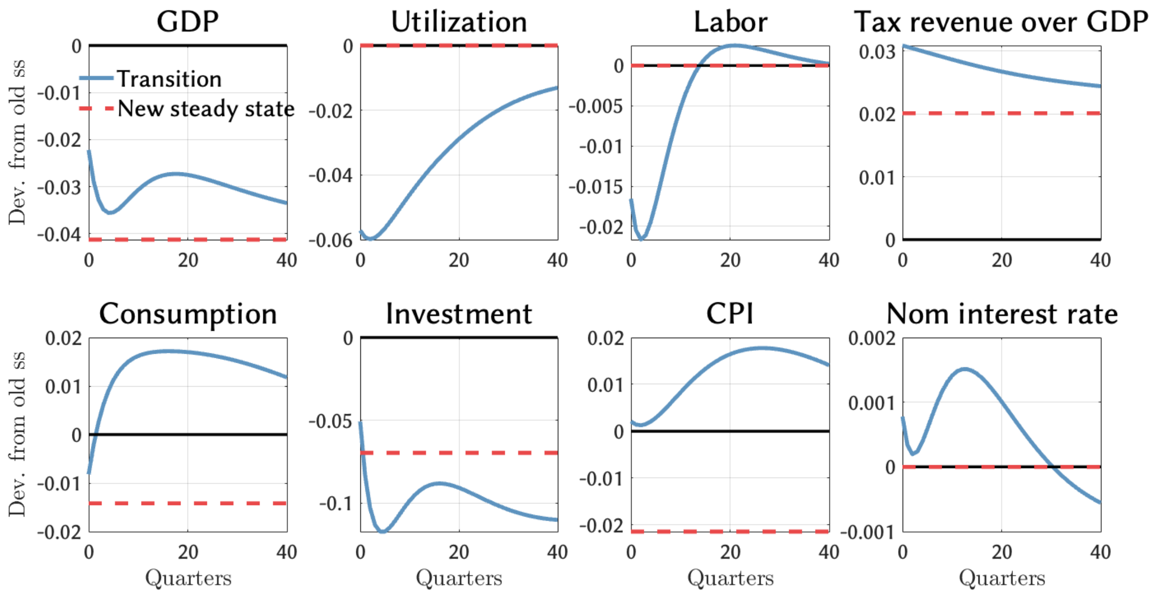
		Carbon intensity ($^{kg}/_{\$}$)			Emissions (%)
		Ψ_{Y_i}	$\Psi_{E_i}\frac{E_i}{Y_i}$	Total	
1	Cement manufacturing	6.17	2.21	8.38	0.9
2	State and local government electric utilities	0.00	4.72	4.72	5.3
3	Federal electric utilities	0.00	4.71	4.72	1.2
4	Electric power generation, transmission, and distribution	0.00	4.62	4.62	31.5
5	Lime and gypsum product manufacturing	2.36	1.46	3.82	0.4
6	Copper, nickel, lead, and zinc mining	0.00	1.30	1.30	0.3
7	Motor vehicle services	0.00	1.20	1.20	16.8
8	Truck transportation	0.00	1.17	1.18	6.4
9	Pipeline transportation	0.00	1.04	1.04	0.8
10	Iron, gold, silver, and other metal ore mining	0.64	0.37	1.01	0.2
11	Fertilizer manufacturing	0.61	0.40	1.01	0.5
12	Wet corn milling	0.00	0.98	0.99	0.2
13	Synthetic dye and pigment manufacturing	0.01	0.81	0.81	0.1
14	Asphalt paving mixture and block manufacturing	0.00	0.77	0.77	0.2
15	State and local government passenger transit	0.02	0.72	0.74	0.2
17	Iron and steel mills and ferroalloy manufacturing	0.52	0.18	0.70	1.5
18	Air transportation	0.00	0.69	0.69	2.5
29	Other basic organic chemical manufacturing	0.05	0.37	0.42	1.1
34	Tenant-occupied housing	0.00	0.33	0.33	2.6
43	Oil and gas extraction	0.12	0.16	0.28	1.7
52	Petroleum refineries	0.12	0.10	0.22	3.2
58	State and local government other services	0.00	0.17	0.17	2.4
72	Owner-occupied housing	0.00	0.11	0.11	2.7
88	Other real estate	0.00	0.08	0.08	1.4
	90% percentile	0.01	0.24	0.30	-
	75% percentile	0.00	0.07	0.07	-
	Median	0.00	0.02	0.02	-
	Average weighted by industry output	0.01	0.17	0.18	-

Notes: Table displays statistics for carbon emissions based on the 2012 input-output tables that distinguish between 404 sectors.The first three columns display the carbon intensity of production, measured as kg of carbon emissions per dollar of production at producer prices. The table distinguishes between direct emissions due to the production process (Ψ_{Y_i}) and emissions due to the combustion of fuels ($\Psi_{E_i}\frac{E_i}{Y_i}$). Total carbon intensity refers to the sum of the first two columns. The last column multiplies the total carbon intensity by a sector's total output and expresses the resulting number as a percent of total U.S. carbon emissions. For motor vehicle services (sector 405), output refers to the implied rental value calculated as private consumption of gasoline plus the rental rate times the stock of motor vehicles. The implied rental value is 2.5% of total U.S. output. See Appendix Section C.7 for details.

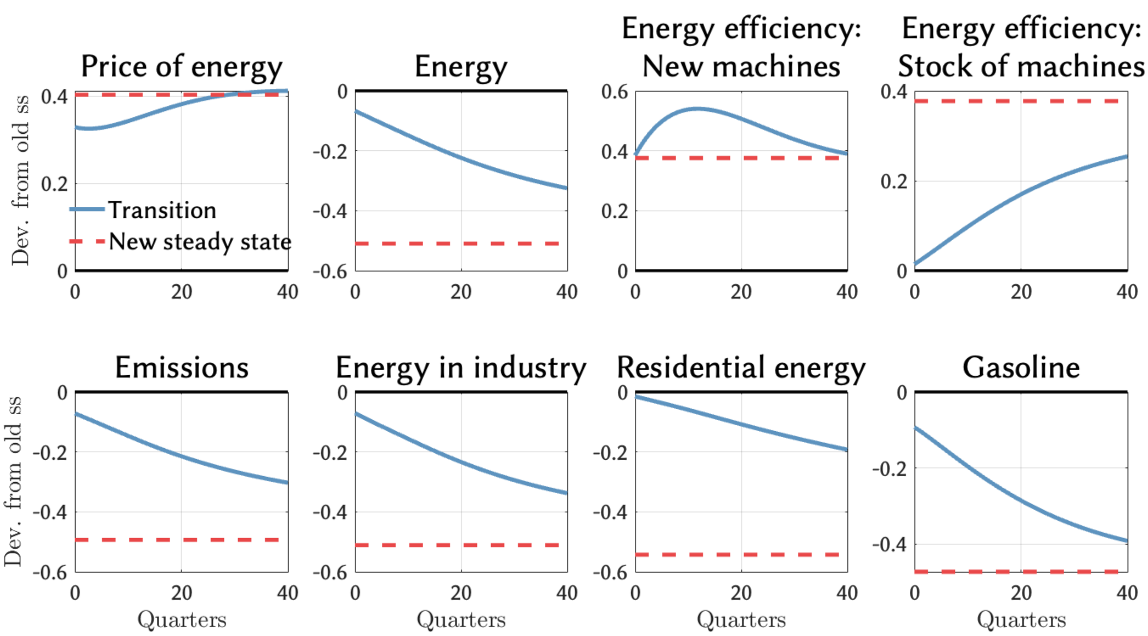
Table 2: Calibration

Description	Parameter	Value	Source / Target
Production			
Curvature of capital in production function	α_i	sec. sp.	I-O tables, 2012, BEA (2021), Karabarbounis and Neiman (2013)
Weight on intermediate goods	ϕ_i	sec. sp.	I-O tables, 2012, BEA (2021)
Weight on energy goods	χ_i	sec. sp.	I-O tables, 2012, BEA (2021)
Input weights for final goods	ω_s^i	sec. sp.	I-O tables, 2012, BEA (2021)
Elast. of subst. value added and intermediates	ξ	0.1	Boehm et al. (2019)
Elast. of subst. across goods	σ	2	Hobijn and Nechio (2019)
Consumption preferences			
Discount factor	β	0.99	Standard value
Consumption basket weights	$\omega_c^j(\iota)$	sec. & inc. sp.	Estimated from CEX (U.S. Department of Labor, 2021) (see text)
Consumption elasticity	$\frac{\partial \ln \omega_c^j(\iota)}{\partial \ln c(\iota)}$	sec. sp.	Estimated from CEX (U.S. Department of Labor, 2021) (see text)
Income			
Share of capital fund per income percentile	$a_k(\iota)$	perc. sp.	Derived from DINA (Piketty et al., 2018)
Labor productivity per income percentile	$a_l(\iota)$	perc. sp.	Derived from DINA (Piketty et al., 2018)
Labor			
Wage stickiness	θ_w	0.85	Grigsby et al. (2021)
Share of workers leaving workforce	ψ_L	0.025	Working life of 40 years
Propensity to change sectors	γ	0.2	Artuç et al. (2010), Caliendo et al. (2019)
Average wage per sector	w_i	sec. sp.	Estimated from CPS data (Flood et al., 2021)
Distribution of income percentiles per sector	$\omega_i^l(\iota)$	sec. & inc. sp.	Estimated from CPS data (Flood et al., 2021)
Capital			
Depreciation rate non-residential capital (p.a.)	δ	0.07	Share non-residential investment in GDP (17%), 2000 - 2019
Depreciation rate housing (p.a.)	δ_h	0.03	Share residential investment in GDP (3%), 2000 - 2019
Depreciation rate motor vehicles (p.a.)	δ_d	0.16	Rates for motor vehicles (Fraumeni, 1997)
Investment adjustment cost	f''	2.50	House and Shapiro (2008)
Utilization adjustment cost	δ''	$\frac{1}{30}$	Short-run energy demand elasticity of 0.15-0.20 (Labandeira et al., 2017)
Fiscal and monetary policy			
Share government consumption	G	0.15	Share government consumption in GDP (15%), 2000 - 2019
Taylor rule persistence coefficient	φ	0.75	Clarida et al. (1997)
Taylor rule inflation coefficient	φ_π	1.5	Clarida et al. (1997)

Notes: Values marked with "sec. sp." are sector- or sector-pair specific. Values marked with "perc. sp.' are specific to each income percentile. Values marked with "sec. & perc. sp." are both sector- and income-percentile-specific.



(a) Economic Variables



(b) Energy-related Variables

Figure 1: Aggregate Response to a Permanent Carbon Tax

Note: The figure plots the simulated time path to a $100 per kg carbon tax as well as the long-run level (approximated by the response after 250 years), both expressed in log deviations from the initial steady state. One period is a quarter.

Exploring short-term impacts The carbon tax raises the cost of energy by more than 30 log points. This slows down economic activity because firms decide to save energy by reducing the time they run machines. The consequence is a 3 log-point decline in GDP during the first year—almost twice the amount of the fall in employment.[28] Despite energy representing a relatively small portion of total sales within the economy, the increase in the price of energy has a substantial impact on economic activity.

To better understand these aggregate dynamics, I consider a version of the model that abstracts from housing and motor vehicles, i.e. all energy is consumed by firms. A first-order approximation to GDP yields[29]

$$\begin{aligned}\widetilde{GDP}_t &= (1-\phi^L)\tilde{E}_t + \phi^L \tilde{L}_t + \left(1-\phi^L+\phi^E\right)\tilde{Z}_t \\ &\quad + \sum_i \frac{E_i}{GDP}\left[\frac{1-\phi_i^L+\phi_i^E}{\phi_i^E} - \frac{1-\phi^L+\phi^E}{\phi^E}\right]\tilde{u}_{i,t},\end{aligned} \tag{4.1}$$

where $\phi^L = \frac{wL}{GDP}$ and $\phi^E = \frac{p^E E}{GDP}$ are the steady-state shares of labor and energy in GDP, and $Z_t = \frac{K_t}{X_t}$ is the energy efficiency of installed machines.

The first row describes GDP dynamics in a one-sector version of the model: Given that energy efficiency of installed machines is pre-determined, short-run dynamics in GDP are driven by fluctuations in energy consumption (E) and employment (L). Energy consumption affects value added because it is tightly linked to utilization and hence, capital services, especially in the short run. Quantitatively, the elasticity of GDP to energy consumption is large and corresponds to the share of capital services in GDP: Holding employment fixed, a 1 log point decrease in energy consumption reduces GDP by about one third of a log point.

This calculation, however, overlooks the presence of multiple sectors. In a multi-sector framework, GDP is less sensitive to the carbon tax because demand switches to low-energy sectors. This is shown by the second row of (4.1). The term $\frac{\phi_i^E}{1-\phi_i^L+\phi_i^E} = 1-\chi_i$ represents the energy intensity of the capital stock in sector i. For sectors heavily reliant on energy, the bracketed term is negative. Since the carbon tax significantly impacts these energy-intensive sectors, they curtail their energy consumption by a larger percentage compared to sectors on average. This results in a positive value for the term in the second row, cushioning the GDP decline.

In essence, energy-intensive sectors carry a disproportionately higher weight in total energy

[28]Note that labor falls in the short run because nominal wages cannot fall sufficiently to cushion the fall in firms' demand for labor. If wages were flexible, labor would stay constant.

[29]See Appendix Section A.5 for derivations of all equations in this section.

consumption than in GDP. As a consequence, substantial cuts in energy usage within these sectors lead to a more pronounced drop in aggregate energy consumption, while the impact on GDP remains somewhat mitigated. Quantitatively, the last term in (4.1) is significant: While the one-sector model forecasts a GDP elasticity to energy consumption of approximately one third, this figure reduces to about 0.2 in the multi-sector model.[30]

Transition to an energy-efficient economy Over time, the link between energy consumption and GDP loosens as firms invest into high-efficient machines. This enables the transition to an economy that is less reliant on energy. As can be observed in Figure 1, despite the steady fall in energy consumption (E) by 50 log points, the drop in GDP in the long run is more modest (around 4 log points) because the energy efficiency of machines (Z) goes up (see also equation (4.1)).

What drives this transition to a more efficient capital stock? The energy efficiency of the existing stock at time t reflects past decisions about the energy efficiency of the machines acquired up to time t, $z_0, z_1, z_2, ...$, factoring in their depreciation:[31]

$$\tilde{Z}_t = \delta\chi \sum_{s=0}^{t-1} (1-\delta)^{t-1-s} \tilde{z}_s. \tag{4.2}$$

The decision of whether to invest in energy-efficient machines or not is forward-looking, and reflects firms' expectations about the path of the energy tax over the lifespan of these machines:

$$\tilde{z}_t = \frac{1-\beta(1-\delta)}{\chi} \sum_{s=t+1}^{\infty} \left[\beta(1-\delta)\right]^{s-t} \left(\Delta\tau_{E,s} - \tilde{r}_s\right). \tag{4.3}$$

Since the increase in the energy tax is permanent, the incentive to invest into energy-efficient machines is particularly strong, which then leads to a more energy-efficient capital stock. How fast these new machines replace the old machines depends on the depreciation rate (as seen in (4.2)) and is, in the full model, therefore faster for motor vehicles than for housing. Consequently, the use of gasoline falls much quicker than the use of residential energy (see Figure 1).

[30] Upon impact labor falls by about 1.7 log points, reducing GDP by about 1 log point all else being equal. The remaining 1.2 log points of the 2.2-log-point drop in GDP therefore stem from the reduction in utilization / energy consumption by about 6 log points. This implies an elasticity of GDP to energy consumption of 0.2 (=1.2/6), all else being equal.

[31] For simplicity, I focus on a one-sector economy in deriving equations (4.2) and (4.3). Appendix Section A.5.2 discusses the case with multiple sectors. In essence, with multiple sectors, changes in aggregate energy efficiency do not only depend on the aggregate energy efficiency of machines installed in previous years—as suggested by (4.2), but also on sector-specific utilization rates that—through their effect on depreciation rates—alter the composition of the aggregate stock of machines.

Carbon emissions follow the transition path for energy and slowly converge to their new steady state that is about 50 log points lower, half of which is reached after about 5 years.

Response of remaining variables The fall in GDP goes along with a large drop in investment of more than 10 log points, but a slight increase in consumption. Investment drops because the carbon tax strongly reduces the return to capital. Over time the capital stock decreases by more than 10 log points. From an intertemporal perspective consumption is therefore relatively cheap in the short run when the capital stock is still high, which explains the initial increase in consumption.[32] The effect on aggregate consumer prices is modest and reflects two counteracting forces: On the one hand, the carbon tax raises consumer prices; on the other hand, the government uses the tax receipts to lower the consumption tax rate (by about 5 percentage points). The central bank slightly raises the interest rate by about 10 basis points to fight the increase in consumer prices. This contributes to the recessionary effect of the carbon tax as well.

5 Distributional consequences

The aggregate picture hides variation across economic sectors. This variation matters for the consumption distribution across households. Workers mostly care about economic conditions in their sector of employment and consumers care about the prices of those goods that they consume. To evaluate the distributional consequences for households, this section calculates the effects on consumption across the income distribution.[33]

5.1 Measuring changes in consumption across households

Consider a carbon tax implemented at time 1. To understand its impact on households' real income, consider consumption of household ι employed in sector i at time t as

$$c_{i,t}(\iota) = \frac{1}{(1+\tau_t^C)p_{c_i,t}(\iota)} \left(a_l(\iota)Y_{i,t}^L + a_{k,t}(\iota)div_t\right).$$

[32]The response of consumption crucially depends on the intertemporal elasticity of substitution, which is set to 1. For lower values, households are less willing to substitute consumption across time and consumption would not go up that much in the beginning.

[33]Alternatively, I could look at the effects on real income. From a welfare point of view, the effects on consumption are more relevant and have been the focus in the literature (Goulder et al., 2019; Rausch et al., 2011).

Consumption is composed of labor income, $a_l(\iota)Y_{i,t}^L \equiv a_l(\iota)w_{i,t}l_{i,t}$, which depends on the sector of employment i, and dividends, $a_{k,t}(\iota)div_t$, which reflects the return on renting out capital net of energy costs and investment.[34] Labor income and dividends are deflated by the household-type specific price index $p_{c_i,t}(\iota)$ and the common consumption tax τ_t^C to obtain consumption.

Conditional on surviving till period t, a household in job ι and sector i sees their consumption grow between period 0 (the non-stochastic steady state) and t by

$$\hat{c}_{i,t}(\iota) = \frac{1+\tau^C}{1+\tau_t^C} \underbrace{\left(\sum_j \omega_c^j(\iota)\left(\hat{p}_{j,t}\right)^{1-\sigma}\right)^{-\frac{1}{1-\sigma}}}_{\text{expenditure channel}} \underbrace{\left\{\frac{a_l(\iota)Y^L}{a_l(\iota)Y^L + a_k(\iota)div}\hat{Y}_{i,t}^L + \frac{a_k(\iota)div}{a_l(\iota)Y^L + a_k(\iota)div}\hat{a}_t^k\widehat{div}_t\right\}}_{\text{income channel}}, \tag{5.1}$$

where $\hat{x}_t = \frac{x_t}{x}$ denotes the gross growth rate of variable x between 0 and t. The growth in consumption is decomposed into two components that reflect changes in consumer prices (expenditure channel) and changes in net income (income channel).

The ***expenditure channel*** captures the expected change in the price of households' consumption basket. Whereas all households face the same prices for the various consumption goods, income-specific preference weights, captured by the pre-shock expenditure shares $\omega_c^j(\iota)$, imply that some households are more exposed to price increases of certain products than others.[35]

The ***income channel*** refers to expected changes in labor income and net capital income (dividends), and can be broken down into two parts: First, households differ in their exposure to fluctuations in labor and capital income, as captured by differences in their pre-shock share of labor income in pre-tax income, $\frac{a_l(\iota)Y^L}{a_l(\iota)Y^L+a_k(\iota)div}$. This is the factor-income channel. Second, households also differ in their sector of employment, as indicated by the i subscript in $Y_{i,t}^L$, and therefore are differently exposed to fluctuations in labor income across sectors.

Equation (5.1) also includes a term for the consumption tax change of the carbon tax, $\frac{1+\tau^C}{1+\tau_t^C}$. While, by construction, changes in the consumption tax do not affect the distribution of consumption, given its size—the consumption tax drops by 5 percentage points when the carbon tax gets introduced—the distributional consequences ultimately depend on how this revenue is redis-

[34] More precisely, labor income for household ι employed in sector i at time t is equal to $w_{i,t}(\iota)l_{i,t}(\iota)$. Due to wage rigidity, this is equal to $a_l(\iota)w_{i,t}l_{i,t}$ only in expectations.

[35] Notice that purchases (rather than rentals) of motor vehicles and purchases of gasoline are assigned to consumption, as in (2.16). In the model, motor vehicles are rented out inclusive of gasoline and housing services are inclusive of natural gas and electricity, and the share of the energy component is fixed across consumers. For the purpose of this measurement exercise I split housing services and motor vehicle services into their components and add the prices of these components to households' price index, using the empirically observed household-specific weights. The prices of the energy component is augmented by the carbon tax that is assumed to be paid by the households, i.e. I implicitly assume an immediate, perfect pass-through for gasoline and natural gas.

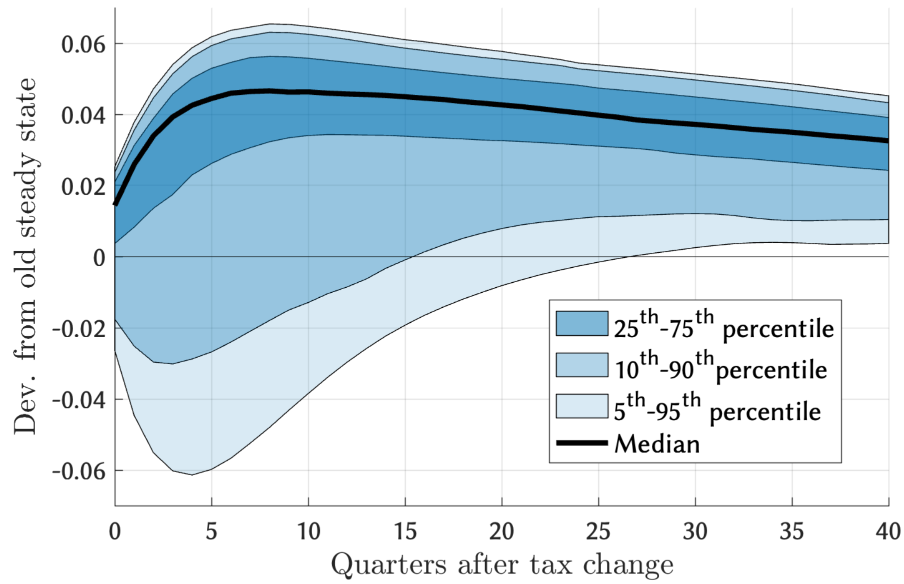


Figure 2: Changes in real consumption across households

Note: The figure plots the cross-sectional distribution of simulated time paths for real consumption per household, $\mathbb{E}_0\left(\hat{c}_t^{i,0}(\iota)\right)$, to a \$100 per kg carbon tax over the first 40 quarters after the introduction of the carbon tax. Percentiles of consumption changes are calculated separately for each period.

tributed in practice.

5.2 Results

Disperion across households Figure 2 displays the distribution of consumption changes (5.1) across households and time. Consumption of the median household jumps up by 2 log points and rises to almost 5 log points within the first year, more than the average consumption response in Figure 1. The figure reveals a larger dispersion of consumption changes below the median than above the median, indicating that consumption changes are left-skewed. About 15% of households see their consumption increase by more than 6 log points, whereas a similar fraction experience a fall in their consumption, with 5% seeing their consumption fall by more than 6 log points. Over time, the cross-sectional dispersion across households becomes smaller and the interdecile range falls below 4 log points after ten years, with almost all households experiencing an increase in consumption in the medium run.

Results by income percentile While Figure 2 illustrates the substantial dispersion in consumption changes across households, it is silent about which households experience consumption gains and which consumption losses. To answer that question, I aggregate the consumption paths in

(5.1) across sectors and calculate the net present value. Figure 3 displays the obtained values across income percentiles for either a 1-year horizon (left) or a 25-year horizon (right) after the shock. Each dot corresponds to the net present value of consumption changes for a particular income percentile, with low-income households shown on the left.

In the short run, the carbon tax is progressive, raising real consumption for the bottom 25% by about 2 log point, but by around 1 log points for the top 10%. The top percentile is the only percentile that experiences a drop in consumption (1.5 log points).[36]

The bottom panel decomposes the change in consumption into the three channels: (i) the expenditure channel, (ii) the labor-income channel and (iii) the factor-income channel. For instance, to isolate the effect of the expenditure channel, I re-calculate households' net present value of consumption changes assuming that they all receive the same (average) labor income and have the same (average) factor income shares. The carbon tax makes the consumption basket of low-income households more expensive, but also improves their labor income prospects compared to those of middle or upper income classes. The strong drop for the top income groups is driven by the factor-income channel. Hence, the expenditure channel is regressive, whereas both sets of income channels are progressive in the short run.

The net present value of consumption over the first 25 years indicates that the incidence of the carbon tax across income percentiles ressembles an inverse u-shape, with households between the 10^{th} and 90^{th} percentile experiencing a consumption increase of almost 3 log points, while the very-low income households and the very high-income households see a somewhat smaller increase. This difference between short-run and long-run effects is driven by all three channels becoming more regressive over time.

5.3 Understanding the distributional effects of a carbon tax

In this section, I discuss each channel separately. Following equation (5.1), I divide the discussion for each channel into two parts: (i) what drives asymmetric movements in sectoral prices, sectoral labor income and aggregate factor income, and (ii) how do income groups differ in their exposure to these dynamics.

[36] The figure displays less variation than Figure 2. This is because sectoral heterogeneity explains an important part of the variation in real consumption changes in Figure 2.

Figure 3: Changes in real consumption by income percentile

Note: The top left (right) panel displays the change in discounted real consumption over the first year (25 years) for the average household in each income percentile, accounting for heterogeneity in both income sources and the consumption basket. The bottom row panels decompose the consumption change into the labor-income channel (assuming the same consumption basket and factor income shares across households), the expenditure channel (assuming the same labor income and factor income shares across households), and the factor income share channel, assuming the same labor income and consumption basket across households.

5.3.1 Expenditure channel

Figure 3 has shown that the expenditure channel is regressive, but somewhat muted in the short run. This suggests that the carbon tax is not immediately passed through to consumer prices, even though firms can freely reset their output prices every period.

In a multi-sector model, a sector's output price does not only respond to its own carbon tax, but also to the carbon tax paid by its suppliers of intermediates. More formally, let $\mathbf{A}$ denote the direct input requirement matrix with element $a_{j,i} \equiv \phi_i \omega_{M_i}^j$ denoting the direct requirement of intermediate good j to produce \$1 worth of good i. Further, let $\lambda_{j,i}$ denote the corresponding direct and indirect requirement, which corresponds to entry (i,j) of the Leontief inverse, $(\mathbf{I}-\mathbf{A})^{-1}$. Then, the price in sector i evolves according to

$$\tilde{p}_{i,t} = \sum_j \lambda_{j,i} \left\{ \Delta\tau_{Y_j,t} + (1-\phi_j)\left(\alpha_j \tilde{r}_{j,t} + (1-\alpha_j)\tilde{w}_{j,t}\right) \right\}. \tag{5.2}$$

That is, the price in sector i reflects production costs across all sectors weighted by their share in sector i's inputs, $\lambda_{j,i}$.[37] The tax on production, $\Delta\tau_{Y_j,t}$ directly shows up in this equation. In contrast, the tax on energy, $\Delta\tau_{E_j,t}$, influences output prices through its effect on the rental price of capital, $\tilde{r}_{j,t}$, but its pass-through strongly depends on how strongly firms shut down their machines in response to the tax, or, more precisely, the supply elasticity of capital services. To see this, it is helpful to look at the market for capital services in more detail. Demand and supply of capital services are described by the following two equations:[38]

$$\tilde{r}_{j,t} = \tilde{p}_{j,t} - \Delta\tau_{Y_j,t} + \tilde{Y}_{j,t} - \left(\tilde{u}_{j,t} + \tilde{K}_{j,t}\right) \tag{5.3}$$

$$\tilde{r}_{j,t} = (1-\chi_j)\left(\tilde{p}_{E_j} + \Delta\tau_{E_j}\right) + \frac{\delta''}{\delta'}\tilde{u}_{j,t}. \tag{5.4}$$

The first equation describes the demand for capital services and states that the rental price of capital must equal the marginal product of capital. The second equation describes the trade-off in choosing the level of utilization: The cost of running machines for longer involves the extra cost of energy needed for running the machine, i.e. a machine's energy share $1-\chi_j = \frac{E_j}{K_j}$ times the change in the energy price inclusive of the tax, $\tilde{p}_{E_j,t} + \Delta\tau_{E_j,t}$, as well as the utilization cost in form of higher depreciation, $\frac{\delta''}{\delta'}\tilde{u}_{j,t}$, whereas the benefit is the additional rental income, $\tilde{r}_{j,t}$. Panel (a) of Figure 4 illustrates the market for capital services. An increase in energy costs shifts the

[37] The fact that supply shocks propogate downstream is also emphasized by Acemoglu et al. (2016) among others.

[38] I assume no investment adjustment costs.

supply curve of capital services upwards, which raises the rental price (see panel (b)). By how much depends on the slopes of the curves: In a model with no utilization ($\delta'' \to \infty$, panel (d)), the supply curve becomes vertical and does not shift, leaving the rental price unchanged. If higher utilization does not raise wear and tear ($\delta'' = 0$, panel (c)), the supply curve becomes horizontal and the upward shift raises the rental price by the increase in the price of energy times the share of energy in capital services, $1 - \chi_j$ (full pass-through). In intermediate cases, the drop in utilization initially dampens the rise in the rental price. Over time, capital is reallocated towards other sectors and the utilization rate goes back to its initial level. Consequently, the rental price more and more reflects the increase in the price of energy (see equation (5.4)).

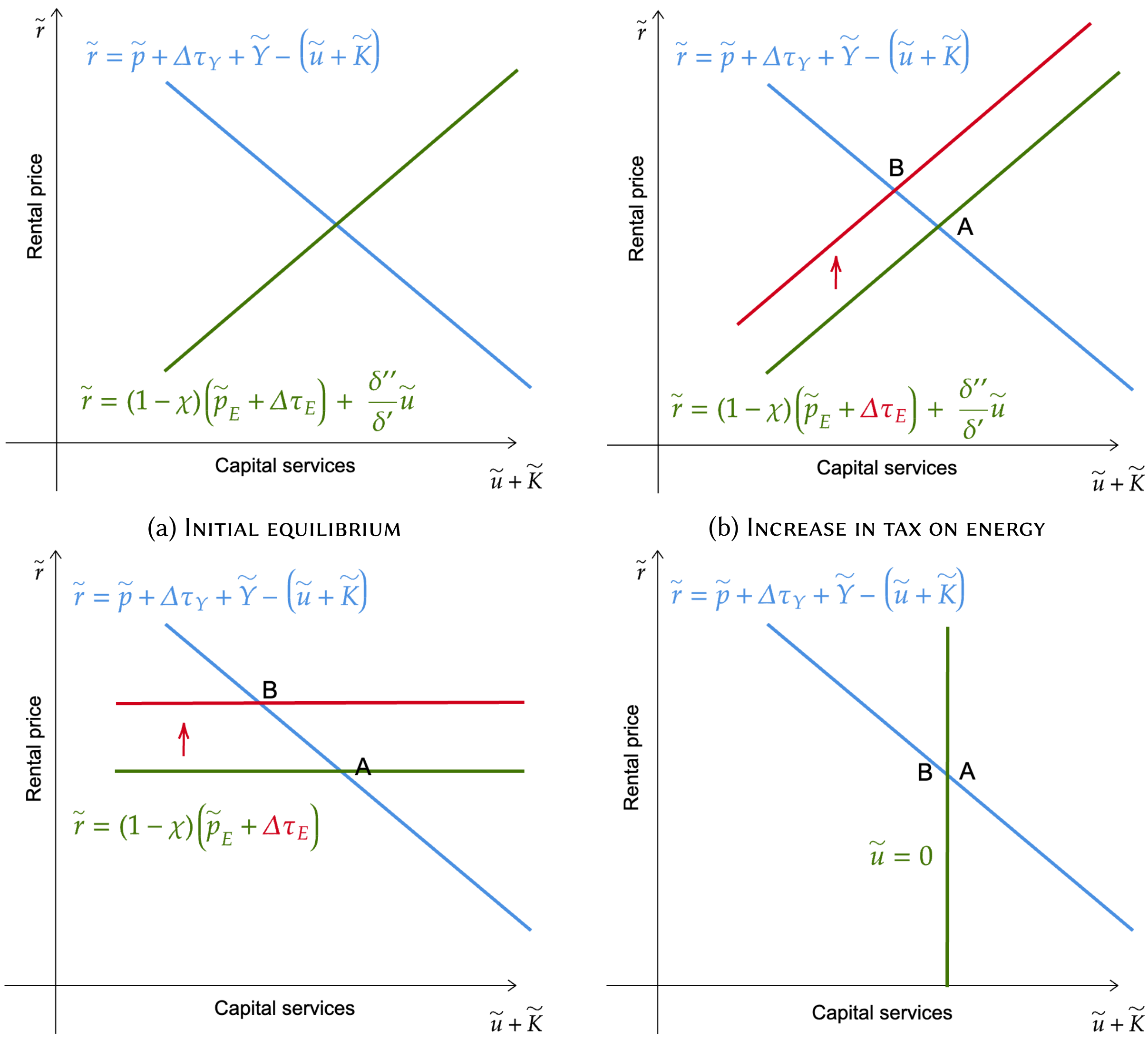
To illustrate this gradual pass-through I regress the response of output prices, $\tilde{p}_{i,t}$, on a sector's (direct and indirect) carbon tax, calculated as $\Delta\tau_i^{total} = \sum_j \lambda_{j,i}\left(\tau_{Y_j} + \frac{E_j}{Y_j}\tau_{E_j}\right)$, where $\frac{E_j}{Y_j} = (1 - \phi_j)\alpha_j(1 - \chi_j)$ is the share of energy in output for sector j:

$$\tilde{p}_{i,t} = \alpha_t + \beta_t \Delta\tau_i^{total} + \epsilon_{i,t} \tag{5.5}$$

I run these cross-sectional regressions separately for each time period t after the implementation of the carbon tax, so that β_t can be interpreted as the pass-through of the carbon tax into output prices at t. Panel (a) of Figure 5 shows that initially, the pass-through into output prices is less than 60% and slowly rises to 80% after 2.5 years. This is consistent with evidence from Ganapati et al. (2020) who find that 70% of energy-driven changes in input costs get passed through to consumers over the first couple of years.[39] This muted response is a direct consequence of the putty-clay feature that makes capital services and energy strong complements in the short run. Since the supply of capital services is rather inelastic in the short run, the tax on energy is only partially passed through to output prices. Over time, as the supply of capital services becomes more elastic through the reallocation of capital across sectors, the pass-through rises. After 5 years it reaches 100%.

Panel (b) displays the exposure of households to the carbon tax through its effect on consumption prices. The exposure is calculated as a weighted average of sectors' tax burden $\Delta\tau_i^{total}$, with weights corresponding to the share of a sector's good in the consumption basket of a specific income percentile. Production of goods in the consumption basket of low-income households is

[39]Note that this pass-through at the product level is conceptually different from the pass-through into inflation, for which both my model and the empirical evidence in Känzig (2021) suggest a more immediate response. Inflation dynamics are influenced by monetary policy and aggregate demand, which are soaked up by the time fixed effects α_t in regression (5.5).



(a) Initial equilibrium

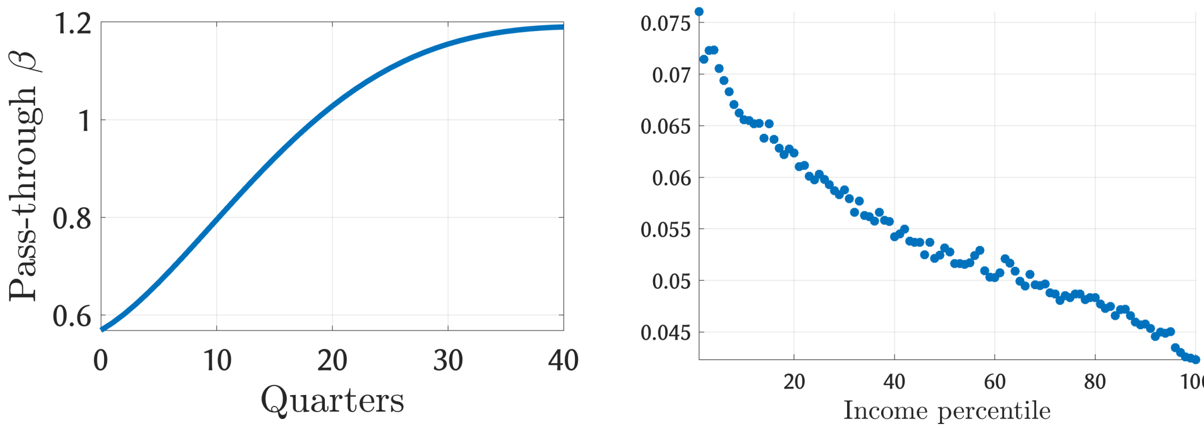
(b) Increase in tax on energy

(c) Perfectly elastic supply

(d) Inelastic supply

Figure 4: Market for capital services

Note: Figure displays the market for capital services based on the log-linearized equations for capital demand (5.3) and capital supply (5.4). For readability the subscripts i and t are suppressed. Panel (a) displays the initial equilibrium. Panel (b) shows the shift in the supply curve in response to an increase in the tax on energy (assuming the sector's output is not taxed through the carbon tax). Panels (c) and (d) repeat the analysis in (b), but assume either no utilization costs (c) or prohibitive utilization costs (d).



(a) Response of output prices to a carbon tax

(b) Tax incidence on consumption

Figure 5: Expenditure channel

Note: Panel (a) displays the estimated β of regression (5.5) for various values of t with $t = 0$ corresponding to the year the carbon tax is introduced. Panel (b) shows, for each income percentile, the tax incidence of the carbon tax, measured in percent. The tax incidence is calculated as a weighted average across taxes for each sector, $\Delta\tau_i^{total}$, with weights corresponding to the share of a sector's output in households' consumption basket.

taxed by more than 7%, whereas the tax rate is closer to 4% for high-income earners. Hence, the expenditure channel is clearly regressive, even though, initially, the limited pass-through mutes the effect on consumer prices: Given an initial pass-through of about 60%, poorer households see their consumption prices go up by 4.2% compared to 2.4% for richer households, a gap of (only) 1.8 percentage points. Over time, this gap becomes larger and the expenditure channel becomes more regressive.[40]

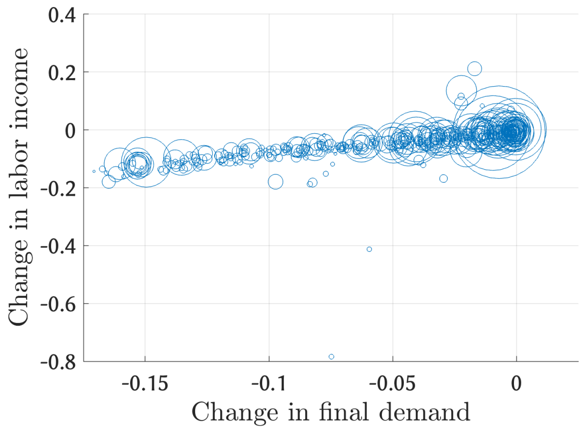
5.3.2 Labor income channel

Figure 3 has shown that high-income households see a somewhat stronger fall in their labor income than low-income households, especially in the short run when labor income across sectors is more dispersed. To understand why, we need to understand what drives differences in labor income across sectors.

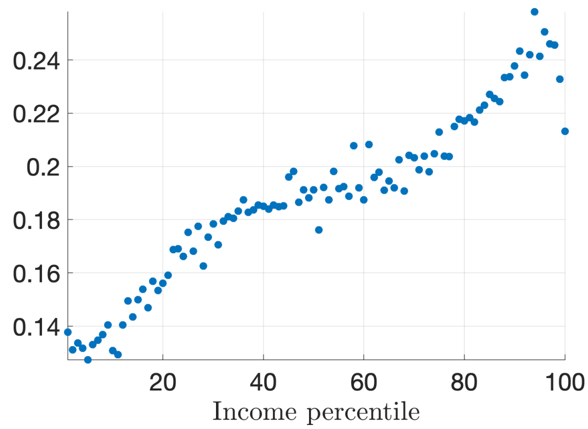
Panel (a) of Figure 6 displays the first-year response of labor income for each sector on the vertical axis against a sector's change in final demand.[41] The size of the circle corresponds to a sector's size in the economy. There is a clear positive relationship with labor income falling in

[40] Note that households' price indices are evaluated at fixed basket weights as in Fajgelbaum and Khandelwal (2016). My model does not take into account that the elasticity of substitution across goods might become larger at longer time horizons, which would dampen the distributional effects of the expenditure channel.

[41] For each sector, I calculate the share of output that either directly or indirectly goes to each final demand component (consumption, investment, government spending). I then use these shares to calculate a sector's change in final demand as a weighted average of the change in consumption, investment and government spending.



(a) Response of labor income across sectors



(b) Share of labor income induced by investment

Figure 6: Labor income channel

Note: Panel (a) displays the first-year response of labor income for each sector on the vertical axis against a sector's change in final demand. Final demand for a sector's products includes both direct sales to final customers and indirect sales. The size of the circle corresponds to a sector's size in the economy. Panel (b) shows, for each income percentile, the share of labor income that is induced by investment. This share is calculated by first calculating for each sector the share of output that either directly or indirectly is sold for final investment, I, and then calculating a weighted average across sectors, with weights corresponding to the sectoral distribution of employment for a given income percentile.

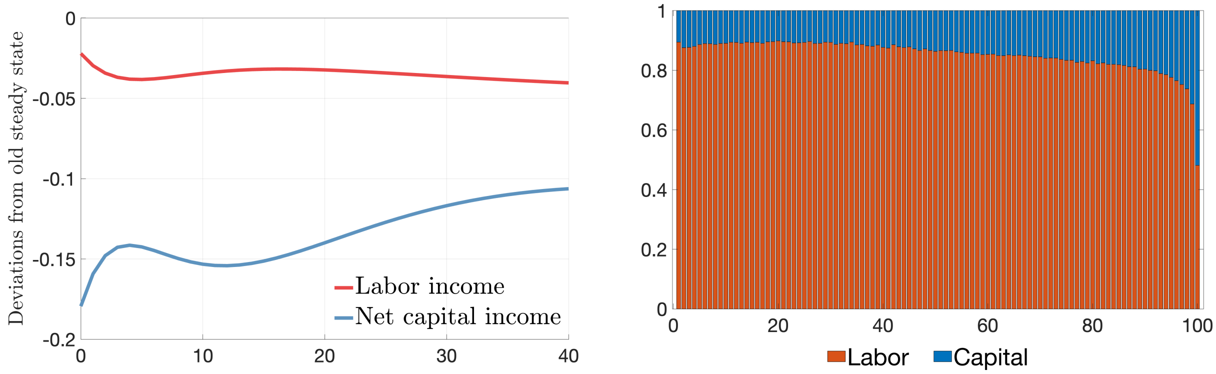
those sectors that see a drop in the demand for their products. The fit is relatively good with an $R^2 = 0.56$ for the weighted regression, indicating that changes in final demand are a key driver in a sector's short-run fluctuations in labor income.

The aggregate responses in Figure 1 indicated that investment drops by about 10 log points in the first few years, whereas final consumption slightly increases. Hence, sectors that produce capital goods, or that produce inputs for sectors that produce capital goods, experience a particularly strong fall in final demand.[42] Consequently, households working in sectors that directly or indirectly produce capital goods see a larger fall in their labor income.

Panel (b) of Figure 6 shows that these workers tend to be high-income workers. It plots, for each income percentile ι, the share of labor income that is induced by investment.[43] Except for the very top income percentiles, a worker's exposure to investment rises with their labor income. For low-income households only 13% of their labor income is derived from sales of capital goods, whereas

[42] The strong cyclicality of investment and capital-goods producing industries is a common feature of most business cycles. In their seminal paper on business cycle fluctuations, Stock and Watson (1999) report that employment in construction, manufacturing and mining is about three times as volatile as employment in other industries. This finding is also consistent with Känzig (2021) who provides empirical evidence that income in demand-sensitive sectors responds more strongly to higher energy prices than income in energy-intensive sectors.

[43] This share is obtained by first calculating for each sector the share of output that either directly or indirectly is sold for final investment, I, and then calculating a weighted average across sectors, with weights corresponding to the sectoral distribution of employment for a given income percentile.



(a) Response of factor income to carbon tax

(b) Factor income shares by income percentile

Figure 7: Factor income channel

Note: Panel (a) shows the response of average labor income and dividends (net capital income) in response to the carbon tax. Panel (b) displays the labor income shares $\frac{a_l(\iota)Y^L}{a_l(\iota)Y^L+a_k(\iota)div}$ and corresponding net capital income shares by income percentile derived from the micro files underlying the distributional national accounts (Piketty et al., 2018).

this number is 25% for workers in the 95$^{\text{th}}$ percentile. This is because low-income workers are overrepresented in service sectors like accommodation and retail trade, whereas middle and upper middle-income workers are overrepresented in manufacturing jobs. The very top earners often work in legal, medical or financial professions that are less sensitive to fluctuations in investment. Consequently, upper middle-income workers are particularly exposed to the carbon tax because the tax leads to a fall in investment that disproportionately hits these workers' labor income.

5.3.3 Factor income channel

Panel (a) of Figure 7 depicts changes in aggregate labor income, $\tilde{Y}_t^L$ and dividends (net capital income), $\widetilde{div}_t$. Both labor income and dividends fall upon impact, but the drop in dividends is substantially larger (more than 15 log points vs. 3 log points). Even over time, dividends remain more affected than labor income: After 10 years dividends is 10.5 log points below its initial steady state, whereas labor income only falls by 4 log points.

Panel (b) displays factor income shares by income percentile derived from the DINA.[44] Net capital income constitutes a larger share of total income for richer households: about 11% for the bottom 10% of households, 20% for the top 10$^{\text{th}}$ percentile and 50% for the top 1%. Hence, richer households are substantially more exposed to fluctuations in capital income. Since the carbon tax reduces dividends more than labor income, the factor income channel is progressive, with higher-income households suffering more than lower-income households.

[44] Consistent with the definition of div, capital income here refers to net capital income (excluding depreciation).

To better understand the dynamics of capital and labor income it is again helpful to consider a simplified model that abstracts from housing and motor vehicles, as in Section 4. Dividends, div, equal capital income, Y^K, less investment, I. The strong fall in dividends is driven by the fall in capital income. At the sectoral level, capital income relates to labor income as follows:

$$\tilde{Y}_{i,t}^K = \tilde{Y}_{i,t}^L + \frac{1-\chi_i}{\chi_i}\left(\tilde{r}_{i,t} - \tilde{p}_{E_i,t} - \Delta\tau_{E_i,t}\right) + \frac{1-\chi_i}{\chi_i}\tilde{Z}_{i,t}. \tag{5.6}$$

Aggregating across sectors yields

$$\begin{aligned}\tilde{Y}_t^K =& \tilde{Y}_t^L + \frac{1-\chi}{\chi}\left(\tilde{r}_t - \tilde{p}_{E,t} - \Delta\tau_{E,t}\right) + \frac{1-\chi}{\chi}\tilde{Z}_t \\ &+ \sum_i \frac{VA_i}{VA}\left[\left(\frac{\phi_i^E+\phi_i^K}{\phi^E+\phi^K} - \frac{\phi_i^L}{\phi^L}\right)\tilde{Y}_{i,t}^L - \left(\frac{\phi_i^E+\phi_i^K}{\phi^E+\phi^K} - \frac{\phi_i^K}{\phi^K}\right)\tilde{u}_{i,t}\right].\end{aligned} \tag{5.7}$$

The first-row describes fluctuatons in capital income in a one-sector version of the model. In plain-vanilla real business cycle models with Cobb-Douglas production function, capital income and labor income move one-for-one with aggregate GDP, that is $\tilde{Y}_t^K = \tilde{Y}_t^L$. In the putty-clay model, this is not necessarily the case because energy and capital are complements in the short run. As a result, an increase in the price of energy (inclusive of the energy tax), $\tilde{p}_{E,t} + \Delta\tau_{E,t}$, lowers capital income relative to labor income. Such an increase in the price of energy can be partially absorbed if it is passed on to consumers through an increase in the rental price of capital, $\tilde{r}_t$. But even with full pass-through—that is, an increase in the rental price by $\tilde{r}_t = (1-\chi)\left(\tilde{p}_{E,t} + \Delta\tau_{E,t}\right)$—capital income falls more than labor income.[45]

Over time, the discrepancy between capital income and labor income at the sectoral level vanishes because the elasticity of substitution between energy and capital converges to one such that the carbon tax affects both capital and labor in the same way. From the expressions for $\tilde{z}_{i,t}$ and $\tilde{Z}_{i,t}$ in (4.3) and (4.2), one obtains that in the long run, energy efficiency of the existing stock of machines rises with the price of energy:

$$\lim_{t\to\infty}\tilde{Z}_{i,t} = \chi_i \lim_{t\to\infty}\tilde{z}_{i,t} = \lim_{t\to\infty}\left(\tilde{p}_{E_i,t} + \Delta\tau_{E,t} - \tilde{r}_t\right).$$

Plugging this into the expression for $\tilde{Y}_{i,t}^K$ yields that capital income and labor income at the sectoral

[45] In the short run, the efficiency of the existing capital stock, $\tilde{Z}_t$, is pre-determined. The composition term in the second row is quantitatively small in the short run because both labor income and utilization move in similar directions across sectors.

level change by the same amount in the long run:

$$\lim_{t\to\infty} \tilde{Y}^K_{i,t} = \lim_{t\to\infty} \tilde{Y}^L_{i,t}.$$

However, in the aggregate, capital income still falls more than labor income because production is shifted towards labor-intensive sectors: Taking the limit of (5.7) yields

$$\lim_{t\to\infty} \tilde{Y}^K_t = \lim_{t\to\infty} \tilde{Y}^L_t + \sum_i \frac{VA_i}{VA}\left(\frac{\phi^K_i}{\phi^K} - \frac{\phi^L_i}{\phi^L}\right) \lim_{t\to\infty} \tilde{Y}^L_{i,t}.$$

Since the elasticity of substitution across sectoral goods is larger than one, $\sigma > 1$, the cost increase for energy- and capital-intensive sectors reduces the value of production in these sectors. Mathematically, this implies a negative correlation between $\frac{\phi^K_i}{\phi^K} - \frac{\phi^L_i}{\phi^L}$ and $\lim_{t\to\infty} \tilde{Y}^L_{i,t}$, such that aggregate capital income falls more than labor income.[46]

5.3.4 Summary

To sum up, the putty-clay feature of the model makes the carbon tax regressive in the short run: Since energy and capital are strong complements and the supply elasticity of capital services is limited in the short run, the tax increase is only partially passed through to output prices. This muted price response helps low-income households who spend a larger share of their income on energy-intensive goods. As a result of the limited pass through, differences in labor income across sectors are largely driven by differences in final demand that sectors face. With capital and energy being strong complements, the tax on energy is effectively a tax on capital so that aggregate investment falls, hurting those sectors that produce capital goods. In the United States, these sectors employ relatively well-off workers. Furthermore, the complementarity between capital and energy also implies that the tax on energy leads to a strong fall in capital income. While over time this complementarity vanishes, capital income still suffers more than labor income because resources are shifted towards more labor-intensive sectors.

6 Model Variations

To better understand the role of certain parameter choices I re-run the model for alternative model variations. For each variation, Table 3 displays one-year responses for GDP, the consumption of

[46] This composition effect would vanish if the elasticity of substitution across sector goods was unity.

Table 3: MODEL VARIATIONS

	Model	ΔGDP	Δc^{B50}	Δc^{T5}	Δc^{T5-B50}
(1)	Baseline	-2.96	1.83	-0.51	-2.34
(2)	No utilization ($\delta'' = \infty$)	-2.17	2.91	-0.20	-3.11
(3)	Flexible utilization ($\delta'' = 0$)	-5.52	-1.27	-3.14	-1.87
(4)	Cobb-Douglas production function	-2.70	0.38	1.47	1.09
(5)	Lump-sum rebate	-3.11	14.15	-4.38	-18.53
(6)	Active monetary policy ($\varphi_{GDP} = 0.125$)	-2.21	2.87	0.38	-2.48
(7)	Damage from CO_2	-2.96	1.84	-0.50	-2.34

Notes: Table disiplays one-year responses for various model variations. It shows the response of GDP, the response of income for the bottom 50%, the response of income for the top 5%, as well as their difference.

the bottom 50% and top 5%, as well their difference.[47]

In the baseline scenario, output falls by 3% in the first year. Consumption for the bottom 50% increases by 1.8%, whereas it falls by half a percent for the top 5%, making the tax progressive.

Rows (2) and (3) highlight the role of utilization. Without a utilization margin, output falls less (2.2%), driven by a fall in employment, and the tax becomes more progressive. Conversely, with completely flexible utilization, output falls by 5.5% as firms shut down their machines for longer, and the tax incidence flattens across the income distribution. The distributional impact is consistent with the discussion around Figure 4: If firms cannot adjust by how much they run their machines, then the carbon tax, which is basically a tax on running machines, falls onto the inelastic side of the market, the capital owners, rather than being passed on to consumers.

To better understand the role of the putty-clay technology row (4) considers an alternative production function that assumes a unit elasticity between capital, labor and energy (see e.g. Känzig, 2021). In this scenario, energy consumption falls immediately by more than 20%. This lowers the marginal product of labor and, as wages are slow to fall, leads to a drop in labor. Output falls a bit less than in the baseline scenario (2.7%). More striking are the different distributional consequences: The carbon tax becomes regressive, with top earners raising their consumption 1 percentage points more than the bottom half. With the alternative production function the carbon tax affects both capital income and labor income symmetrically: Both are a constant share of GDP and therefore fall by the same amount as output falls. This makes the income channel less progressive compared to the baseline. In addition, the pass-through into consumer goods is more

[47] Appendix Section D displays impulse responses for the various model variations and more details on their implementation.

immediate and exacerbates the expenditure channel.[48] As a result, the tax is regressive. Taken together, the putty-clay technology generates a similar output response, but it strongly affects the distributional consequences of the carbon tax.

Row (5) displays the effects if the government chooses to rebate the carbon tax revenue in a lump-sum fashion rather than by reducing the consumption tax. This alternative policy makes the tax a lot more progressive, with the bottom half of the income distribution seeing their consumption increase by more than 14%, whereas it falls by more than 4% for the top 5%. Maybe somewhat surprisingly, this redistribution towards the bottom of the distribution slightly amplifies the recession rather than mitigating it. At first sight, this seems to contradict the demand channel emphasized by Auclert et al. (2023) and Chan et al. (2024) who argue that temporary energy price hikes have recessionary effects because they reduce real income for poor, high-MPC households. Following this logic, redistributing resources towards the bottom of the distribution should be expansionary. However, my setup differs from theirs in two important ways: First, I consider a permanent shock rather than a transitory shock. MPCs out of permanent shocks are equal to one across the income distribution. Hence, shifting resources from high-income to low-income households does not affect the aggregate MPC. In contrast, Känzig (2021) considers a highly transitory carbon tax where the government reduces the initial rate by 10% every quarter. This assumption, paired with a regressive tax due to redistribution of the carbon tax revenue towards savers and a unit elasticity between capital, labor and energy, lowers the aggregate MPC and strongly amplifies the recession. Second, my model takes non-homothetic preferences into account. Low-income households consume goods that are relatively less labor-intensive.[49] Shifting resources towards low-income households therefore reduces the aggregate demand for labor (see e.g Hall, 2009, for a similar mechanism in response to government spending shocks). Taken together, the recessionary effect of the carbon tax becomes larger when the government rebates the revenue in a lump-sum fashion rather than using it to reduce the consumption tax.

Row (6) considers the role of monetary policy by imposing an alternative monetary policy rule that responds to both inflation and GDP fluctuations. Assuming that the central bank lowers the interest rate by 1 percentage point for every half a percent annualized drop in GDP, output only falls by 2% upon impact rather than 3%. Monetary policy therefore strongly matters for the aggregate response. However, it matters less for the distributional effect as the carbon tax remains

[48] The labor income channel is still progressive in this scenario: The strong fall in energy consumption leads to a reduction in labor in energy-producing industries that employ middle to upper-middle class workers.

[49] According to my calibration the labor intensity of goods consumed by the bottom 10% is 49%, but it is 52.5% for the top 10%.

progressive.

The baseline model abstracts from climate change dynamics. In row (7) I consider a model variation where productivity $A_{i,t}$ is a function of the atmospheric carbon concentration as in Golosov et al. (2014). The short-run effects of the carbon tax are virtually identical. However, as shown in the Appendix, GDP only falls by 1.3% in the very long run and aggregate consumption is even 2% higher than in the initial equilibrium, but these changes take time. For instance, after 100 years, the difference in GDP between the two scenarios is only half a percentage point.

7 Conclusion

How does a carbon tax cascade through the economy and affect sectors and households? The multi-sector putty-clay model presented in this paper suggests that, in the short run, high-income households suffer more than low-income because the carbon tax is essentially a tax on capital services. Labor income also falls, but this effect is concentrated in capital-producing rather than carbon-intensive sectors. In the United States, this makes the income channel of the carbon tax progressive, hurting well-paid jobs in capital-producing sectors and high-income earners that gain a large share of their income from capital. In the short run, the progressive income channel even outbalances the regressive expenditure channel. Over time, as wages across sectors adjust and the capital stock becomes more energy-efficient, the tax incidence throughout the income distribution flattens out.

Key model elements and predictions, such as the low short-run elasticity of energy demand, the effect of carbon pricing on stock markets / capital income and the less-than-one-for-one pass-through into consumer prices, are borne out by the data. Still we know far too little about how shocks to energy prices percolate through the production network and affect different households in the data. The model's granular input-output structure with segmented factor markets and household heterogeneity makes rich predictions about the movements of output, employment and capital returns across sectors and household income that lend themselves to be tested empirically. Recent advances in identifying macroeconomic shocks in general (Nakamura and Steinsson, 2018) and energy shocks in particular (Känzig, 2021) would make such an endeavor a fruitful avenue for future research.

The model results also point towards additional considerations for economists and policy makers alike. For instance, the carbon tax leads to a strong fall in the valuation of an economy's capital stock. This could, in principle, destabilize the financial system. While the model in this paper abstracts from any imperfections in financial markets, adding financial frictions along the lines of Bernanke et al. (1999) could be a useful exercise to better gauge the interactions between carbon taxation and financial instability.

Finally, the model predictions are not set in stone and necessarily depend on the response of fiscal and monetary policy, in particular how the government rebates the carbon tax revenue and how the monetary authority responds to the increase in inflation. For instance, policy makers enacted a wide array of policies to contain the recent energy crisis in the wake of the Russian - Ukranian war. The model might be particularly useful in gauging the success of these policies that are often targeted towards specific sectors or income groups.

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FOR ONLINE PUBLICATION

Appendix to:

Aggregate and distributional effects of a carbon tax

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Contents

A Details on some model derivations

A.1 Labor supply

To account for short-run movements in labor, I add wage rigidity as in House et al. (2018) who extend the framework in Erceg et al. (2000) to the case of inelastic labor supply. Labor supply is then governed by a New Keynesian wage Phillips curve that relates movements in labor to changes in the expected growth rate of wage inflation.

To be more specific, workers supply a fixed amount of labor, l^S, in their sector. For each job ι, and each sector i, there is a labor union with market power that acts in the interest of its workers in setting a wage rate and choosing how much each household works. Job-specific labor $l_{i,t}\left(\iota\right)$ is employed by competitive labor-aggregating firms who, in turn, sell aggregate effective labor to goods-producing firms. Labor-aggregating firms behave competitively and choose hours per job $l_{i,t}\left(\iota\right)$ to maximize their profits

$$W_{i,t}l_{i,t} - \int_0^1 W_{i,t}(\iota)l_{i,t}(\iota)d\varsigma.$$

Here $W_{i,t}$ is the nominal wage charged for a unit of effective labor while $W_{i,t}\left(\iota\right)$ is the nominal wage paid for an hour of labor in job ι. Labor-aggregating firms take $W_{i,t}$ and $W_{i,t}\left(\iota\right)$ as given. Effective labor $l_{i,t}$ is produced from the following combination of jobs $l_{i,t}\left(\iota\right)$:

$$l_{i,t} = \zeta + \left(\int_0^1 \left(\omega_i^l(\iota)\right)^{\frac{1}{\psi_w}} a_l(\iota)\left([l_{i,t}(\iota) - \zeta]\right)^{\frac{\psi_w - 1}{\psi_w}} d\iota\right)^{\frac{\psi_w}{\psi_w - 1}}, \tag{A.1}$$

where $\omega_i^l(\iota)$ is a preference weight for job ι in sector i, $\psi_w > 1$ is the elasticity of substitution across jobs, $a_l(\iota)$ is labor productivity of job ι, and $\zeta > 0$. This specification is a variaton of the Stone-Geary preferences to allow for a non-unitary elasticity of substitution between labor types. For $\zeta = 0$, this specification would collapse to a CES specificaton along the lines of Erceg et al. (2000). Then, demand for each job satisfies

$$l_{i,t}(\iota) = \zeta + \omega_i^l(\iota)\left(\frac{W_{i,t}(\iota)}{a_l(\iota)W_{i,t}}\right)^{-\psi_w}(l_{i,t} - \zeta) \tag{A.2}$$

where the labor aggregating firms take total labor demand, $l_{i,t}$ as given. The aggregate wage index is $W_{i,t} = \left(\int_0^1 \left(\frac{W_{i,t}(\iota)}{a_l(\iota)}\right)^{1-\psi_w} \omega_i^l(\iota)d\iota\right)^{\frac{1}{1-\psi_w}}$.

Labor unions set wages $W_{i,t}(\iota)$ to maximize the total amount paid to their workforce taking the demand curve (A.2) as given. Wages are set according to a Calvo mechanism with a wage reset probability given by $1-\theta_w$. Thus, a union that gets to reset their wage at time t chooses a productivity-adjusted reset wage $\frac{W^*_{i,t}(\iota)}{a_l(\iota)}$ to maximize their real wage payments over the life of the wage contract,

$$\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\theta_w\beta)^j\frac{W^*_{i,t}(\iota)}{P_{t+j}}\left(\zeta+\omega_i^l(\iota)\left(\frac{W^*_{i,t}(\iota)}{a_l(\iota)W_{i,t+j}}\right)^{-\psi_w}(l_{i,t+j}-\zeta)\right)\right].$$

All labor unions that can adjust at time t optimally choose the same productivity-adjusted reset wage so $\frac{W^*_{i,t}(\iota)}{a_l(\iota)}=W^*_{i,t}$. This reset wage is given by[1]

$$\left(W^*_{i,t}\right)^{\psi_w}=\frac{\psi_w-1}{\zeta}\frac{\mathbb{E}_t\sum_{j=0}^{\infty}(\theta_w\beta)^j(l_{i,t+j}-\zeta)W^{\psi_w}_{i,t+j}P^{-1}_{t+j}}{\mathbb{E}_t\sum_{j=0}^{\infty}(\theta_w\beta)^jP^{-1}_{t+j}} \tag{A.3}$$

Wages for effective labor adjust according to

$$W_{i,t}=(1-\theta_w)W^*_{i,t}+\theta_wW_{i,t-1}. \tag{A.4}$$

Log-linearizing (A.3) and (A.4), I get the wage Phillips curve that describes labor supply:

$$\tilde{\pi}^w_{i,t}=\frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w}\tilde{l}_{i,t}+\beta\mathbb{E}_t\left[\tilde{\pi}^w_{i,t+1}\right],$$

Notice that if wages were fully flexible ($\theta_w\to 0$) then hours per worker would be constant.

A.2 Capital funds' optimization problem

At each date t, capital funds choose div_t, B_t, $X_{i,t+1}$, $K_{i,t+1}$, $z_{i,t}$, $x_{i,t}$, and $u_{i,t}$ to maximize the expected discounted sum of their dividends,

$$\mathbb{E}_t\left(\sum_{s=0}^{\infty}\beta^s\frac{\Phi_{t+s}}{\Phi_t}div_{t+s}\right),$$

[1]This equation shows why the restriction $\zeta>0$ is needed. A CES-specification would imply that the optimal wage set by trade unions is the cost of supplying labor times a gross markup. Since the cost of supplying an additional unit of labor is zero up to the fixed amount of labor supply, the optimal wage would be zero.

subject to the definition of dividends (λ_t)

$$div_t = \sum_{i \in \mathcal{K}} \left\{ u_{i,t} r_{i,t} K_{i,t} - p_{I_i,t} x_{i,t} z_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t}\right) u_{i,t} X_{i,t} \right\} + B_t - B_{t-1} \frac{1 + i_{t-1}}{\pi_t},$$

the law of motion for the number of machines usage ($-\psi_{i,t}\lambda_t$)

$$X_{i,t+1} = (1 - \delta_i(u_{i,t}))X_{i,t} + x_{i,t}\left(1 - f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right), \tag{A.5}$$

the law of motion for capital capacity ($\nu_{i,t}\lambda_t$)

$$K_{i,t+1} = (1 - \delta_i(u_{i,t}))K_{i,t} + x_{i,t} a_i z_{i,t}^{\chi_i}\left(1 - f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right). \tag{A.6}$$

and the definition of investment ($\mu_{i,t}\lambda_t$)

$$I_{i,t} = x_{i,t} z_{i,t}$$

The first order conditions are (div_t, B_t, $X_{i,t+1}$, $K_{i,t+1}$, $z_{i,t}$, $x_{i,t}$, $I_{i,t}$ and $u_{i,t}$)

$$\lambda_t = \Phi_t \tag{A.7}$$

$$\lambda_t = \beta(1 + i_t)\mathbb{E}_t\left(\frac{\lambda_{t+1}}{\pi_{t+1}}\right) \tag{A.8}$$

$$\psi_{i,t}\lambda_t = \beta\mathbb{E}_t\left(u_{i,t+1}\left(p_{E_i,t+1} + \tau_{E_i,t+1}\right)\lambda_{t+1} + (1 - \delta_i(u_{i,t}))\psi_{i,t+1}\lambda_{t+1}\right) \tag{A.9}$$

$$\nu_{i,t}\lambda_t = \beta\mathbb{E}_t\left(r_{i,t+1}u_{i,t+1}\lambda_{t+1} + (1 - \delta_i(u_{i,t}))\nu_{i,t+1}\lambda_{t+1}\right) \tag{A.10}$$

$$\lambda_t p_{I_i,t} = \nu_{i,t}\lambda_t\chi_i a_i z_{i,t}^{\chi_i - 1}\left(1 - f_{i,t}\right) + \mu_{i,t}\lambda_t \tag{A.11}$$

$$\lambda_t p_{I_i,t} z_{i,t} = \lambda_t\left(-\psi_{i,t} + \nu_{i,t} a_i z_{i,t}^{\chi_i}\right)\left(1 - f_{i,t}\right) + \mu_{i,t}\lambda_t z_{i,t} \tag{A.12}$$

$$\mu_{i,t}\lambda_t = -\lambda_t x_{i,t}\left(-\psi_{i,t} + \nu_{i,t} a_i z_{i,t}^{\chi_i}\right)\frac{1}{I_{i,t-1}} f'_{i,t} \tag{A.13}$$

$$+ \beta\mathbb{E}_t\left(\lambda_{t+1} x_{i,t+1}\left(-\psi_{i,t+1} + \nu_{i,t+1} a_i z_{t+1}^{\chi_i}\right)\frac{I_{i,t+1}}{I_{i,t}^2} f'_{i,t+1}\right) \tag{A.14}$$

$$r_{i,t}K_{i,t} = \left(p_{E_i,t} + \tau_{E_i,t}\right)X_{i,t} + \delta'_i(u_{i,t})\left(-\psi_{i,t}X_{i,t} + \nu_{i,t}K_{i,t}\right) \tag{A.15}$$

Combining equations (A.7) and (A.9) and solving forward yields

$$\psi_{i,t} = \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t}(1 - \delta_{i,t+s})^{s-1} u_{i,t+s}\left(p_{E_i,t+s} + \tau_{E_i,t+s}\right)$$

Combining equations (A.7) and (A.10) and solving forward yields

$$\nu_{i,t} = \mathbb{E}_t \sum_{s=1}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t} (1-\delta_{i,t+s})^{s-1} u_{i,t+s} r_{i,t+s}$$

Combining equations (A.11) and (A.12) yields

$$\psi_{i,t} = (1-\chi_i)\nu_{i,t} a_i z_{i,t}^{\chi_i}.$$

Combining equations (A.12) and (A.13) yields

$$\begin{aligned} p_{I_i,t} z_{i,t} &= \left(-\psi_{i,t} + \nu_{i,t} a_i z_{i,t}^{\chi_i}\right) \left(1 - f_{i,t} - \frac{I_{i,t}}{I_{i,t-1}} f'_{i,t}\right) \\ &\quad + z_{i,t} \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} x_{i,t+1} \left(-\psi_{i,t+1} + \nu_{i,t+1} a_i z_{t+1}^{\chi_i}\right) \frac{I_{i,t+1}}{I_{i,t}^2} f'_{i,t+1} \right), \end{aligned}$$

which, combined with the previous equation to replace $\psi_{i,t}$ yields

$$p_{I_i,t} = \chi_i \nu_{i,t} a_i z_{i,t}^{\chi_i - 1} \left(1 - f_{i,t} - \frac{I_{i,t}}{I_{i,t-1}} f'_{i,t}\right) + \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} x_{i,t+1} \chi_i \nu_{i,t+1} a_i z_{t+1}^{\chi_i} \frac{I_{i,t+1}}{I_{i,t}^2} f'_{i,t+1} \right).$$

A.3 Aggregation across households

Household heterogeneity shows up in two parts of the model: consumption and the stochastic discount factor.

A.3.1 Consumption

This block is described by the following equations:

- Budget constraint

$$(1+\tau_t^C) p_{c_i,t}(\iota) c_{i,t}(\iota) = w_{i,t}(\iota) l_{i,t} + a_{k,t}(\iota) div_t$$

Log-linearizing yields[2]

$$(1+\tau^C) c_i(\iota) \left(\tilde{p}_{c_i,t}(\iota) + \tilde{c}_{i,t}(\iota)\right) + c_i(\iota) \Delta\tau_t^C = w(\iota) l \left(\tilde{w}_{i,t} + \tilde{l}_{i,t}\right) + a_k(\iota) div \left(\tilde{a}_{k,t}(\iota) + \widetilde{div}_t\right)$$

[2]In steady state, $a_l(\iota) w_i = a_l(\iota) w = w(\iota)$.

Here, we have

$$p_{c_i,t}(\iota) = \left(\sum_{j=1}^{J} \omega_{c_i,t}^{j}(\iota) p_{j,t}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$
$$\tilde{p}_{c_i,t}(\iota) = \sum_{j=1}^{J} \omega_c^j(\iota)\tilde{p}_{j,t}.$$

and

$$a_{k,t}(\iota) = \frac{a_k(\iota)}{\sum_i \int_0^1 n_{i,t}(\iota) a_k(\iota)}$$
$$\tilde{a}_{k,t}(\iota) = -\sum_i \left(n_i \int_0^1 \omega_i^l(\iota) a_k(\iota) d\iota\right) \times \tilde{n}_{i,t} \equiv \tilde{a}_{k,t}.$$

Hence, we can write

$$(1+\tau^C)c(\iota)\left(\sum_{j=1}^{J} \omega_c^j(\iota)\tilde{p}_{j,t} + \tilde{c}_{i,t}(\iota)\right) = -c_i(\iota)\Delta\tau_t^C + w(\iota)l\left(\tilde{w}_{i,t} + \tilde{l}_{i,t}\right)$$
$$+ a_k(\iota)div\left(-\sum_i n_i \left(\int_0^1 \omega_i^l(\iota) a_k(\iota) d\iota\right)\tilde{n}_{i,t} + \widetilde{div}_t\right)$$

Dividing both sides by consumption yields

$$(1+\tau^C)\left(\sum_{j=1}^{J} \omega_c^j(\iota)\tilde{p}_{j,t} + \tilde{c}_{i,t}(\iota)\right) = -\Delta\tau_t^C + ls(\iota)\left(\tilde{w}_{i,t} + \tilde{l}_{i,t}\right)$$
$$+ (1 - ls(\iota))\left(-\sum_i n_i \left(\int_0^1 \omega_i^l(\iota) a_k(\iota) d\iota\right)\tilde{n}_{i,t} + \widetilde{div}_t\right)$$

where $ls(\iota) = \frac{w(\iota)l}{w(\iota)l + a_k(\iota)div}$ is the share of labor income in total pre-tax income

- Demand for sector goods

$$y_{c_i,t}^j(\iota) = \omega_c^j(c_{i,t}(\iota))\left(\frac{p_{j,t}}{p_{c_i,t}(\iota)}\right)^{-\sigma} c_{i,t}(\iota).$$

Aggregating across households[3]

$$\sum_i \int_0^1 n_{i,t}(\iota) y^j_{c_i,t}(\iota) d\iota = \sum_i \int_0^1 n_{i,t}(\iota) \omega^j_c(c_{i,t}(\iota)) \left(\frac{p_{j,t}}{p_{c_i,t}(\iota)}\right)^{-\sigma} c_{i,t}(\iota) d\iota$$

$$p_{j,t} y^j_{c,t} = \sum_i n_{i,t} \int_0^1 \omega^l_i(\iota) \omega^j_c(c_{i,t}(\iota)) \left(\frac{p_{j,t}}{p_{c_i,t}(\iota)}\right)^{1-\sigma} p_{c_i,t}(\iota) c_{i,t}(\iota) d\iota$$

Log-linearizing

$$\begin{aligned}\frac{y^j_c}{c}\tilde{y}^j_{c,t} = &\sum_i n_i \int_0^1 \left(\omega^l_i(\iota)\omega^j_c(\iota)\frac{c(\iota)}{c}\left(1 + \frac{\partial \omega^j_c(c(\iota))}{\partial c(\iota)}\frac{c(\iota)}{\omega^j_c(\iota)}\right) \times \tilde{c}_{i,t}(\iota)\right) d\iota \\ &- \sigma \frac{y^j_c}{c}\tilde{p}_{j,t} \\ &+ \sigma \sum_i n_i \int_0^1 \left(\omega^l_i(\iota)\omega^j_c(\iota)\frac{c(\iota)}{c} \times \tilde{p}_{c_i,t}(\iota)\right) d\iota \\ &+ \sum_i \left(\int_0^1 \omega^l_i(\iota)\omega^j_c(\iota)\frac{c(\iota)}{c}\right) d\iota \times n_i \tilde{n}_{i,t}.\end{aligned}$$

where $\omega^j_c(\iota)$ is the share of expenditure that households in job ι spend on good j in the non-stochastic steady state.

In practice, I insert the budget constraint in the demand equation to replace $\tilde{c}_{i,t}(\iota)$ and use $\tilde{p}_{c_i,t}(\iota) = \sum_{j=1}^J \omega^j_c(\iota)\tilde{p}_{j,t}$ to replace $\tilde{p}_{c_i,t}(\iota)$ in the demand equation. This gives me a demand equation that does not depend on any movements in ι-specific variables.

A.3.2 Stochastic discount factor

The price-adjusted marginal utility that enters the stochastic discount factor of the capital fund is

$$\Phi_t = \frac{1+\tau^C}{1+\tau^C_t} \int_0^1 \sum_i a_{k,t}(\iota) \frac{c_i(\iota)}{p_{c_i,t}(\iota) c_{i,t}(\iota)} n_{i,t}(\iota) d\iota$$

[3] Note that $n_{i,t}(\iota) = n_{i,t}\omega^l_i(\iota)$, where fluctuations in $\omega^l_i(\iota)$ across jobs are due to wage stickiness: Jobs whose wage is stuck below the optimal wage see an increase in demand. In a first-order approximation, we can treat $\omega^l_i(\iota)$ as constant because it is constant in expectations.

Log-linearizing yields

$$\Phi\left(\frac{\Delta\tau_t^C}{1+\tau^C}+\tilde{\Phi}_t\right)=\Phi\tilde{a}_{k,t}$$
$$+\sum_i n_i\left(\int_0^1 a_k(\iota)\omega_i^l(\iota)d\iota\right)\times\tilde{n}_{i,t}$$
$$-\int_0^1\sum_i a_k(\iota)n_{i'}\omega_i^l(\iota)\times\left(\tilde{p}_{c_i,t}(\iota)+\tilde{c}_{i,t}(\iota)\right)d\iota.$$

Since $\tilde{a}_{k,t}=-\sum_i\left(n_i\int_0^1\omega_i^l(\iota)a_k(\iota)d\iota\right)\times\tilde{n}_{i,t}$, we have

$$\Phi\left(\frac{\Delta\tau_t^C}{1+\tau^C}+\tilde{\Phi}_t\right)=\sum_i n_i\left(\int_0^1\omega_i^l(\iota)a_k(\iota)\left(1-\Phi\right)d\iota\right)\times\tilde{n}_{i,t}-\int_0^1\sum_i a_k(\iota)n_i\omega_i^l(\iota)\times\left(\tilde{p}_{c_i,t}(\iota)+\tilde{c}_{i,t}(\iota)\right)d\iota,$$

where I use that $c_i(\iota)=c(\iota)$ in steady state. Notice that $\Phi=1$. Hence,

$$\tilde{\Phi}_t=-\frac{\Delta\tau_t^C}{1+\tau^C}-\int_0^1\sum_i a_k(\iota)n_i\omega_i^l(\iota)\times\left(\tilde{p}_{c_i,t}(\iota)+\tilde{c}_{i,t}(\iota)\right)d\iota,$$

Again, I insert the budget constraint in the demand equation to replace $\tilde{c}_{i,t}(\iota)$ and use $\tilde{p}_{c_i,t}(\iota)=\sum_{j=1}^J\omega_c^j(\iota)\tilde{p}_{j,t}$ to replace $\tilde{p}_{c_i,t}(\iota)$ in the demand equation. This gives me an equation that does not depend on any movements in ι-specific variables.

A.4 Aggregate GDP

Consistent with the NIPA GDP is composed of consumption, C_t, investment, $p_{I,t}I_t$, and government spending, $p_{G,t}G_t$.

$$GDP_t=C_t+p_{I,t}I_t+p_{G,t}G_t.$$

Consumption and invstment are defined as follows: Consumption excludes consumption of motor vehicle services (sector J), but includes purchases of motor vehicles and consumption of gasoline

$$C_t=\sum_{i=1}^J\int_0^1 n_{i,t}(\iota)\left[p_{c,t}(\iota)c_{i,t}(\iota)-p_{J,t}y_{c_i,t}^J(\iota)\right]d\iota+p_{I_J,t}I_{J,t}+p_{E_J,t}E_{J,t},$$

Aggregate investment excludes purchases of motor vehicles:

$$p_{I,t}I_t = \sum_{i=1}^{J-1} p_{I_i,t}x_{i,t}z_{i,t}.$$

To rewrite the expression for consumption, start from the budget constraint for household ι in sector i:

$$(1+\tau_t^C)p_{c_i,t}(\iota)c_{i,t}(\iota) = a_l(\iota)w_{i,t}l_{i,t} + a_k(\iota)div_t$$

Aggregating across households and sectors yields

$$(1+\tau_t^C)\sum_{i=1}^{J}\int_0^1 n_{i,t}(\iota)p_{c_i,t}(\iota)c_{i,t}(\iota)d\iota = w_tL_t + div_t$$

Replace dividend payments by

$$div_t = \sum_{i=1}^{J}\left\{u_{i,t}r_{i,t}K_{i,t} - p_{I_i,t}x_{i,t}z_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t}\right)E_{i,t}\right\}.$$

and tax payments by

$$\tau_t^C\sum_{i=1}^{J}\left[\int_0^1 n_{i,t}(\iota)p_{c_i,t}(\iota)c_{i,t}(\iota)d\iota\right] = p_{G,t}G - \sum_{i=1}^{J}\left(\tau_{E_i,t}E_{i,t} + \tau_{Y_i}Y_{i,t}\right).$$

to obtain

$$\begin{aligned}\sum_{i=1}^{J}\int_0^1 n_{i,t}(\iota)p_{c_i,t}(\iota)c_{i,t}(\iota)d\iota &= w_tL_t - p_{G,t}G + \sum_{i=1}^{J}\left(\tau_{E_i,t}E_{i,t} + \tau_{Y_i}Y_{i,t}\right)\\ &+ \sum_{i=1}^{J}\left\{u_{i,t}r_{i,t}K_{i,t} - p_{I_i,t}x_{i,t}z_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t}\right)E_{i,t}\right\}.\end{aligned}$$

Taken together,

$$
\begin{aligned}
GDP_t &= C_t + p_{I,t} I_t + p_{G,t} G_t \\
&= \sum_{i=1}^{J} \int_0^1 n_{i,t}(\iota) p_{c_i,t}(\iota) c_{i,t}(\iota) d\iota - \sum_{i=1}^{J} \int_0^1 n_{i,t}(\iota) p_{J,t} y^J_{c_i,t}(\iota) d\iota + p_{E_j,t} E_{J,t} + \sum_{i=1}^{J} p_{I_i,t} x_{i,t} z_{i,t} + p_{G,t} G_t \\
&= w_t L_t - p_{G,t} G_t + \sum_{i=1}^{J} \left(\tau_{E_i,t} E_{i,t} + \tau_{Y_i} Y_{i,t} \right) + \sum_{i=1}^{J} \left\{ u_{i,t} r_{i,t} K_{i,t} - p_{I_i,t} x_{i,t} z_{i,t} - p_{E_i,t} E_{i,t} + \tau_{E_i,t} E_{i,t} \right\} \\
&- \sum_{i=1}^{J} n_{i,t} p_{J,t} y^J_{c_i,t} + p_{E_J,t} E_{J,t} + \sum_{i=1}^{J} p_{I_i,t} x_{i,t} z_{i,t} + p_{G,t} G_t \\
&= w_t L_t + \sum_{i=1}^{J} \tau_{Y_i} Y_{i,t} + \sum_{i=1}^{J} \left\{ u_{i,t} r_{i,t} K_{i,t} - p_{E_i,t} E_{i,t} \right\} - \sum_{i=1}^{J} n_{i,t} p_{J,t} y^J_{c_i,t} + p_{E_J,t} E_{J,t}
\end{aligned}
$$

Next, the sector providing motor vehicle services only requires capital services. The zero-profit condition in that sector requires that factor payments to capital are equal to total revenue, $u_{J,t} r_{J,t} K_{J,t} = \sum_{i=1}^{J} n_{i,t} p_{J,t} y^J_{c_i,t}$. Then, noting that sector J does not pay any taxes on output, one obtains

$$
GDP_t = \sum_{i=1}^{J-1} \left\{ u_{i,t} r_{i,t} K_{i,t} - p_{E_i,t} E_{i,t} + w_{i,t} L_{i,t} + \tau_{Y_i} Y_{i,t} \right\}
$$

This expression for GDP follows the income approach, where the first two terms refer to capital income, the third term to labor income and the last term to taxes on products.

Using the zero profit conditions for sectors $j = 1, ..., J-1$,

$$
\left(p_{i,t} - \tau_{Y_{i,t}} \right) Y_{i,t} = u_{i,t} K_{i,t} r_{i,t} + w_{i,t} L_{i,t} + p_{M_i,t} M_{i,t},
$$

one obtains

$$
GDP_t = \sum_{i=1}^{J-1} \left(p_{i,t} Y_{i,t} - p_{M_i,t} M_{i,t} - p_{E_i,t} E_{i,t} \right).
$$

Rewriting in aggregate terms:

$$
GDP_t = p_t Y_t - p_{M,t} M_t - p_{E,t} E_t.
$$

This expression for GDP follows the production approach.

Summarizing, real GDP can be calculated from either the production side, the expenditure side or the income side

$$
\begin{aligned}
GDP_t^{Prod} &= p_t Y_t - p_{M,t} M_t - p_{E,t} E_t \\
GDP_t^{Exp} &= C_t + p_{I,t} I_t + p_{G,t} G_t \\
GDP_t^{Inc} &= r_t u_t K_t - p_{E,t} E_t + w_t L_t + \tau_{Y,t} Y_t
\end{aligned}
$$

Note that this definition of real GDP takes relative price changes across goods into account, similar to the chain-linking method used by statistical agencies. This differs from the more traditional method that values production across sectors at a constant set of relative prices.

A.5 Derivations in Sections 4 and 5

To derive the expressions I assume a model with no housing and no durable consumption goods. This implies that only firms directly consume energy. I further assume that the baseline depreciation rate is the same across sectors, as done in the calibration of the full model.

A.5.1 Expression for GDP

Real GDP is given by

$$
GDP_t = \sum_i p_i Y_{i,t} - p_{M,i} M_{i,t} - p_{E,i} E_{i,t}.
$$

Log-linearizing and noting that all prices are normalized to one in steady state yields

$$
GDP\widetilde{GDP}_t = \sum_i Y_i \tilde{Y}_{i,t} - M_i \tilde{M}_{i,t} - E_i \tilde{E}_{i,t}.
$$

Inserting the equation describing the production of sector goods

$$
\tilde{Y}_{i,t} = (1-\phi_i)\left[\alpha_i\left(\tilde{u}_{i,t} + \tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right] + \phi_i \tilde{M}_{i,t}
$$

yields

$$
GDP\widetilde{GDP}_t = \sum_i Y_i (1-\phi_i)\left[\alpha_i\left(\tilde{u}_{i,t} + \tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right] + Y_i \phi_i \tilde{M}_{i,t} - M_i \tilde{M}_{i,t} - E_i \tilde{E}_{i,t}.
$$

Further, notice that $E_i = (1-\chi_i)\alpha_i(1-\phi_i)Y_i$ and $M_j = \phi_j Y_j$ to get

$$\widetilde{GDPGDP}_t = \sum_i (1-\phi_i)Y_i \left[\alpha_i \left(\tilde{u}_{i,t} - (1-\chi_i)\tilde{E}_{i,t} + \tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right].$$

Using the definition of energy, $\tilde{E}_{i,t} = \tilde{u}_{i,t} + \tilde{X}_{i,t}$, to replace $\tilde{u}_{i,t}$:

$$\widetilde{GDPGDP}_t = \sum_i (1-\phi_i)Y_i \left[\alpha_i \left(\chi_i\tilde{E}_{i,t} + \tilde{K}_{i,t} - \tilde{X}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right].$$

Define real value added in sector i as

$$VA_{i,t} = r_i K_{i,t} - p_{E_i}E_{i,t} + w_i L_{i,t}.$$

Using its steady-state counterpart and noting that $r_i K_i = \alpha_i(1-\phi_i)Y_i$, $E_i = (1-\chi_i)\alpha_i(1-\phi_i)Y_i$ and $w_i L_i = (1-\alpha_i)(1-\phi_i)Y_i$, I obtain

$$VA_i = \left[1-(1-\chi_i)\alpha_i\right](1-\phi_i)Y_i.$$

GDP is then given by

$$\widetilde{GDPGDP}_t = \sum_i \frac{VA_i}{1-(1-\chi_i)\alpha_i}\left[\alpha_i \left(\chi_i\tilde{E}_{i,t} + \tilde{K}_{i,t} - \tilde{X}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right].$$

In Section C.2.2 I show that $\phi_i^L \equiv \frac{w_i L_i}{VA_i} = \frac{1-\alpha_i}{1-(1-\chi_i)\alpha_i}$ and $\phi_i^E \equiv \frac{p_{E_i}E_i}{VA_i} = \frac{(1-\chi_i)\alpha_i}{1-(1-\chi_i)\alpha_i}$. Then, GDP can be rewritten as

$$\widetilde{GDPGDP}_t = \sum_i VA_i \left[(1-\phi_i^L)\tilde{E}_{i,t} + (1-\phi_i^L+\phi_i^E)\tilde{Z}_{i,t} + \phi_i^L\tilde{L}_{i,t}\right],$$

with $Z_{i,t} \equiv \frac{K_{i,t}}{X_{i,t}}$ denoting energy efficiency of stock i.

To aggregate up, rewrite the first term as

$$\begin{aligned}
\sum_i VA_i(1-\phi_i^L)\tilde{E}_{i,t} &= \tilde{E}_t\left(GDP - wL\right) + \sum_i VA_i(1-\phi_i^L)\tilde{E}_{i,t} - \tilde{E}_t\left(GDP - wL\right) \\
&= \tilde{E}_t\left(GDP - wL\right) + \sum_i VA_i(1-\phi_i^L)\tilde{E}_{i,t} - \sum_i \frac{E_i}{E}\tilde{E}_{i,t}\left(GDP - wL\right) \\
&= \tilde{E}_t\left(GDP - wL\right) + \sum_i VA_i\left[(1-\phi_i^L) - \frac{E_i}{VA_i}\frac{GDP - wL}{E}\right]\tilde{E}_{i,t} \\
&= \tilde{E}_t\left(GDP - wL\right) + \sum_i VA_i\left[(1-\phi_i^L) - \phi_i^E\frac{1-\phi^L}{\phi^E}\right]\tilde{E}_{i,t} \\
&= \tilde{E}_t\left(GDP - wL\right) + \sum_i VA_i\left[\frac{1-\phi_i^L}{\phi_i^E} - \frac{1-\phi^L}{\phi^E}\right]\phi_i^E\tilde{E}_{i,t}.
\end{aligned}$$

Similarly, I can write

$$\begin{aligned}
\sum_i VA_i(1-\phi_i^L+\phi_i^E)\tilde{X}_{i,t} &= \tilde{X}_t\left(GDP - wL + E\right) + \sum_i VA_i(1-\phi_i^L+\phi_i^E)\tilde{X}_{i,t} - \tilde{X}_t\left(GDP - wL + E\right) \\
&= \tilde{X}_t\left(GDP - wL + E\right) + \sum_i VA_i\left[\frac{1-\phi_i^L+\phi_i^E}{\phi_i^X} - \frac{1-\phi^L+\phi^E}{\phi^X}\right]\phi_i^X\tilde{X}_{i,t}.
\end{aligned}$$

Note that $X_i = E_i = \phi_i^E VA_i$ and this becomes

$$\sum_i VA_i(1-\phi_i^L+\phi_i^E)\tilde{X}_{i,t} = \tilde{X}_t\left(GDP - wL + E\right) + \sum_i VA_i\left[\frac{1-\phi_i^L}{\phi_i^E} - \frac{1-\phi^L}{\phi^E}\right]\phi_i^E\tilde{X}_{i,t}.$$

Finally, the terms with $\tilde{K}_{i,t}$ and $\tilde{L}_{i,t}$ aggregate up:

$$\begin{aligned}
\sum_i VA_i(1-\phi_i^L+\phi_i^E)\tilde{K}_{i,t} &= \sum_i r_i K_i\tilde{K}_{i,t} = rK\tilde{K}_t \\
\sum_i VA_i\phi_i^L\tilde{L}_{i,t} &= \sum_i w_i L_i\tilde{L}_{i,t} = wL\tilde{L}_t.
\end{aligned}$$

Hence, GDP can be rewritten as

$$\widetilde{GDP}_t = (1-\phi^L)\tilde{E}_t + \left(1-\phi^L+\phi^E\right)\tilde{Z}_t + \phi^L\tilde{L}_t + \sum_i \frac{E_i}{GDP}\left[\frac{1-\phi_i^L}{\phi_i^E} - \frac{1-\phi^L}{\phi^E}\right]\tilde{u}_{i,t}.$$

In a one-sector model, the last term cancels out.

A.5.2 Expressions for Z_t and z_t

Energy efficiency of installed machines in sector i at time t is $Z_{i,t} \equiv \frac{K_{i,t}}{X_{i,t}}$. Log-linearizing and inserting the law of motions describing $\tilde{K}_{i,t}$ and $\tilde{X}_{i,t}$ yields

$$\begin{aligned}\tilde{Z}_{i,t} &= \tilde{K}_{i,t} - \tilde{X}_{i,t} \\ &= \Big[(1-\delta)\,\tilde{K}_{i,t-1} + \delta\left(\tilde{x}_{i,t-1} + \chi_i \tilde{z}_{i,t-1}\right) - \delta' \tilde{u}_{i,t-1}\Big] - \Big[(1-\delta)\,\tilde{X}_{i,t-1} + \delta \tilde{x}_{i,t-1} - \delta' \tilde{u}_{i,t-1}\Big] \\ &= (1-\delta)\,\tilde{Z}_{i,t-1} + \delta \chi_i \tilde{z}_{i,t-1},\end{aligned}$$

where I use that the baseline depreciation rate, δ, is the same across sectors. Aggregate energy efficiency is

$$\begin{aligned}\tilde{Z}_t &= \tilde{K}_t - \tilde{X}_t \\ &= \sum_i \left(\frac{r_i K_i}{rK}\tilde{K}_{i,t} - \frac{X_i}{X}\tilde{X}_{i,t}\right) \\ &= \sum_i \left(\frac{r_i K_i}{rK}\tilde{K}_{i,t} - \frac{1-\chi_i}{1-\chi}\frac{r_i K_i}{rK}\tilde{X}_{i,t}\right) \\ &= \sum_i \frac{r_i K_i}{rK}\left(\tilde{Z}_{i,t} + \frac{\chi_i - \chi}{1-\chi}\tilde{X}_{i,t}\right).\end{aligned}$$

Inserting the law of motions for $Z_{i,t}$ and $X_{i,t}$ yields

$$\begin{aligned}\tilde{Z}_t &= \sum_i \frac{r_i K_i}{rK}\left[(1-\delta)\tilde{Z}_{i,t-1} + \delta \chi_i \tilde{z}_{i,t-1} + \frac{\chi_i - \chi}{1-\chi}\Big((1-\delta)\tilde{X}_{i,t-1} + \delta \tilde{x}_{i,t-1} - \delta' \tilde{u}_{i,t-1}\Big)\right] \\ &= (1-\delta)\tilde{Z}_{t-1} + \delta \sum_i \frac{r_i K_i}{rK}\left[\chi_i \tilde{z}_{i,t-1} + \frac{\chi_i - \chi}{1-\chi}\tilde{x}_{i,t-1}\right] - \sum_i \frac{r_i K_i}{rK}\frac{\chi_i - \chi}{1-\chi}\delta' \tilde{u}_{i,t-1}.\end{aligned}$$

Next, define the aggregate energy efficiency of new machines as the capital capacity of all new machines bought divided by the number of new machines:

$$z_t^{\chi} = \frac{\sum_i a_i x_{i,t} z_{i,t}^{\chi_i}}{\sum_i x_{i,t}}.$$

Log-linearizing yields

$$\chi \tilde{z}_t = \sum_i \frac{a_i x_i z_i^{\chi_i}}{\sum_j a_j x_j z_j^{\chi_j}}\left(\tilde{x}_{i,t} + \chi_i \tilde{z}_{i,t}\right) - \sum_i \frac{x_i}{x}\tilde{x}_{i,t}.$$

Notice that in steady state, $a_i x_i z_i^{\chi_i} = \delta K_i$ and $x_i = \delta X_i$. Further, with a common depreciation rate, $r_i = r$. Hence, this simplifies to

$$\chi \tilde{z}_t = \sum_i \frac{r_i K_i}{rK} \chi_i \tilde{z}_{i,t} + \sum_i \left(\frac{r_i K_i}{rK} - \frac{X_i}{X} \right) \tilde{x}_{i,t}.$$

Since $\frac{r_i K_i}{rK} - \frac{X_i}{X} = \frac{r_i K_i}{rK} \frac{\chi_i - \chi}{1-\chi}$, the expression for aggregate energy efficiency can be rewritten as

$$\tilde{Z}_t = (1-\delta)\tilde{Z}_{t-1} + \delta \chi \tilde{z}_{t-1} - \sum_i \frac{r_i K_i}{rK} \frac{\chi_i - \chi}{1-\chi} \delta' \tilde{u}_{i,t-1},$$

with

$$\chi \tilde{z}_t = \sum_i \frac{r_i K_i}{rK} \left[\chi_i \tilde{z}_{i,t} + \frac{\chi_i - \chi}{1-\chi} \tilde{x}_{i,t} \right].$$

One can simplify this further by noting that

$$\begin{aligned} \frac{r_i K_i}{rK} \frac{\chi_i - \chi}{1-\chi} = \frac{r_i K_i}{rK} - \frac{X_i}{X} &= \left(\frac{1-\phi_i^L + \phi_i^E}{1-\phi^L+\phi^E} - \frac{\phi_i^E}{\phi^E} \right) \frac{VA_i}{VA} \\ &= \left(\frac{1-\phi_i^L + \phi_i^E}{\phi_i^E} - \frac{1-\phi^L+\phi^E}{\phi^E} \right) \frac{\phi_i^E}{1-\phi^L+\phi^E} \frac{VA_i}{VA} \\ &= \left(\frac{1-\phi_i^L}{\phi_i^E} - \frac{1-\phi^L}{\phi^E} \right) \frac{E_i}{rK}. \end{aligned}$$

Further, since $\delta' = \frac{r}{\nu}$, I end up with

$$\tilde{Z}_t = (1-\delta)\tilde{Z}_{t-1} + \delta \chi \tilde{z}_{t-1} - \sum_i \left(\frac{1-\phi_i^L}{\phi_i^E} - \frac{1-\phi^L}{\phi^E} \right) \frac{E_i}{\nu K} \tilde{u}_{i,t-1}.$$

The term in parentheses is negative for sectors operating energy-intensive machines. That is, if utilization disproportionately falls in sectors with energy-intensive machines, then this reduces aggregate energy efficiency all else equal. In essence, this is driven by a shift in the composition of the aggregate stock of machines. If sectors with energy-intensive machines cut their utilization by more, then machines in these sectors depreciate less than the average machine in the economy. Consequently, over time, energy-intensive machines take up a larger share of the aggregate stock of machines, making the aggregate stock less energy efficient.

The optimal choice of z states $\chi_i \tilde{z}_{i,t} = \tilde{\psi}_{i,t} - \tilde{\nu}_{i,t}$. Inserting the Euler equations for the number of machines and the capital stock

$$\chi_i \tilde{z}_{i,t} = (1-\beta(1-\delta))\left(\tilde{p}_{E_i,t+1} + \Delta\tau_{E_i,t+1} - \tilde{r}_{i,t+1}\right) + \beta(1-\delta)\chi_i \tilde{z}_{i,t}.$$

Solving forward yields

$$\tilde{z}_{i,t} = \frac{1-\beta(1-\delta)}{\chi_i} \sum_{s=t+1}^{\infty} \left[\beta(1-\delta)\right]^{s-t} \left(\tilde{p}_{E_i,s} + \Delta\tau_{E_i,s} - \tilde{r}_{i,s}\right).$$

In a one-sector model, the expressions for $\tilde{Z}_t$ and $\tilde{z}_t$ simplify to

$$\tilde{Z}_t = (1-\delta)\tilde{Z}_{t-1} + \delta\chi\tilde{z}_{t-1},$$

with

$$\tilde{z}_t = \frac{1-\beta(1-\delta)}{\chi} \sum_{s=t+1}^{\infty} \left[\beta(1-\delta)\right]^{s-t} \left(\Delta\tau_{E,s} - \tilde{r}_s\right).$$

The law of motion of aggregate energy efficiency can be rewritten as an infinite sum:

$$\tilde{Z}_t = \delta\chi \sum_{s=0}^{t-1} (1-\delta)^{t-1-s} \tilde{z}_s.$$

A.5.3 Expressions for factor income

Labor income in sector i is given by

$$Y^L_{i,t} = w_{i,t} L_{i,t},$$

which can be log-linearized to

$$\tilde{Y}^L_{i,t} = \tilde{w}_{i,t} + \tilde{L}_{i,t}$$

Capital income in sector i is given by

$$Y^K_{i,t} = u_{i,t} r_{i,t} K_{i,t} - (p_{E_i,t} + \tau_{E_i,t}) u_{i,t} X_{i,t},$$

which can be log-linearized to

$$\tilde{Y}^K_{i,t} = \frac{1}{\chi_i}\left(\tilde{u}_{i,t} + \tilde{r}_{i,t} + \tilde{K}_{i,t} - (1-\chi_i)\left(\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t} + \tilde{u}_{i,t} + \tilde{X}_{i,t}\right)\right).$$

Re-arranging yields

$$\tilde{Y}^K_{i,t} = \tilde{u}_{i,t} + \frac{1}{\chi_i}\left(\tilde{r}_{i,t} + \tilde{K}_{i,t} - (1-\chi_i)\left(\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t} + \tilde{X}_{i,t}\right)\right).$$

Using $\tilde{u}_{i,t} = \tilde{Y}^L_{i,t} - \tilde{r}_{i,t} - \tilde{K}_{i,t}$ and $\tilde{Z}_{i,t} = \tilde{K}_{i,t} - \tilde{X}_{i,t}$ gives

$$\tilde{Y}^K_{i,t} = \tilde{Y}^L_{i,t} + \frac{1-\chi_i}{\chi_i}\left(\tilde{r}_{i,t} - \tilde{p}_{E_i,t} - \Delta\tau_{E_i,t} + \tilde{Z}_{i,t}\right).$$

Next, I aggregate across sectors. For this, let $\phi^K_i \equiv \frac{Y^K_i}{VA_i} = \frac{\alpha_i\chi_i}{1-(1-\chi_i)\alpha_i}$:

$$\begin{aligned}
\sum_i Y^K_i \tilde{Y}^K_{i,t} &= \sum_i Y^K_i \left\{\tilde{Y}^L_{i,t} + \frac{1-\chi_i}{\chi_i}\left(\tilde{r}_{i,t} - \tilde{p}_{E_i,t} - \Delta\tau_{E_i,t} + \tilde{Z}_{i,t}\right)\right\} \\
Y^K \tilde{Y}^K_t &= \sum_i VA_i \left\{\phi^K_i \tilde{Y}^L_{i,t} + \phi^E_i\left(\tilde{r}_{i,t} - \tilde{p}_{E_i,t} - \Delta\tau_{E_i,t} + \tilde{Z}_{i,t}\right)\right\}.
\end{aligned}$$

The first term on the RHS can be rewritten as

$$\begin{aligned}
\sum_i VA_i \phi^K_i \tilde{Y}^L_{i,t} &= \frac{Y^K}{Y^L}\sum_i Y^L_i \tilde{Y}^L_{i,t} + \sum_i VA_i\phi^K_i \tilde{Y}^L_{i,t} - \frac{Y^K}{Y^L}\sum_i Y^L_i \tilde{Y}^L_{i,t} \\
&= Y^K \tilde{Y}^L_t + \sum_i VA_i\left(\phi^K_i - \frac{Y^K}{Y^L}\phi^L_i\right)\tilde{Y}^L_{i,t} \\
&= Y^K \tilde{Y}^L_t + Y^K \sum_i VA_i\left(\frac{\phi^K_i}{Y^K} - \frac{\phi^L_i}{Y^L}\right)\tilde{Y}^L_{i,t} \\
&= Y^K \tilde{Y}^L_t + Y^K \sum_i \frac{VA_i}{VA}\left(\frac{\phi^K_i}{\phi^K} - \frac{\phi^L_i}{\phi^L}\right)\tilde{Y}^L_{i,t}.
\end{aligned}$$

The next term is

$$\begin{aligned}
\sum_i VA_i \phi^E_i \tilde{r}_{i,t} &= E\sum_i \frac{r_i K_i}{rK}\tilde{r}_{i,t} + \sum_i VA_i\phi^E_i\tilde{r}_{i,t} - E\sum_i \frac{r_i K_i}{rK}\tilde{r}_{i,t} \\
&= E\tilde{r}_t + \sum_i VA_i\left(\phi^E_i - \frac{E}{rK}\left(\phi^E_i + \phi^K_i\right)\right)\tilde{r}_{i,t} \\
&= E\tilde{r}_t + E\sum_i VA_i\left(\frac{\phi^E_i}{E} - \frac{\phi^E_i + \phi^K_i}{rK}\right)\tilde{r}_{i,t} \\
&= E\tilde{r}_t + E\sum_i \frac{VA_i}{VA}\left(\frac{\phi^E_i}{\phi^E} - \frac{\phi^E_i + \phi^K_i}{\phi^E + \phi^K}\right)\tilde{r}_{i,t}
\end{aligned}$$

Recalling the expression for $\tilde{Z}_t$ from above:

$$\tilde{Z}_t = \sum_i \frac{r_i K_i}{rK} \left(\tilde{Z}_{i,t} + \frac{\chi_i - \chi}{1-\chi} \tilde{X}_{i,t} \right),$$

I can write

$$\begin{aligned}
\sum_i VA_i \phi_i^E \tilde{Z}_{i,t} &= E\tilde{Z}_t + \sum_i VA_i \phi_i^E \tilde{Z}_{i,t} - E \sum_i \frac{r_i K_i}{rK} \left(\tilde{Z}_{i,t} + \frac{\chi_i - \chi}{1-\chi} \tilde{X}_{i,t} \right) \\
&= E\tilde{Z}_t + E \sum_i \frac{VA_i \phi_i^E}{VA \phi^E} \tilde{Z}_{i,t} - E \sum_i \frac{VA_i}{VA} \left[\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} \tilde{Z}_{i,t} + \left(\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} - \frac{\phi_i^E}{\phi^E} \right) \tilde{X}_{i,t} \right] \\
&= E\tilde{Z}_t + E \sum_i \frac{VA_i}{VA} \left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} \right) \left(\tilde{Z}_{i,t} + \tilde{X}_{i,t} \right).
\end{aligned}$$

Hence, I obtain

$$\sum_i VA_i \phi_i^E \tilde{r}_{i,t} + \sum_i VA_i \phi_i^E \tilde{Z}_{i,t} = E\left(\tilde{r}_t + \tilde{Z}_t \right) + E \sum_i \frac{VA_i}{VA} \left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} \right) \left(\tilde{r}_{i,t} + \tilde{K}_{i,t} \right).$$

Finally, the aggregate price of energy inclusive of the tax is:

$$\sum_i VA_i \phi_i^E \left(\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t} \right) = E\left(\tilde{p}_{E,t} + \Delta\tau_{E,t} \right).$$

Taken together, I obtain

$$\begin{aligned}
\tilde{Y}_t^K =& \tilde{Y}_t^L + \frac{1-\chi}{\chi} \left(\tilde{r}_t - \tilde{p}_{E,t} - \Delta\tau_{E,t} + \tilde{Z}_t \right) \\
&+ \sum_i \frac{VA_i}{VA} \left[\left(\frac{\phi_i^K}{\phi^K} - \frac{\phi_i^L}{\phi^L} \right) \tilde{Y}_{i,t}^L + \frac{1-\chi}{\chi} \left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} \right) \left(\tilde{r}_{i,t} + \tilde{K}_{i,t} \right) \right]
\end{aligned}$$

It is possible to rewrite the second line. For this, note that $\frac{1-\chi}{\chi} = \frac{\phi^E}{\phi^K}$ and that $\tilde{r}_{i,t} + \tilde{K}_{i,t} = \tilde{Y}^L_{i,t} - \tilde{u}_{i,t}$. Then

$$
\begin{aligned}
&\sum_i \frac{VA_i}{VA}\left[\left(\frac{\phi_i^K}{\phi^K} - \frac{\phi_i^L}{\phi^L}\right)\tilde{Y}^L_{i,t} + \frac{\phi^E}{\phi^K}\left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K}\right)\left(\tilde{Y}^L_{i,t} - \tilde{u}_{i,t}\right)\right] \\
&\quad = \sum_i \frac{VA_i}{VA}\left[\left(\frac{\phi_i^K}{\phi^K} - \frac{\phi_i^L}{\phi^L} + \frac{\phi_i^E}{\phi^K} - \frac{\phi^E}{\phi^K}\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K}\right)\tilde{Y}^L_{i,t} - \frac{\phi^E}{\phi^K}\left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K}\right)\tilde{u}_{i,t}\right] \\
&\quad = \sum_i \frac{VA_i}{VA}\left[\left(-\frac{\phi_i^L}{\phi^L} + \frac{\phi_i^E + \phi_i^K}{\phi^K}\left(1 - \frac{\phi^E}{\phi^E + \phi^K}\right)\right)\tilde{Y}^L_{i,t} - \frac{\phi^E}{\phi^K}\left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K}\right)\tilde{u}_{i,t}\right] \\
&\quad = \sum_i \frac{VA_i}{VA}\left[\left(\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} - \frac{\phi_i^L}{\phi^L}\right)\tilde{Y}^L_{i,t} - \frac{\phi^E}{\phi^K}\left(\frac{\phi_i^E}{\phi^E} - \frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K}\right)\tilde{u}_{i,t}\right] \\
&\quad = \sum_i \frac{VA_i}{VA}\left[\left(\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} - \frac{\phi_i^L}{\phi^L}\right)\tilde{Y}^L_{i,t} - \left(\frac{\phi_i^E + \phi_i^K}{\phi^E + \phi^K} - \frac{\phi_i^K}{\phi^K}\right)\tilde{u}_{i,t}\right]
\end{aligned}
$$

A.5.4 Expression for the sectoral price

From the demand equations for capital services, labor and intermediate goods, I derive the following expression for sector i's output price:

$$\tilde{p}_{i,t} = \Delta\tau_{Y_i,t} + (1-\phi_i)\left(\alpha_i \tilde{r}_{i,t} + (1-\alpha_i)\tilde{w}_{i,t}\right) + \phi_i \tilde{p}^M_{i,t}.$$

The price index for the intermediate good used by sector i is:

$$\tilde{p}^M_{i,t} = \sum_j \omega_i^j \tilde{p}_{j,t},$$

where $\omega^j_{M_i}$ denotes the steady-state expenditure share of firms in sector i on intermediates from sector j, with $\sum_j \omega^j_{M_i} = 1$. Inserting this into the expression for $\tilde{p}_{i,t}$ yields

$$\tilde{p}_{i,t} = \Delta\tau_{Y_i,t} + (1-\phi_i)\left(\alpha_i \tilde{r}_{i,t} + (1-\alpha_i)\tilde{w}_{i,t}\right) + \phi_i \sum_j \omega^j_{M_i} \tilde{p}_{j,t}.$$

Let $a_{j,i} \equiv \phi_i \omega^j_{M_i}$ denote the direct requirement of good j to produce \$1 worth of good i. In matrix notation, $\mathbf{A}$ denotes the $I \times I$ direct requirement matrix with $a_{j,i}$ corresponding to element (i,j). Then, rewriting the expression in matrix notation yields

$$\tilde{\mathbf{p}}_{\mathbf{t}} = \boldsymbol{\Delta}\tau_{\mathbf{Y},\mathbf{t}} + (\mathbf{I} - \phi)\left(\alpha \tilde{\mathbf{r}}_{\mathbf{t}} + (\mathbf{I} - \alpha)\,\tilde{\mathbf{w}}_{\mathbf{t}}\right) + \mathbf{A}\tilde{\mathbf{p}}_{\mathbf{t}},$$

where $\mathbf{I}$ is the identity matrix, ϕ and α denote diagonal matrices, and $\tilde{\mathbf{p}}_\mathbf{t}$, $\boldsymbol{\Delta}\tau_{\mathbf{Y},\mathbf{t}}$, $\tilde{\mathbf{r}}_\mathbf{t}$ and $\tilde{\mathbf{w}}_\mathbf{t}$ are vectors. Then, this can be rewritten as

$$\tilde{\mathbf{p}}_\mathbf{t} = (\mathbf{I} - \mathbf{A})^{-1} \left(\boldsymbol{\Delta}\tau_{\mathbf{Y},\mathbf{t}} + (\mathbf{I} - \phi) \left(\alpha \tilde{\mathbf{r}}_\mathbf{t} + (\mathbf{I} - \alpha) \tilde{\mathbf{w}}_\mathbf{t} \right) \right),$$

where $(\mathbf{I} - \mathbf{A})^{-1}$ is the Leontieff matrix. Let entry (i, j) of the Leontief inverse, denoted by $\lambda_{j,i}$, indicate the amount of j that is needed to produce \$1 of good i, taking both direct and indirect effects (through intermediates) into account. Then, the equation can be written as

$$\tilde{p}_{i,t} = \sum_j \lambda_{j,i} \left\{ \Delta\tau_{Y_j,t} + (1 - \phi_j) \left(\alpha_j \tilde{r}_{j,t} + (1 - \alpha_j) \tilde{w}_{j,t} \right) \right\}.$$

The rental price of capital, $\tilde{r}_{i,t}$, is influenced by the tax on energy, $\Delta\tau_{E_i,t}$. This can directly be seen from the equation describing the optimal supply of capital services:

$$\tilde{r}_{i,t} = (1 - \chi_i) \left(\tilde{p}_{E_i} + \Delta\tau_{E_i} \right) + \frac{\delta_i''}{\delta_i'} \tilde{u}_{i,t} - (1 - \chi_i) \tilde{\psi}_{i,t} + \tilde{\nu}_{i,t}.$$

In the absence of investment adjustment costs, $(1 - \chi_i)\tilde{\psi}_{i,t} = \tilde{\nu}_{i,t}$ and this simplifies to

$$\tilde{r}_{i,t} = (1 - \chi_i) \left(\tilde{p}_{E_i} + \Delta\tau_{E_i} \right) + \frac{\delta_i''}{\delta_i'} \tilde{u}_{i,t}.$$

B Steady state & log-linearized equations

B.1 Steady state

I solve the model in a neighborhood around a non-stochastic steady state with zero inflation. Real prices are nominal prices deflated by the aggregate consumption price index, $P_{C,t}$. They are denominated by lower-case letters. Utilization is normalized to 1 such that $X_i = E_i$ for all $i \in \mathcal{K}$.

Output prices I adjust A_i to ensure that all sector prices are equal to one, $p_i = 1$. That directly implies that $p_I = p_G = p_{M_i} = p_{E_i} = 1$. Notice that $p_C = 1$ by definition.

The capital-labor aggregate in sector i is defined as $F_i = A_i (K_i)^{\alpha_i} L_i^{1-\alpha_i}$ and its price index is

$$p_{F_i}^{1-\xi} = \frac{(p_i - \tau_{Y_i})^{1-\xi} - \phi p_{M_i}^{1-\xi}}{1 - \phi_i}.$$

Since prices are all equal to 1, this then implies that ω_s^i measures the share of final demand component s falling on good i, which can be read off the I-O tables.

Output per sector To solve for Y_i, it is useful to aggregate the investment goods across sectors. The model is written such that each sector potentially has its own investment good, where $\omega_{I_j}^i$ is the preference weight on good i in the production of investment good for sector j. In the data I do not observe which sector goods are used to produce investment goods for which sectors. Instead, the input-output tables provide the sectorial composition of the inputs for two separate investment goods: residential investment and non-residential investment. I assume that residential investment is used in the three housing sectors, whereas non-residential investment is used in all other sectors. I denote residential investment by I_h and non-residential investment by I_n. In addition, purchases of motor vehicles are denoted by I_d.

Then, the market clearing condition for sector i is

$$Y_i = y_C^i + y_G^i + y_{I_d}^i + y_{I_h}^i + y_{I_n}^i + \sum_{j=1}^{J} y_{M_j}^i + \sum_{j=1}^{J} y_{E_j}^i,$$

where $y_{I_h}^i$ is the quantity of good i used for residential investment and $q_{I_n}^i$ the quantity of good i used for non-residential investment. Using the demand for sector i's good, e.g. $y_G^i = \omega_G^i G \frac{p_G}{p_i}$ for government spending, this gives

$$p_i Y_i = \omega_C^i C + \omega_G^i p_G G + \sum_{s=d,n,h} \omega_{I_s}^i p_{I_s} I_s + \left(\sum_{j=1}^{J} \omega_{M_j}^i p_{M_j} M_j + \omega_{E_j}^i p_{E_j} E_j \right). \tag{B.1}$$

I normalize aggregate GDP to $GDP = 1$ by adjusting aggregate labor. Then, for instance, G is the share of government spending in GDP. I directly match the shares of government spending, G, purchases of motor vehicles I_d, residential investment I_h, and non-residential investment I_n, in GDP to their counterparts in the data. Note that ω_C^i is observed in the input-output tables.

Next, I derive expressions for C, M_j and E_j.

- M_j: From the demand equation for intermediates, I obtain:

$$M_i = \phi_i Y_i \left(\frac{p_i - \tau_{Y_i}}{p_{M_i}} \right)^{\xi} \tag{B.2}$$

- E_j: Energy is used for either production or for consumption (motor vehicles). The demand for energy used for production is proportional to a sector's output. Combining the optimal choice for the number of new machines, x_i, and their energy efficiency, z_i, yields an expression for ψ_i

$$\psi_i = \nu_i a_i z_i^{\chi_i} - p_{I_i} z_i \qquad \text{and} \qquad p_{I_i} z_i = \nu_i \chi_i a_i z_i^{\chi_i}$$
$$\rightarrow \quad \psi_i = (1-\chi_i)\nu_i a_i z_i^{\chi_i}$$

Combining the laws of motion for capital and energy, to get

$$\delta_i K_i = x_i a_i z_i^{\chi_i} \qquad \text{and} \qquad \delta_i E_i = x_i$$
$$\rightarrow \quad K_i = E_i a_i z_i^{\chi_i}$$

Inserting this in the expression for ψ_i to replace $a_i z_i^{\chi_i}$ yields

$$\frac{\psi_i}{\nu_i} = (1-\chi_i)\frac{K_i}{E_i}. \tag{B.3}$$

Combining the capital Euler equation and the machine Euler equation

$$\left(\frac{1}{\beta} - 1 + \delta_i\right)\nu_i = r_i \qquad \text{and} \qquad \left(\frac{1}{\beta} - 1 + \delta_i\right)\psi_i = p_{E_i} + \tau_{E_i}$$
$$\rightarrow \quad \frac{\psi_i}{\nu_i} = \frac{p_{E_i} + \tau_{E_i}}{r_i}.$$

Combining the two expressions for $\frac{\psi_i}{\nu_i}$ gives an expression for the share of energy in total expenditure on energized capital:

$$1 - \chi_i = \frac{\left(p_{E_i} + \tau_{E_i}\right) E_i}{r_i K_i}. \tag{B.4}$$

Note that the steady-state equivalent of the demand for capital is

$$K_i = \frac{\alpha_i \left(p_i - \tau_{Y_i}\right)\left((1-\phi_i)Y_i\right)^{\frac{1}{\xi}} F_i^{\frac{\xi-1}{\xi}}}{r_i}$$

Optimal demand for F_i requires

$$\left(\frac{(1-\phi_i)Y_i}{F_i}\right)^{\frac{1}{\xi}} = \frac{p_{F_i}}{p_i - \tau_{Y_i}}.$$

Hence, I have

$$K_i = \frac{\alpha_i}{r_i} p_{F_i} F_i. \tag{B.5}$$

This implies that $r_i K_i = \alpha_i p_{F_i} F_i = \alpha_i (1-\phi_i) p_{F_i} \left(\frac{p_{F_i}}{p_i - \tau_{Y_i}}\right)^{-\xi} Y_i$, which can be inserted into (B.4) to obtain

$$\left(p_{E_i} + \tau_{E_i}\right) E_i = (1-\chi_i)\alpha_i(1-\phi_i) p_{F_i} \left(\frac{p_i - \tau_{Y_i}}{p_{F_i}}\right)^{\xi} Y_i. \tag{B.6}$$

- C: Finally, consumption is derived from

$$C = 1 - p_{I_n} I_n - p_{I_h} I_h - p_G G. \tag{B.7}$$

Given these expressions, I can solve the system of equations given by (B.1) for all $i = 1, ..., J$ to determine sectoral output Y_i across sectors. Given Y_i, I derive M_i from (B.2), E_i from (B.6), C from (B.7) and F_i from

$$F_i = (1-\phi_i) Y_i \left(\frac{p_{F_i}}{p_i - \tau_{Y_i}}\right)^{-\xi}.$$

Depreciation rates At this stage, I can solve for the depreciaton rates for non-residential investment and residential investment using (C.3) and (C.4) (see below).

Shadow prices, capital and investment The Euler equation for the number of machines yields an expression for the shadow cost of energy, ψ_i:

$$\psi_i = \left(\frac{1}{\beta} - 1 + \delta_i\right)^{-1} \left(p_{E_i} + \tau_{E_i}\right). \tag{B.8}$$

Combining the optimal choice for the number of new machines, x_i, and their energy efficiency, z_i, yields an expression for z_i as a function of the shadow price of energy, ψ_i:

$$\psi_i = \nu_i a_i z_i^{\chi_i} - p_{I_i} z_i \qquad \text{and} \qquad p_{I_i} z_i = \nu_i \chi_i a_i z_i^{\chi_i}$$
$$\rightarrow \quad z_i = \frac{\chi_i}{1-\chi_i} \frac{\psi_i}{p_{I_i}} \tag{B.9}$$

From the optimal choice for z_i, I derive the shadow price of capital capacity, $\nu_i = p_{I_i} z_i^{1-\chi_i} \frac{1}{\chi_i a_i}$. Replacing z_i and ψ_i, and setting $a_i = (1-\chi_i)^{\chi_i - 1} \chi_i^{-\chi_i} \left(\frac{1}{\beta} - 1 + \delta_i\right)^{\chi_i - 1}$ yields:

$$\nu_i = p_{I_i}^{\chi_i} \left(p_{E_i} + \tau_{E_i}\right)^{1-\chi_i}.$$

Given ν_i, I obtain an expression for the rental rate of capital capacity, r_i, from the capital Euler equation:

$$r_i = \left(\frac{1}{\beta} - 1 + \delta_i\right) \nu_i. \tag{B.10}$$

Finally, K_i is derived from (B.5), and x_i is taken from the law of motion for the number of machines

$$x_i = \delta_i E_i.$$

Investment is then simply $I_i = x_i z_i$. Dividend payments are

$$div = \sum_{i=1}^{J} \left\{ r_i K_i - p_{I_i} I_i - \left(p_{E_i} + \tau_{E_i}\right) E_i \right\}.$$

Hours per worker and hourly wages I set hours per worker, l_j, constant across sectors and set their value such that aggregate GDP is equal to 1. Wages per hour are taken from the data. Then, the number of workers per sector, n_i, is given by

$$n_i = \frac{(1-\alpha_i)\frac{F_i}{w_i}}{\sum_j (1-\alpha_j)\frac{F_j}{w_j}}. \tag{B.11}$$

Total hours per sector are

$$L_i = (1-\alpha_i)\frac{F_i}{w_i}$$

and hours per worker are $l_i = \frac{L_i}{n_i}$.

Consumption taxes Consumption taxes are set to ensure that the government's budget is balanced:

$$\tau^C = \frac{p_G G - \tau^{carb} carb}{C}.$$

B.2 Log-linearized equilibrium conditions

1. Budget constraint (for each $i = 1, ..., J$ and each ι)

$$(1+\tau_t^C)p_{c_i,t}(\iota)c_{i,t}(\iota) = w_{i,t}(\iota)l_{i,t} + a_{k,t}(\iota)div_t$$

$$(1+\tau^C)\left(\sum_{j=1}^{J}\omega_c^j(\iota)\tilde{p}_{j,t} + \tilde{c}_{i,t}(\iota)\right) = -\Delta\tau_t^C + ls(\iota)\left(\tilde{w}_{i,t} + \tilde{l}_{i,t}\right)$$
$$+ (1-ls(\iota))\left(-\sum_i n_i\left(\int_0^1 \omega_i^l(\iota)a_k(\iota)d\iota\right)\tilde{n}_{i,t} + \widetilde{div}_t\right)$$

2. Dividends[4]

$$div_t = \sum_{i=1}^{J}\left\{u_{i,t}r_{i,t}K_{i,t} - p_{I_i,t}I_{i,t} - \left(p_{E_i,t} + \tau_{E_i,t}\right)u_{i,t}X_{i,t}\right\}$$
$$\widetilde{div}_t = \sum_{i=1}^{J}\frac{r_iK_i}{div}\left\{\tilde{r}_{i,t} + \tilde{K}_{i,t} + \chi_i\tilde{u}_{i,t} - \frac{\chi_i\delta_i}{\frac{1}{\beta}-1+\delta_i}\left(\tilde{p}_{I_i,t} + \tilde{I}_{i,t}\right) - (1-\chi_i)\left(\frac{p_{E_i}\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t}}{p_{E_i} + \tau_{E_i}} + \tilde{X}_{i,t}\right)\right\}$$

3. Labor supply (for each $i = 1, ..., J$)

$$\tilde{\pi}_{i,t}^w = \frac{(1-\theta_w\beta)(1-\theta_w)}{\theta_w}\tilde{l}_{i,t} + \beta\mathbb{E}_t\left[\tilde{\pi}_{i,t+1}^w\right]$$

4. Definition wage inflation (for each $i = 1, ..., J$)

$$\tilde{\pi}_{i,t}^w = \tilde{\pi}_t + \tilde{w}_{i,t} - \tilde{w}_{i,t-1}$$

[4] Log-linearizing yields

$$div\widetilde{div}_t = \sum_{i\in\mathcal{K}}\left\{r_iK_i\left(\tilde{u}_{i,t} + \tilde{r}_{i,t} + \tilde{K}_{i,t}\right) - p_{I_i}I_i\left(\tilde{p}_{I_i,t} + \tilde{I}_{i,t}\right) - (p_{E_i} + \tau_{E_i})E_i\left(\frac{p_{E_i}\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t}}{p_{E_i} + \tau_{E_i}} + \tilde{X}_{i,t} + \tilde{u}_{i,t}\right)\right\}.$$

In steady state, $\frac{p_{I_i}I_i}{r_iK_i} = \frac{\chi_i}{1-\chi_i}\psi_i\frac{\delta_iE_i}{r_iK_i} = \frac{\chi_i\delta_i\nu_i}{r_i} = \frac{\chi_i\delta_i}{\frac{1}{\beta}-1+\delta_i}$, as well as $\frac{(p_{E_i}+\tau_{E_i})E_i}{r_iK_i} = 1-\chi_i$. Then

$$div\widetilde{div}_t = \sum_{i\in\mathcal{K}}r_iK_i\left\{\left(\tilde{u}_{i,t} + \tilde{r}_{i,t} + \tilde{K}_{i,t}\right) - \frac{\chi_i\delta_i}{\frac{1}{\beta}-1+\delta_i}\left(\tilde{p}_{I_i,t} + \tilde{I}_{i,t}\right) - (1-\chi_i)\left(\frac{p_{E_i}\tilde{p}_{E_i,t} + \Delta\tau_{E_i,t}}{p_{E_i} + \tau_{E_i}} + \tilde{X}_{i,t} + \tilde{u}_{i,t}\right)\right\}.$$

5. Life-time value of sector i

$$V_{i,t} = \mathcal{U}(c_{i,t}) + \beta(1-\psi)\mathbb{E}_t V_{i,t+1}$$
$$\Delta V_{i,t} = \frac{\int_0^1 \omega_i^l(\iota)c(\iota)}{\int_0^1 \omega_i^l(\iota')c(\iota')d\iota'}\tilde{c}_{i,t}(\iota)d\iota + \beta(1-\psi)\Delta V_{i,t+1}$$

6. Sector choice (for each $j = 1, ..., J$)[5]

$$\sum_{j=1}^{J} \mu_{j,t} = 1 \qquad \forall i = 1$$
$$\frac{1}{\gamma}\left(\ln \mu_{i,t} - \ln \mu_{j,t}\right) = V_{i,t} - \kappa_i - V_{1,t} + \kappa_1 \qquad \forall i \neq 1$$
$$\sum_{j=1}^{J} n_j \tilde{\mu}_{j,t} = 0 \qquad \forall i = 1$$
$$\frac{1}{\gamma}\left(\tilde{\mu}_{i,t} - \tilde{\mu}_{1,t}\right) = \Delta V_{i,t} - \Delta V_{1,t} \qquad \forall i \neq 1$$

7. Law of motion for households (for each $i = 1, ..., J$)

$$\sum_{j} n_{j,t} = 1 \qquad \forall i = 1$$
$$n_{i,t} = (1-\psi)n_{i,t-1} + \psi\mu_{i,t}. \qquad \forall i \neq 1$$
$$\sum_{j} n_j \tilde{n}_{j,t} = 0 \qquad \forall i = 1$$
$$\tilde{n}_{i,t} = (1-\psi)\tilde{n}_{i,t-1} + \psi\tilde{\mu}_{i,t} \qquad \forall i \neq 1$$

[5] Comparing the share of households that choose i over j, we get

$$\frac{\mu_{i,t}}{\mu_{j,t}} = \frac{\exp\left\{V_{i,t} - \kappa_i\right\}^{\gamma}}{\exp\left\{V_{j,t} - \kappa_j\right\}^{\gamma}}.$$

Taking logs and dividing by γ:

$$\frac{1}{\gamma}\left(\ln \mu_{i,t} - \ln \mu_{j,t}\right) = V_{i,t} - \kappa_i - V_{j,t} + \kappa_j.$$

8. Law of motion for number of machines (for each $i = 1, ..., J$)

$$X_{i,t+1} = (1-\delta_{i,t})X_{i,t} + x_{i,t}\left(1 - f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right)$$
$$\tilde{X}_{i,t+1} = (1-\delta_i)\tilde{X}_{i,t} + \delta_i \tilde{x}_{i,t} - \delta_i' \tilde{u}_{i,t}$$

9. Machine Euler equation (for each $i = 1, ..., J$)[6]

$$\psi_{i,t}\Phi_t = \beta \mathbb{E}_t \left\{ \Phi_{t+1} \left[u_{i,t+1} \left(p_{E_i,t+1} + \tau_{E_i,t+1} \right) + (1 - \delta_i(u_{i,t}))\psi_{i,t+1} \right] \right\}$$
$$\tilde{\psi}_{i,t} + \tilde{\Phi}_t = \tilde{\Phi}_{t+1} + (1 - \beta(1-\delta_i)) \frac{p_{E_i}\tilde{p}_{E_i,t+1} + \Delta\tau_{E_i,t+1}}{p_{E_i} + \tau_{E_i}} + \beta(1-\delta_i)\tilde{\psi}_{i,t+1}$$

10. Law of motion for capital capacity (for each $i = 1, ..., J$)

$$K_{i,t+1} = (1-\delta_{i,t})K_{i,t} + x_{i,t} a_i z_{i,t}^{\chi_i}\left(1 - f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right)$$
$$\tilde{K}_{i,t+1} = (1-\delta_i)\tilde{K}_{i,t} + \delta_i \left(\tilde{x}_{i,t} + \chi_i \tilde{z}_{i,t}\right) - \delta_i' \tilde{u}_{i,t}$$

11. Capital Euler equation (for each $i = 1, ..., J$)

$$\nu_{i,t}\Phi_t = \beta \mathbb{E}_t \left\{ \Phi_{t+1} \left[u_{i,t+1} r_{i,t+1} + (1-\delta_{i,t})\nu_{i,t+1} \right] \right\}$$
$$\tilde{\nu}_{i,t} + \tilde{\Phi}_t = \tilde{\Phi}_{t+1} + (1 - \beta(1-\delta_i))\, \tilde{r}_{i,t+1} + \beta(1-\delta_i)\tilde{\nu}_{i,t+1}$$

12. Optimal choice of z (for each $i = 1, ..., J$)

$$\psi_{i,t} = (1-\chi_i)\nu_{i,t} a_i \chi_i z_{i,t}^{\chi_i}$$
$$\tilde{\psi}_{i,t} = \tilde{\nu}_{i,t} + \chi_i \tilde{z}_{i,t}$$

[6] Log-linearizing

$$\psi_i \Phi \left(\tilde{\psi}_{i,t} + \tilde{\Phi}_t\right) = \beta\Phi\left(\tilde{\Phi}_{t+1} + p_{E_i}\tilde{p}_{E_i,t+1} + \Delta\tau_{E_i,t+1} + (1-\delta_i)\tilde{\psi}_{i,t+1} + \left(p_{E_i} + \tau_{E_i} - \delta_i'\psi_i\right)\tilde{u}_{i,t}\right).$$

Notice that $\delta_i' = \frac{r_i}{\nu_i}$ and $\frac{\nu_i}{r_i} = \frac{p_{E_i} + \tau_{E_i}}{\psi_i}$ so that the $\tilde{u}_{i,t}$ term drops.

13. Optimal choice of x (for each $i = 1, ..., J$)[7]

$$p_{I_i,t} = \chi_i \nu_{i,t} a_i z_{i,t}^{\chi_i - 1} \left(1 - f_{i,t} - \frac{I_{i,t}}{I_{i,t-1}} f'_{i,t}\right) + \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} x_{i,t+1} \chi_i \nu_{i,t+1} a_i z_{t+1}^{\chi_i} \frac{I_{i,t+1}}{I_{i,t}^2} f'_{i,t+1}\right)$$
$$\tilde{p}_{I_i,t} = \tilde{\nu}_{i,t} - (1 - \chi_i) \tilde{z}_{i,t} - f'' \left[(1+\beta) \tilde{I}_{i,t} - \tilde{I}_{i,t-1} - \beta \tilde{I}_{i,t+1}\right]$$

14. Definition of investment in sector i (for each $i = 1, ..., J$)

$$I_{i,t} = x_{i,t} z_{i,t}$$
$$\tilde{I}_{i,t} = \tilde{x}_{i,t} + \tilde{z}_{i,t}$$

15. Definition of energy (for each $i = 1, ..., J$)

$$E_{i,t} = u_{i,t} X_{i,t}$$
$$\tilde{E}_{i,t} = \tilde{u}_{i,t} + \tilde{X}_{i,t}$$

16. Optimal utilization (for each $i = 1, ..., J$)[8]

$$r_{i,t} = (p_{E_i,t} + \tau_{E_i,t}) \frac{X_{i,t}}{K_{i,t}} + \delta'_i(u_{i,t}) \left(-\psi_{i,t} \frac{X_{i,t}}{K_{i,t}} + \nu_{i,t}\right)$$
$$\tilde{r}_{i,t} = (1 - \chi_i) \frac{p_{E_i} \tilde{p}_{E_i} + \Delta \tau_{E_i}}{p_{E_i} + \tau_{E_i}} + \frac{\delta''_i}{r_i} \chi_i \tilde{u}_{i,t} - (1 - \chi_i) \tilde{\psi}_{i,t} + \tilde{\nu}_{i,t}.$$

[7] Log-linearizing yields

$$p_I \tilde{p}_{I_i,t} = \chi_i \nu_i a_i z_i^{\chi_i - 1} \left(\tilde{\nu}_{i,t} + (\chi_i - 1) \tilde{z}_{i,t}\right) - f'' p_I \left[(1+\beta) \tilde{I}_{i,t} - \tilde{I}_{i,t-1} - \beta \tilde{I}_{i,t+1}\right]$$

Notice that the optimal choice of z implies $p_I = \chi_i \nu_i a_i z_i^{\chi_i - 1}$:

$$\tilde{p}_{I_i,t} = \tilde{\nu}_{i,t} - (1 - \chi_i) \tilde{z}_{i,t} - f'' \left[(1+\beta) \tilde{x}_{i,t} - \tilde{x}_{i,t-1} - \beta \tilde{x}_{i,t+1}\right]$$

[8] Log-linearizing yields

$$r_i \tilde{r}_{i,t} = (p_{E_i} \tilde{p}_{E_i} + \Delta \tau_{E_i}) \frac{X_i}{K_i} + (p_{E_i} + \tau_{E_i}) \frac{X_i}{K_i} \left(\tilde{X}_{i,t} - \tilde{K}_{i,t}\right) + \delta''_i (-\psi_i \frac{X_i}{K_i} + \nu_i) \tilde{u}_{i,t} - \delta'_i \psi_i \frac{X_i}{K_i} \left(\tilde{\psi}_{i,t} + \tilde{X}_{i,t} - \tilde{K}_{i,t}\right) + \delta'_i \nu_i \tilde{\nu}_{i,t}.$$

Using $(p_{E_i} + \tau_{E_i}) \frac{X_i}{r_i K_i} = 1 - \chi_i$, $-\psi_i \frac{X_i}{K_i} = -(1 - \chi_i) \nu_i$ and $\delta'_i = \frac{r_i}{\nu_i}$, this becomes

$$\tilde{r}_{i,t} = (1 - \chi_i) \frac{p_{E_i} \tilde{p}_{E_i} + \Delta \tau_{E_i}}{p_{E_i} + \tau_{E_i}} + (1 - \chi_i) \left(\tilde{X}_{i,t} - \tilde{K}_{i,t}\right) + \frac{\delta''_i}{r_i} \chi_i \tilde{u}_{i,t} - (1 - \chi_i) \left(\tilde{\psi}_{i,t} + \tilde{X}_{i,t} - \tilde{K}_{i,t}\right) + \tilde{\nu}_{i,t}.$$

17. Consumption Euler equation

$$\Phi_t = \beta(1+i_t)\mathbb{E}_t\left[\Phi_{t+1}\frac{P_{c,t}}{P_{c,t+1}}\right]$$
$$0 = -\tilde{\Phi}_t + \tilde{\Phi}_{t+1} + \Delta i_t - \tilde{\pi}_{t+1}$$

18. Taylor rule

$$\Delta i_t = \varphi_i \Delta i_{t-1} + (1-\varphi_i)\phi_\pi \tilde{\pi}_t$$

19. Production of sector goods (for each $i = 1, ..., J$)

$$Y_{i,t} = \left\{(1-\phi_i)^{\frac{1}{\xi}}\left[A_i\left(u_{i,t}K_{i,t}\right)^{\alpha_i} L_{i,t}^{1-\alpha_i}\right]^{\frac{\xi-1}{\xi}} + \phi_i^{\frac{1}{\xi}} M_{i,t}^{\frac{\xi-1}{\xi}}\right\}^{\frac{\xi}{\xi-1}}$$
$$\widetilde{Y}_{i,t} = (1-\phi_i)\left[\alpha_i\left(\tilde{u}_{i,t}+\tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right] + \phi_i\tilde{M}_{i,t}$$

20. Demand for capital (for each $i = 1, ..., J$)

$$r_{i,t}u_{i,t}K_{i,t} = \left(p_{i,t} - \tau_{Y_i,t}\right)\alpha_i\left(\frac{(1-\phi_i)Y_{i,t}}{A_i\left(K_{i,t}\right)^{\alpha_i} L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_i\left(u_{i,t}K_{i,t}\right)^{\alpha_i} L_{i,t}^{1-\alpha_i}$$
$$\tilde{r}_{i,t} + \tilde{u}_{i,t} + \tilde{K}_{i,t} = \frac{p_i\tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\tilde{Y}_{i,t} + \frac{\xi-1}{\xi}\left(\alpha_i\left(\tilde{u}_{i,t}+\tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right)$$

21. Demand for labor (for each $i = 1, ..., J$)

$$w_{i,t}L_{i,t} = \left(p_{i,t} - \tau_{i,t}\right)(1-\alpha_i)\left(\frac{(1-\phi_i)Y_{i,t}}{A_i\left(u_{i,t}K_{i,t}\right)^{\alpha_i} L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_i u_{i,t} K_{i,t}^{\alpha_i} L_{i,t}^{1-\alpha_i}$$
$$\tilde{w}_{i,t} + \tilde{L}_{i,t} = \frac{p_i\tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\tilde{Y}_{i,t} + \frac{\xi-1}{\xi}\left(\alpha_i\left(\tilde{u}_{i,t}+\tilde{K}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right)$$

22. Demand for intermediates (for each $i = 1, ..., J$)

$$p_{M_i,t} = \left(p_{i,t} - \tau_{i,t}\right)\left(\frac{\phi_i Y_{i,t}}{M_{i,t}}\right)^{\frac{1}{\xi}}$$
$$\tilde{p}_{M_i,t} = \frac{p_i\tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\left(\tilde{Y}_{i,t} - \tilde{M}_{i,t}\right)$$

23. Market clearing sector goods (for each $i = 1, ..., J$)

$$\sum_s y^i_{s,t} = Y_{i,t}$$
$$\sum_s \frac{y^i_s}{Y_i} \tilde{y}^i_{s,t} = \tilde{Y}_{i,t}$$

24. Production of composite goods (for $s = I_j, E_j, M_j$ and G)

$$s_t = \left(\sum_{i=1}^{J} \left(\omega^i_s\right)^{\frac{1}{\sigma}} \left(y^i_{s,t}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$
$$\tilde{s}_t = \sum_{i=1}^{J} \omega^i_s \tilde{y}^i_{s,t}$$

25. CPI as numeraire

$$1 = \sum_i n_{i,t} \int_0^1 \omega^l_i(\iota) c(\iota) p_{c_i,t}(\iota) d\iota$$
$$0 = \sum_i n_i \int_0^1 \omega^l_i \left[\sum_j \omega^j_c(\iota) \left(\frac{\partial \omega^j_c(c(\iota))}{\partial c(\iota)} \frac{c(\iota)}{\omega^j_c(\iota)} c(\iota) \tilde{c}_{i,t}(\iota) \right) \right] + (1-\sigma) y^j_c \tilde{p}_{j,t}$$

26. Demand for sector goods (for each $s = I_j, M_j, E_j, G$ and c)

- For c (for each $i = 1, ..., J$)

$$y^j_{c_i,t}(\iota) = \omega^j_c(c_{i,t}(\iota)) \left(\frac{p_{j,t}}{p_{c_i,t}(\iota)} \right)^{-\sigma} c_{i,t}(\iota)$$
$$\begin{aligned} \frac{y^j_c}{C} \tilde{y}^j_{c,t} = & \sum_i n_i \int_0^1 \left(\omega^l_i(\iota) \omega^j_c(\iota) \frac{c(\iota)}{C} \left(1 + \frac{\partial \omega^j_c(c(\iota))}{\partial c(\iota)} \frac{c(\iota)}{\omega^j_c(\iota)} \right) \times \tilde{c}_{i,t}(\iota) \right) d\iota \\ & - \sigma \frac{y^j_c}{C} \tilde{p}_{j,t} \\ & + \sigma \sum_{k=1}^{J} \left[\int_0^1 \left(\sum_i n_i \omega^l_i(\iota) \right) \omega^j_c(\iota) \frac{c(\iota)}{C} \omega^k_c(\iota) d\iota \right] \times \tilde{p}_{k,t} \\ & + \sum_i n_i \left(\int_0^1 \omega^l_i(\iota) \omega^j_c(\iota) \frac{c(\iota)}{C} \right) d\iota \times \tilde{n}_{i,t}. \end{aligned}$$

- For remaining ($s = I_j, E_j, M_j, G$) (for each $i = 1, ..., J$)

$$p_{i,t}^{\sigma} y_{s,t}^{i} = \omega_s^i p_{s,t}^{\sigma} s_t$$
$$\sigma \tilde{p}_{i,t} + \tilde{y}_{s,t}^{i} = \sigma \tilde{p}_{s,t} + \tilde{s}_t$$

27. Market clearing labor (for each i)

$$L_{i,t} = n_{i,t} l_{i,t}$$
$$\tilde{L}_{i,t} = \tilde{n}_{i,t} + \tilde{l}_{i,t}$$

28. Carbon tax (for all $i = 1, ..., J$)

$$\tau_{i,t}^{E} = \tau_t^{carb} \Psi_{E_i} \qquad \tau_{Y_i,t} = \tau_t^{carb} \Psi_{Y_i}$$
$$\Delta \tau_{E_i,t} = \Delta \tau_t^{carb} \Psi_{E_i} \qquad \Delta \tau_{Y_i,t} = \Delta \tau_t^{carb} \Psi_{Y_i}$$

29. Carbon emissions

$$carb_t = \sum_{j=1}^{J} \left(\Psi_{E_j} E_{j,t} + \Psi_{Y_j} Y_{j,t} \right)$$
$$\Delta carb_t = \sum_{j=1}^{J} \left(\Psi_{E_j} E_j \tilde{E}_{j,t} + \Psi_{Y_j} Y_j \tilde{Y}_{j,t} \right)$$

30. Average marginal utility of owners of capital fund

$$\Phi_t = \frac{1 + \tau^C}{1 + \tau_t^C} \int_0^1 \sum_i a_{k,t}(\iota) \frac{c_i(\iota)}{p_{c_i,t}(\iota) c_{i,t}(\iota)} n_{i,t}(\iota) d\iota$$
$$\tilde{\Phi}_t = -\frac{\Delta \tau_t^C}{1 + \tau^C} - \int_0^1 \sum_i a_k(\iota) n_i \omega_i^l(\iota) \times \left(\tilde{p}_{c_i,t} + \tilde{c}_{i,t}(\iota) \right) d\iota$$

C Calibration

C.1 Classification of goods

I classify sectors in the BEA I-O tables as follows:

Energy products As in Ingwersen et al. (2022), I classify the following sectors as energy-producing sectors (BEA industry codes in parentheses):

- Oil and gas extraction (211000)
- Coal mining (212100)
- Electric power generation (221100)
- Federal electric utilities (S00101)
- State and local government electric utilities (S00202)
- Natural gas distribution (221200)
- Petroleum refineries (324110)

The set of these sectors is called $\mathcal{E}$. All other products are classified as non-energy products.

Motor vehicle services Carbon emissions by motor vehicles account for a substantial share in U.S. total emissions. Emissions by motor vehicles owned by firms or government enterprises are assigned to the firm's output in the U.S. Environmentally-Extended Input-Output tables and are therefore included in the firms' carbon intensity. However, a large share of motor vehicles are owned by private households and these emissions are not directly included in the U.S. Environmentally-Extended Input-Output tables.

To account for these emissions in the framework of my model I create an additional, artificial sector. Instead of assigning motor vehicle purchases to final consumption, I consider these purchases as investment by a sector called 'motor vehicle services'. This sector employs no labor and requires no non-energy intermediates. Sales by the following sectors to private households count towards investment to this sector:

- Transportation equipment manufacturing (336)

- Motor vehicle and motor vehicle parts and supplies merchant wholesalers (4231)
- Motor vehicle and parts dealers (441)

Any purchases of gasoline (sector Petroleum Refineries, 324110) by private households are assigned to the production of motor vehicle services. See Section C.7 for more details on how this artificial sector is set up.

Housing Housing by private households is composed of

- Owner-occupied housing (531HSO)
- Tenant-occupied housing (531HST)
- Other real estate (531ORE)

Investment into real estate corresponds to residential investment in the BEA I-O tables. The set of these sectors is called $\mathcal{H}$. The corresponding energy goods are coal mining (212100), natural gas distribution (221200) and electric power generation (221100).

C.2 Shares used for calibration

C.2.1 Investment and government purchases

The following shares used for the calibration of the initial steady state are calculated as averages over 2000 - 2019 using the National Input and Production Account table 1.5.5. 'Gross Domestic Product, Expanded Detail' in current USD:

- Government consumption in GDP $\left(\frac{G}{GDP}\right)$: (Federal consumption expenditure (ll.55 + 58) and state-and-local consumption expenditure (l.61)) over GDP (l.1). 15.0%
- Non-residential investment in GDP $\left(\frac{I_n}{GDP}\right)$: (Non-residential fixed investment (l.28), change in private inventories (l.42), federal gross investment (ll.56 + 59) and state-and-local gross investment (l.62)) over GDP (l.1). 17.1%
- Residential investment in GDP $\left(\frac{I_h}{GDP}\right)$: Residential fixed investment (l.41) over GDP (l.1). 4.1%

Purchases of motor vehicles, $\left(\frac{I_d}{GDP}\right)$, are taken from the use tables after redefinitions at producer prices (summary level), averaged over 2000 - 2019. All sales to private households of the following sectors count towards purchases of motor vehicles: Motor vehicles, bodies and trailers, and parts (3361MV), Other transportation equipment (3364OT), and Motor vehicle and parts dealers (441). The summary tables do not provide any values for sector 423100. That sector accounts for about 5.4% of all investment into transportation in the detailed benchmark tables 2007 and 2012. I therefore multiply the average value found in the summary tables by 1.054. The final value for the investment share in GDP is $\frac{I_d}{GDP} = 2.3\%$.

C.2.2 Labor shares, energy shares and shares of intermediates

In this section I derive the production parameters, α_i, ϕ_i and χ_i. For the derivation of α_i, ϕ_i and χ_i for motor vehicle services, which are not part of the I-O tables, see Section C.7.

Denote by a_i^j element i, j in the requirement matrix $\mathbf{A}$ and indicates the direct requirement of product j to produce \$1 worth of product i. For each sector, I calculate the following shares from the input-output tables, after redefinition, producer prices, detailled level:[9]

- Use of non-energy intermediates in sector i over sector i's output:

$$\text{ms}_i = \sum_{j \notin \mathcal{E}} a_i^j$$

- Use of non-energy intermediates in sector i over sector i's output:

$$\text{es}_i = \sum_{j \in \mathcal{E}} a_i^j$$

- Labor share:

$$\text{ls}_i = \frac{\text{Compensation of employees}_i}{\text{Value added}_i}$$

Next, I solve for the parameters α_i, χ_i and ϕ_i as a function of these shares in the initial steady state, using the normalization that all prices are equal to 1 and taxes are zero.

The parameter ϕ_i is equal to the share of intermediates, ms_i (see (B.2)):

$$\phi_i = \frac{M_i}{Y_i}$$

[9] *'IOUse_After_Redefinitions_PRO_DET.xlsx'*

To derive an expression for α_i, note that value added in sector i is

$$VA_i = A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i} - E_i = A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}\left(1 - \frac{E_i}{Y_i}\frac{Y_i}{A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}}\right).$$

Since ϕ_i is the share of intermediates, it follows that $A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i} = (1-\phi_i)Y_i$. Also, the share of energy in output is $es_i \equiv \frac{E_i}{Y_i}$. Hence, value added can be rewritten as

$$VA_i = A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}\left(1 - \frac{es_i}{1-\phi_i}\right).$$

Demand for labor in steady state is

$$\frac{w_iL_i}{A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}} = (1-\alpha_i).$$

The labor share is

$$ls_i \equiv \frac{w_iL_i}{VA_i} = \frac{w_iL_i}{A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}}\frac{A_i(u_iK_i)^{\alpha_i}L_i^{1-\alpha_i}}{VA_i} = \frac{1-\alpha_i}{1-\frac{es_i}{1-\phi_i}}.$$

Solving for α_i yields

$$\alpha_i = 1 - ls_i\left(1 - \frac{es_i}{1-\phi_i}\right).$$

Finally, to get an expression for χ_i, note that from equation (B.6), I have

$$es_i \equiv \frac{E_i}{Y_i} = (1-\chi_i)\alpha_i(1-\phi_i).$$

Solving for χ_i yields

$$\chi_i = 1 - \frac{es_i}{\alpha_i(1-\phi)}.$$

The production parameters α_i, χ_i and ϕ_i are set to match the labor income shares and the expenditure shares on energy and intermediate goods in each sector in the 2012 I-O tables. Aggregating the share of value added distributed to employees across sectors yields a labor share that is somewhat too low compared to shares reported in the literature. This is because the input-output tables subsume the compensation for labor provided by the self-employed into gross operating surplus. To correct this, I scale the sector values for $1-\alpha_i$ to match the aggregate labor share of 0.63 reported by Karabarbounis and Neiman (2013).

C.3 Carbon intensity by industry sector

The US Environmentally-Extended Input-Output (USEEIO) tables are published by the US Environment Protection Agency (EPA). They "meld data on economic transactions between about 400 industry sectors with envrionmental data for these sectors covering land, water, energy and mineral usage and emissions of greenhouse gases, criteria air pollutants, nutrients and toxics, to build a life-cycle model of close to 400 U.S. goods and services." (Yang et al., 2017). Collected data represent the year 2013 and is therefore best used in junction with the 2012 BEA input-output table. For the purpose of my study I focus on a small subset of data that form the USEEIO tables: greenhouse gas emissions related to CO_2. The underlying source is the US greenhouse gas inventory for which the EPA collects data from other U.S. government agencies, academic institutions, industry associations and environmental organizations.[10]

The environmental matrix published in the USEEIO tables ('USEEIOv1.1_Matrices', tab B) includes *direct* carbon emissions (in kg) for the production of $1 of each commodity that can be found in the BEA input-output tables. The USEEIO tables distinguish between carbon emitted through the use of fossil fuels and carbon emitted through the production process. Since my model is cast in terms of industries rather than commodities, I pre-multiply the reported emission values by the industry-by-commodity make table (that indicates which commodities account for what share of each industry's output) to get emissions per $1 of industry output. The resulting values are displayed in Table 1 in the main text.

C.4 Sector wage rates

I calculate hourly wages from the CPS public data files via IPUMS (Flood et al., 2021), coverting 2003 - 2019. For each industry, I calculate total wages received divided by total hours worked. Depending on the employment contract, I use usually weekly earnings and usual hours worked per week, or hourly wages if paid by the hour.

C.5 Utilization cost

I assume that the depreciation rate in sector i depends on the utilization rate in sector i, $\delta_{i,t} = \delta_i(u_{i,t})$. For the log-linearized solution, I need to specify the first and second derivatives of these functions, evaluated at their steady state, δ_i' and δ_i''.

[10] `http://www3.epa.gov/climatechange/ghgemissions/usinventoryreport.html`

The steady state determines δ_i': Optimal utilization requires

$$r_i = (p_{E_i} + \tau_{E_i})\frac{X_i}{K_i} + \delta_i'\left(-\psi\frac{X_i}{K_i} + \nu_i\right).$$

In steady state, $(p_{E_i} + \tau_{E_i})\frac{X_i}{K_i} = (1-\chi_i)r_i$, $-\psi_i\frac{X_i}{K_i} = -(1-\chi_i)\nu_i$ and $\nu_i = 1$. Hence,

$$r_i = (1-\chi_i)r_i + \delta_i'\chi_i$$

and $\delta_i' = r_i$.

Given δ_i', δ_i'' determines the response of utilization, and hence energy consumption, to a change in the price of energy, $\tilde{p}_{E_i,t} - \Delta\tau_{E_i,t}$. To calibrate δ_i'', I first impose that this second derivative is the same across all sectors, $\delta'' = \delta_i''$. Then, I compute the price elasticity of energy demand in the model and compare it to estimates in the literature. For this, I focus on the short-run elasticity because this short-run elasticity is mainly determined by variation in utilization rather than purchases and use of new machines.

The meta-analysis by Labandeira et al. (2017) finds a short-run price elasticity of energy demand of -0.2 to -0.15. I adjust δ'' in the model so that my model reproduces this figure. In particular, I run the following regression using the simulated data from the model:

$$\tilde{E}_{i,1-4} = \alpha + \beta\left(\tilde{p}_{E_i,1-4} + \Delta\tau_{E_i,1-4}\right) + \varepsilon_i,$$

where $\tilde{E}_{i,1-4}$ is energy consumption in sector i, expressed in log deviations from steady state and averaged over the first 4 quarters, and $\tilde{p}_{E_i,1-4}+\Delta\tau_{E_i,1-4}$ is the price of energy (including the energy tax), also expressed in log deviations from steady state and averaged over the first 4 quarters. I choose a horizon of 4 quarters because Labandeira et al. (2017) define the short-run price elasticity as the elasticity within a year. Choosing $\delta'' = \frac{1}{30}$ yields a slope coefficient of $\hat{\beta} = -0.175$.

C.6 Depreciation rates

I assume three different types of depreciation rates, one for non-residential investment (δ_n), one for residential investment (δ_h), and one for motor vehicles (δ_d).

I solve for δ_n and δ_h to match the shares of non-residential and residential investment in GDP. Inserting the law of motion for energy, $x_i = \delta_i E_i$ into the definition $I_i = z_i x_i$ and solving for z_i

yields

$$z_i = \frac{I_i}{\delta_i E_i}.$$

Combining this with the expression for z_i as a function of the shadow price of energy, $z_i = \frac{\chi_i}{1-\chi_i}\frac{\psi_i}{p_{I_i}}$, (B.9), yields (noting that $p_{I_i} = 1$)

$$p_{I_i} I_i = \frac{\chi_i}{1-\chi_i}\psi_i \delta_i E_i. \tag{C.1}$$

Using the expression for the shadow price of energy, $\frac{\psi_i}{\nu_i} = (1-\chi_i)\frac{K_i}{E_i}$, (B.3), to remove $\psi_i E_i$

$$p_{I_i} I_i = \delta_i \chi_i K_i \nu_i.$$

Since $K_i = \frac{\alpha_i}{r_i} Y_i$ and using (B.10) to remove $\frac{\nu_i}{r_i} = \left(\frac{1}{\beta} - 1 + \delta_i\right)^{-1}$, I get

$$p_{I_i} I_i = \frac{\delta_i}{\frac{1}{\beta} - 1 + \delta_i}\chi_i \alpha_i Y_i.$$

Letting $\mathcal{H}$ denote the set of sectors that rely on residential investment and $\mathcal{N}$ the set of sectors that rely on non-residential investment, then non-residential investment is given by

$$p_{I_n} \equiv \sum_{i\in\mathcal{N}} p_{I_n} I_i = \frac{\delta_n}{\frac{1}{\beta} - 1 + \delta_n}\sum_{i\in\mathcal{N}}\chi_i \alpha_i Y_i, \tag{C.2}$$

Solving for δ_n yields

$$\delta_n = \frac{1-\beta}{\beta}\left[\frac{\sum_{i\in\mathcal{N}}\chi_i\alpha_i Y_i}{p_{I_n} I_n} - 1\right]^{-1}. \tag{C.3}$$

Similarly, the depreciaton rate for housing is derived from

$$\delta_h = \frac{1-\beta}{\beta}\left[\frac{\sum_{i\in\mathcal{H}}\chi_i\alpha_i Y_i}{p_{I_h} I_h} - 1\right]^{-1}. \tag{C.4}$$

For motor vehicles I do not observe any rental value in the data. I therefore use the annual depreciation rates of 16% reported by Fraumeni (1997).

C.7 Sector 405: Services of motor vehicles

To account for carbon emissions stemming from motor vehicles, I add a fictional additional sector, called motor vehicle services, to the input-output tables.

In this setup, households no longer directly purchase motor vehicles and gasoline, but rent motor vehicle services from firms. In the input-output tables, I therefore remove motor vehicle purchases by households, I_d, and consumption of gasoline by households, E_d, from households' consumption and assign these values to investment and energy consumption for sector 405.

The main challenge is that I do not observe the rental value of motor vehicle services in the data. But this value can be derived from investment and energy consumption in that sector together with an assumption on the depreciation rate. I next derive the rental value of motor vehicle services, Y_d, together with the energy share, χ_d.

Production of motor vehicle services only requires capital (+ gasoline), i.e. $\alpha_d = 1$ and $\phi_d = 0$. To calculate χ_d, start from (C.1)

$$I_d = \frac{\chi_d}{1-\chi_d}\psi_d \delta_d E_d.$$

Use the energy Euler equation (with $p_{E_d} = 1$ and $\tau_{E_d} = 0$), $\psi_d = \left(\frac{1}{\beta} - 1 + \delta_d\right)^{-1}$ to replace ψ_d and solve for χ_d:

$$\chi_d = 1 - \frac{\delta E_d}{\delta E_d + \left(\frac{1}{\beta} - 1 + \delta_d\right) I_d},$$

which gives χ_d as a function of households' gasoline consumption E_d and households' purchases of motor vehicles, I_d, together with their depreciation rate, δ_d.

In this setup, households' consumption of motor vehicle services is not equal to households' purchases of motor vehicles and gasoline because motor vehicles are durables. Instead, households' consumption of motor vehicle services is

$$Y_d = r_d K_d = \frac{E_d}{1-\chi_d} = \frac{\frac{1}{\beta} - 1 + \delta_d}{\delta_d} I_d + E_d.$$

C.8 Basket weights, industry distribution and factor income shares across households

To capture household heterogeneity in expenditure shares and income, I make use of three datasets: the CPS, the BLS' Consumer Expenditure Survey (CEX, U.S. Department of Labor, 2021), and the distributional national accounts (DINA, Piketty et al., 2018).

In all surveys, I follow Heathcote et al. (2017) and restrict the sample to the active working-age population, defined as households that earn at least \$15'000 for a two-adult household, which corresponds to one person working full-time at minimum wage, and \$11'250 for a single-adult household, i.e. 30 hours per week at minimum wage. For both the CEX and DINA, I classify households into income percentiles based on total household income adjusted for family size.[11] For the CPS, I classify workers into income percentiles based on their labor income.

C.8.1 Consumer Expenditure Survey (CEX)

I use the CEX data to calculate spending across consumption goods by income group. The BLS collects data on households' spending patterns for around 650 categories (UCCs) through two surveys in the CEX: quarterly interviews and weekly diaries. To gain a complete picture of expenditure and income, the two surveys need to be integrated. While the two surveys complement each other, they use independent samples and cover mostly different products. In some cases, expenditure on certain products are recorded in both surveys and I then use the source selection file provided by the BLS that indicates which survey is used for which UCC.

Since samples across the two surveys differ I cannot directly merge the two surveys. Instead, for each survey, I first split households into 100 bins of household income before taxes (variable FINCBTXM in the interview survey and variable FINCBEFX in the diary survey) adjusted for household size. I calculate, for each income bin, each survey and each product, expenditure as a share of household income. I then combine the two surveys to get, for each bin, expenditures for every product, expressed as a share of income. I re-scale expenditure shares for each bin to ensure that they sum up to 1. I apply this procedure to the surveys from 2004 - 2019 and then take a simple average across years.

I concord the UCCs used in the CEX to personal consumption expenditure (PCE) categories provided by the BLS. Based on the resulting matrix of expenditure shares, $\omega_c^j(\iota)$, for each j and

[11] To calculate households' size I weigh children under the age of 15 by 0.5 and babys under the age of 2 by 0.25. In the DINA, there is no distinction between babys and children and I weigh all children by 0.5.

Table A1: ELASTICITY OF EXPENDITURE SHARE TO CONSUMPTION

Top positive		Top negative	
PCE category	$\hat{\beta}$	PCE category	$\hat{\beta}$
Foundations and grantmaking and giving services to households	2.27	Group housing	−1.86
Domestic services	2.21	Tobacco	−1.27
Water transportation	1.98	Rental of tenant-occupied nonfarm housing	−0.94
Housing at schools	1.79	Lubricants and fluids	−0.90
Social advocacy and civic and social organizations	1.71	Pork	−0.82
Repair and hire of footwear	1.43	Eggs	−0.77
Hotels and motels	1.36	Fresh milk	−0.73
Spectator sports	1.35	Mineral waters, soft drinks, and vegetable juices	−0.70
Membership clubs and participant sports centers	1.33	Poultry	−0.70
Railway transportation	1.31	Beef and veal	−0.69

Notes: Table displays the top 10 PCE categories with the lowest and the highest elasticities of the consumption expenditure share with respect to consumption. The elasticity estimates correspond to $\hat{\beta}$ from regression (C.5).

percentile ι I estimate the elasticity of expenditure shares to consumption, $\frac{\partial \omega_c^j(c(\iota))}{\partial c(\iota)} \frac{c(\iota)}{\omega_c^j(\iota)}$, using the following regression:

$$\ln \omega_c^j(\iota) = \alpha_j + \beta_j \ln c(\iota) + \varepsilon_j(\iota). \tag{C.5}$$

A positive estimate for β_j indicates that good j constitutes a larger share of consumption expenditure for high-consumption households than low-consumption households. More precisely, as aggregate consumption increases by 1%, the share spent on good j increases by $\beta_j\%$ (not: percentage points). Table A1 displays the PCE category name and elasticity estimates for the ten highest and lowest estimates. Richer households spend a larger fraction on philantropy, travelling and entertainment, whereas poorer households spend more on rent and food.

In a final step, I concord the PCE categories to the industry classification used by the BEA. For this, I use a BEA bridge table that indicates which commodities are used for each PCE category. In some cases, the BLS uses a more aggregate PCE classification than the BEA. In that case, I rely on a classification file provided by the BEA that contains a mapping between PCE categories used by the BLS and PCE categories used by the BEA.

Re-adjusting the basket weights to ensure consistency In the model, summing up the expenditures on good j for each income percentile should yield the same value as the aggregate consumption expenditure on good j, $\int_0^1 \omega_c^j(\iota)c(\iota)d\iota = y_c^j$. Since data on aggregate consumption expenditure on good j is from the BEA and the expenditure shares by income percentile are derived from BLS data, there are discrepancies, in some cases even quite large, mainly due to different coverages between the CEX and the BEA personal consumption expenditure data (Passero et al., 2014).

I adjust the basket weights by iterating over the following expressions until convergence:

$$\omega_c^j(\iota) = \omega_c^j(\iota)\frac{y_c^j}{\int_0^1 \omega_c^j(\iota)c(\iota)d\iota}$$
$$\omega_c^j(\iota) = \omega_c^j(\iota)\left(\sum_j \omega_c^j(\iota)\right)^{-1}.$$

On the RHS I start with the basket weights, $\omega_c^j(\iota)$, as measured in the data. The first equation adjusts those basket weights to ensure that aggregating across income percentiles yields consumption values that correspond to those found in the input-output tables for each commodity j. I then use these adjusted basket weights and re-adjust them to ensure that they sum up to one for each income percentile. Starting from these re-adjusted basket weights, I go back to the first equation and so on. After about 50 iterations, this converges to basket weights that fully satisfy the second add-up constraint and almost fully satisfy the first add-up constraint.

Note that two commodities j have positive consumption values in the BEA data, but not in the CEX data ('Scientific research and development services' and 'Funds, trusts, and other financial vehicles'). For these categories, before applying the procedure just described, I first assign them consumption elasticities of expenditure shares based on similar categories ('Management consulting services' and 'Direct life insurance carriers'). Then, I populate $\omega_c^j(\iota)$ to be consistent with this elasticity and the aggregate value of consumption on that category, y_c^j.

Re-adjusting elasticities to ensure consistencies The income (or: consumption) elasticities in Table A1 are elasticities of expenditure *shares* to consumption. In theory, they should average out to zero: As a household becomes richer, expenditure shares of certain products increase whereas expenditure shares of other products decrease. In practice, regression (C.5) is estimated separately on each product and as a result, this does not guarantee that elasticities average out. Therefore, I adjust the final estimates as follows:

$$\beta_j = \beta_j - \sum_j \beta_j \omega_c^j,$$

where ω_c^j is the share households spend on product j.

C.8.2 Distributional National Accounts (DINA)

Piketty et al. (2018) combine tax, survey and national accounts data to estimate the distribution of national income in the United States. I access their micro-files that contain information on income and its components for a synthetic set of individuals.

For each income percentile I calculate pre-tax factor incomes and express them as shares of total pre-tax income. Pre-tax factor income for labor and capital are the "sum of all the income flows accruing to the individual owners of the factors of production, labor and capital, before taking into account the operation of pensions and the tax and transfer system" (Piketty et al., 2018). By construction, adding up the income across individuals in these micro-files adds up to national income, which is GDP minus capital depreciaton plus net income received from abroad.

Capital income (`pkinc`) in these micro-files therefore reflects capital income net of capital depreciation and maps in steady state into dividends, which are capital income net of investment, in the model. To be more precise, capital factor income includes: housing asset income, business income, equity asset income, pension and insurance asset income, as well as sales taxes and subsidies allocated to capital. I exclude any interest income and interest payments (mortgages) because these are not part of the model (`fkfix` and `fkdeb`).

Labor income (`plinc`) includes income from employment and the labor component of mixed income.

Finally, I calculate averages income shares for each income percentile over the years 2002 - 2019, excluding the Great Recession years 2008 and 2009 that were characterized by strong declines in capital income.

C.8.3 Current Population Survey (CPS)

I use data on the CPS social and economic supplement via IPUMS (Flood et al., 2021), which has information on household income. I restrict my sample to the working-age population as described above and focus on employed workers. I restrict the sample to start in 2003 because of a substantial revision of industry codes between 2002 and 2003. I then sort workers into income bins based on their labor income and calculate the distribution across industries for each income bin, $\omega_i^l(\iota)$ with $\int_0^1 \omega_i^l(\iota) d\iota = 1$. I concord CPS industries to BEA IO industries as explained above. I adjust the weights $\omega_i^l(\iota)$ to ensure that all income percentiles have a mass of 0.01, $\sum_i n_i \omega_i^l(\iota) = 0.01$. I proceed similar to the adjustment of the basket weights for the CEX. In particular, I iteratate over

$$\omega_i^l(\iota) = \omega_i^l(\iota)\frac{0.01}{\sum_i n_i \omega_i^l(\iota)}$$
$$\omega_i^l(\iota) = \omega_i^l(\iota)\left(\omega_i^l(\iota)d\iota\right)^{-1}$$

until convergence.

D Model variations

Figures display the impulse responses to a permanent carbon tax shock for various model variations: (i) baseline, (ii) no utilization ($\delta'' \to \infty$), (iii) flexible utilization ($\delta'' = 0$), (iv) Cobb-Douglas production function, (v) lump-sum rebate, (vi) active monetary policy and (vii) damage from CO_2

Here, I briefly discuss how I implement these scenarios.

Adjusting the utilization cost In practice, I remove the utilization margin from the model in scenario (ii) and set $\delta'' = \frac{1}{1000}$ in scenario (iii).

Lump-sum transfers In this scenario, I assume that the carbon tax is rebated in a lump-sum way:

$$(1-\tau^C)p_{c_i,t}(\iota)c_{i,t}(\iota) = w_{i,t}(\iota)l_{i,t} + a_{k,t}(\iota)div_t + \tau_t^{rebate},$$

with the rebate determined by the government budget constraint:

$$\tau_t^{rebate} = \tau_t^{carb}carb_t + \tau^C\sum_{i=1}^{J}\left[\int_0^1 n_{i,t}(\iota)p_{c_i,t}(\iota)c_{i,t}(\iota)d\iota\right] - p_{G,t}G.$$

Active monetary policy In this scenario, I assume that the Taylor also responds to GDP:

$$i_t = \varphi i_{t-1} + (1-\varphi)\left(\bar{i} + \varphi_\pi \pi_t + \varphi_{GDP}\frac{GDP_t - \overline{GDP}}{\overline{GDP}}\right).$$

I set $\varphi_\pi = 1.5$ and $\varpi_{GDP} = 0.125$.

Cobb-Douglas production function In this scenario, I assume the following production function:

$$Y_{i,t} = \left\{ (1-\phi_i)^{\frac{1}{\xi}} \left[A_i K_{i,t}^{\alpha_i \chi_i} E_{i,t}^{\alpha_i(1-\chi_i)} L_{i,t}^{1-\alpha_i} \right]^{\frac{\xi-1}{\xi}} + \phi_i^{\frac{1}{\xi}} M_{i,t}^{\frac{\xi-1}{\xi}} \right\}^{\frac{\xi}{\xi-1}},$$

Dividends are composed of the returns on renting out capital net of investment, and the return on bonds:

$$div_t = \sum_{i=1}^{J} \{ r_{i,t} K_{i,t} - p_{I_i,t} I_{i,t} \} + B_t - B_{t-1} \frac{1+i_{t-1}}{\pi_t}.$$

Capital funds then choose $K_{i,t+1}$, $I_{i,t}$ and B_t to maximize the expected discounted sum of their dividends net of taxes,

$$\mathbb{E}_t \left(\sum_{s=0}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t} div_{t+s} \right)$$

subject to the law of motions for capital

$$K_{i,t+1} = (1-\delta_i) K_{i,t} + I_{i,t} \left(1 - f_{i,t}\right).$$

The first order conditions are (div_t, B_t, $K_{i,t+1}$ and $I_{i,t}$)

$$\begin{aligned}
\lambda_t &= \Phi_t \\
\lambda_t &= \beta(1+i_t) \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\pi_{t+1}} \right) \\
\nu_{i,t} \lambda_t &= \beta \mathbb{E}_t \left(r_{i,t+1} \lambda_{t+1} + (1-\delta_i) \nu_{i,t+1} \lambda_{t+1} \right) \\
p_{I_i,t} \lambda_t &= \lambda_t \nu_{i,t} (1 - f_{i,t}) - \lambda_t \nu_{i,t} \frac{I_{i,t}}{I_{i,t-1}} f'_{i,t} + \beta \mathbb{E}_t \left(\lambda_{t+1} \nu_{i,t+1} \left(\frac{I_{i,t+1}}{I_{i,t}} \right)^2 f'_{i,t+1} \right).
\end{aligned}$$

Using the first equation to replace λ yields

$$\begin{aligned}
\Phi_t &= \beta(1+i_t) \mathbb{E}_t \left(\frac{\Phi_{t+1}}{\pi_{t+1}} \right) \\
\nu_{i,t} \Phi_t &= \beta \mathbb{E}_t \left[\Phi_{t+1} \left(r_{i,t+1} + (1-\delta_i) \nu_{i,t+1} \right) \right] \\
p_{I_i,t} &= \nu_{i,t} \left[1 - f_{i,t} - \frac{I_{i,t}}{I_{i,t-1}} f'_{i,t} \right] + \beta \mathbb{E}_t \left(\frac{\Phi_{t+1}}{\Phi_t} \left(\frac{I_{i,t+1}}{I_{i,t}} \right)^2 f'_{i,t+1} \right).
\end{aligned}$$

The maximization problem of the first-stage producers is:

$$\max_{L_{i,t},K_{i,t},E_{i,t}M_{i,t}} \left\{ \left(p_{i,t} - \tau_{Y_i,t}\right) Y_{i,t} - w_{i,t}L_{i,t} - r_{i,t}K_{i,t} - p_{E_i,t}p_{M_i,t}M_{i,t} \right\}.$$

The first-order conditions are

$$r_{i,t}K_{i,t} = \left(p_{i,t} - \tau_{Y_i,t}\right)\alpha_i \left(\frac{(1-\phi_i)Y_{i,t}}{A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{(1-\alpha_i)\chi_i}L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{\alpha_i(1-\chi_i)}L_{i,t}^{1-\alpha_i}$$

$$w_{i,t}L_{i,t} = \left(p_{i,t} - \tau_{Y_i,t}\right)(1-\alpha_i) \left(\frac{(1-\phi_i)Y_{i,t}}{A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{(1-\alpha_i)\chi_i}L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{\alpha_i(1-\chi_i)}L_{i,t}^{1-\alpha_i}$$

$$\left(p_{i,t}^E + \tau_{E_i,t}\right)E_{i,t} = \left(p_{i,t} - \tau_{Y_i,t}\right)(1-\alpha_i) \left(\frac{(1-\phi_i)Y_{i,t}}{A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{(1-\alpha_i)\chi_i}L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_iK_{i,t}^{\alpha_i\chi_i}E_{i,t}^{\alpha_i(1-\chi_i)}L_{i,t}^{1-\alpha_i}$$

Taken together, this affects the set of log-linearized equilibrium conditions as follows: The following set of equations is removed:

1. Law of motion for number of machines
2. Machine Euler equation
3. Optimal choice of z
4. Optimal choice of x
5. Definition of energy E
6. Definition of I
7. Optimal choice of u

The following set of equations are changed

1. Dividends

$$div_t = \sum_{i=1}^{J} \left\{ r_{i,t}K_{i,t} - p_{I_i,t}I_{i,t} \right\}$$

$$\widetilde{div}_t = \sum_{i=1}^{J} \frac{r_iK_i}{div}\left(\tilde{r}_{i,t} + \tilde{K}_{i,t}\right) - \frac{p_{I_i}I_i}{div}\left(\tilde{p}_{I_i,t} + \tilde{I}_{i,t}\right)$$

2. Law of motion for capital (for each $i = 1, ..., J$)

$$K_{i,t+1} = (1-\delta_{i,t})K_{i,t} + I_{i,t}\left(1 - f\left(\frac{I_{i,t}}{I_{i,t-1}}\right)\right)$$
$$\tilde{K}_{i,t+1} = (1-\delta_i)\tilde{K}_{i,t} + \delta_i \tilde{I}_{i,t}$$

3. Production of sector goods (for each $i = 1, ..., J$)

$$Y_{i,t} = \left\{(1-\phi_i)^{\frac{1}{\xi}}\left[A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{\alpha_i(1-\chi_i)} L_{i,t}^{1-\alpha_i}\right]^{\frac{\xi-1}{\xi}} + \phi_i^{\frac{1}{\xi}} M_{i,t}^{\frac{\xi-1}{\xi}}\right\}^{\frac{\xi}{\xi-1}}$$
$$\widetilde{Y}_{i,t} = (1-\phi_i)\left[\alpha_i\left(\chi_i\tilde{K}_{i,t} + (1-\chi_i)\tilde{E}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right] + \phi_i\tilde{M}_{i,t}$$

4. Demand for capital (for each $i = 1, ..., J$)

$$r_{i,t}K_{i,t} = (p_{i,t} - \tau_{Y_i,t})\,\alpha_i\left(\frac{(1-\phi_i)Y_{i,t}}{A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{(1-\alpha_i)\chi_i} L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{\alpha_i(1-\chi_i)} L_{i,t}^{1-\alpha_i}$$
$$\tilde{r}_{i,t} + \tilde{K}_{i,t} = \frac{p_i\tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\tilde{Y}_{i,t} + \frac{\xi-1}{\xi}\left(\alpha_i\left(\chi_i\tilde{K}_{i,t} + (1-\chi_i)\tilde{E}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right)$$

5. Demand for labor (for each $i = 1, ..., J$)

$$w_{i,t}L_{i,t} = (p_{i,t} - \tau_{Y_i,t})\,(1-\alpha_i)\left(\frac{(1-\phi_i)Y_{i,t}}{A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{(1-\alpha_i)\chi_i} L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{\alpha_i(1-\chi_i)} L_{i,t}^{1-\alpha_i}$$
$$\tilde{w}_{i,t} + \tilde{L}_{i,t} = \frac{p_i\tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\tilde{Y}_{i,t} + \frac{\xi-1}{\xi}\left(\alpha_i\left(\chi_i\tilde{K}_{i,t} + (1-\chi_i)\tilde{E}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right)$$

The following set of equations is added

1. Optimal choice of investment

$$p_{I_i,t} = \nu_{i,t}\left[1 - f_{i,t} - \frac{I_{i,t}}{I_{i,t-1}}f'_{i,t}\right] + \beta\mathbb{E}_t\left(\frac{\Phi_{t+1}}{\Phi_t}\left(\frac{I_{i,t+1}}{I_{i,t}}\right)^2 f'_{i,t+1}\right)$$
$$\tilde{p}_{I_i,t} = \tilde{\nu}_{i,t} - f''\left[(1+\beta)\tilde{I}_{i,t} - \tilde{I}_{i,t-1} - \beta\tilde{I}_{i,t+1}\right]$$

2. Demand for energy (for each $i = 1, ..., J$)

$$\left(p^E_{i,t} + \tau_{E_i,t}\right) E_{i,t} = \left(p_{i,t} - \tau_{Y_i,t}\right)(1-\alpha_i)\left(\frac{(1-\phi_i)Y_{i,t}}{A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{(1-\alpha_i)\chi_i} L_{i,t}^{1-\alpha_i}}\right)^{\frac{1}{\xi}} A_i K_{i,t}^{\alpha_i\chi_i} E_{i,t}^{\alpha_i(1-\chi_i)} L_{i,t}^{1-\alpha_i}$$

$$\frac{p_i^E \tilde{p}^E_{i,t} + \Delta\tau_{E_i,t}}{p_i^E + \tau_{E_i}} + \tilde{E}_{i,t} = \frac{p_i \tilde{p}_{i,t} - \Delta\tau_{Y_i,t}}{p_i - \tau_{Y_i}} + \frac{1}{\xi}\tilde{Y}_{i,t} + \frac{\xi-1}{\xi}\left(\alpha_i\left(\chi_i\tilde{K}_{i,t} + (1-\chi_i)\tilde{E}_{i,t}\right) + (1-\alpha_i)\tilde{L}_{i,t}\right)$$

The steady state is equivalent to the steady state with putty-clay technology, but $K_{i,t}$ in the putty-clay model corresponds to $K_{i,t}^{\chi_i} E_{i,t}^{1-\chi_i}$ in the model with Cobb-Douglas aggregator. We also have that $\chi_i r_i K_i$ in the putty-clay model corresponds to $r_i K_i$ in the model with Cobb-Douglas aggregator.

Climate damage I introduce a climate damage function as in Golosov et al. (2014). In particular, the productivity parameter $A_{i,t}$ in the production of the sector goods now depends on the atmospheric carbon concentration:

$$A_{i,t} = e^{-\varsigma\Upsilon_t}\bar{A}_i,$$

Here, $\bar{A}_i$ being a constant productivity shifter in sector i. The function $e^{-\sigma\Upsilon_t}$ captures climate damages, where Υ_t is the atmospheric carbon concentration. This generates a feedback loop between carbon in the atmosphere and the economy. More production raises the level of carbon in the atmosphere, which, in turn, reduces output, e.g. through extreme weather events. The size of this output reduction is governed by the parameter σ as in Golosov et al. (2014).

The level of atmospheric carbon concentration follows the law of motion

$$\Upsilon_t = (1-\upsilon)\Upsilon_{t-1} + \upsilon_0 carb_t^{global},$$

where υ_0 is the share of carbon emissions that do not immediately leave the atmosphere and υ is the decay rate of existing emissions in the atmosphere. Global carbon emissions are composed of U.S. carbon emissions and non-U.S. carbon emissions. I assume that non-U.S. carbon emissions stay at their steady-state level.

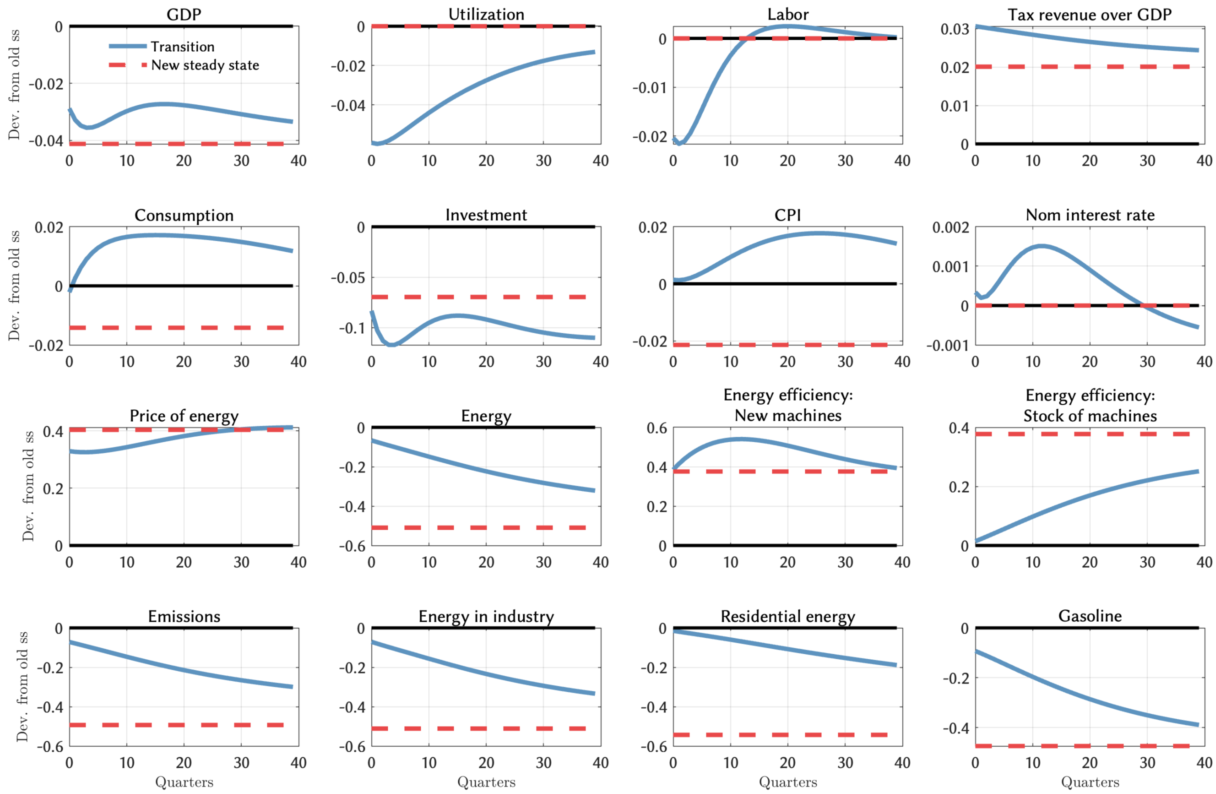
Following Golosov et al. (2014) and Känzig (2021), I set $1-\upsilon = 0.9994$, which implies a half-life of 300 years for carbon that reaches the atmosphere (Archer, 2005), $\upsilon_0 = \frac{0.5}{(1-\upsilon)^{120}} = 0.5359$, which is consistent with half of the emitted carbon being removed from the atmosphere after 30 years (or: 120 quarters) (IPCC, 2007), and $\varsigma = 5.3 \times 10^{-5}$ (Golosov et al., 2014). This damage function implies that doubling the carbon concentration in the atmosphere compared to 2012 levels lowers productivity by about 4.5%.[12,13] Finally, I set the share of U.S. emissions in total emissions to 15%, corresponding to the share observed in 2012.

Log-linearizing the two new equations yields

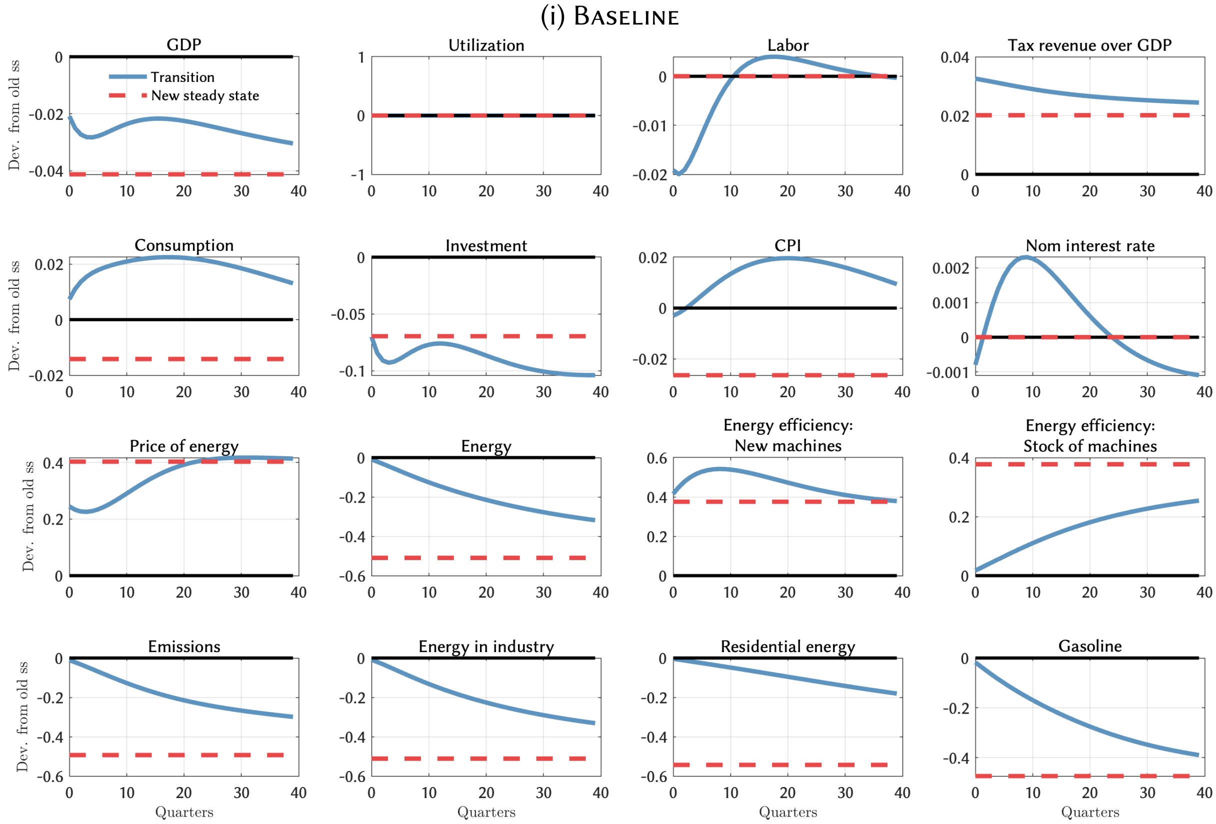
$$\tilde{A}_{i,t} = -\varsigma \Delta \Upsilon_t$$
$$\Delta \Upsilon_t = (1-\upsilon)\Delta \Upsilon_{t-1} + 0.15\upsilon_0 \Delta carb_t.$$

[12]Figure 1 in Golosov et al. (2014) indicates that doubling the carbon concentration from 850 gigatons of carbon (approximately the global level of concentration around 2012) to 1'700 gigatons of carbon reduces productivity by about 4.5%.

[13]The parameterization of the damage function depends on the units of carbon emissions. In the steady state, I set $carb = 1.407$ gigatons, which corresponds to the carbon emissions in the United States as of 2012 (or: 5.206 gigatons of carbon dioxide).

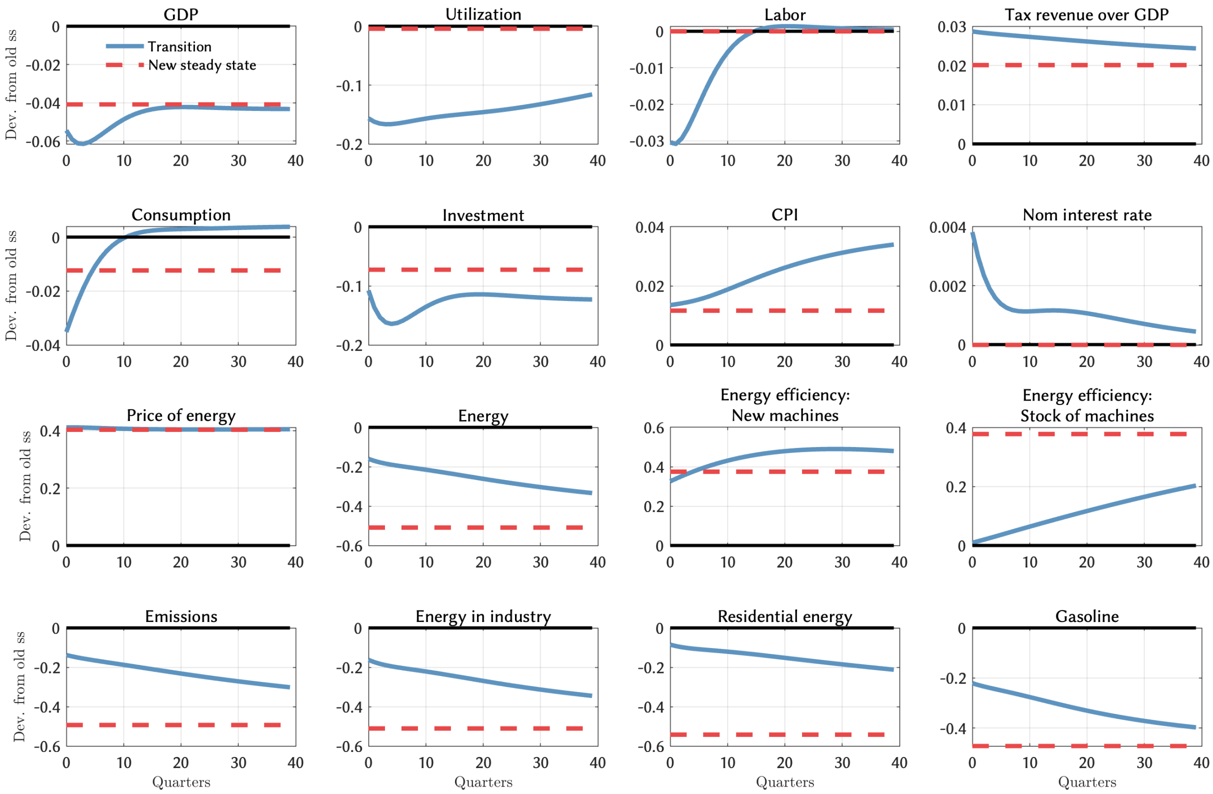


(i) Baseline

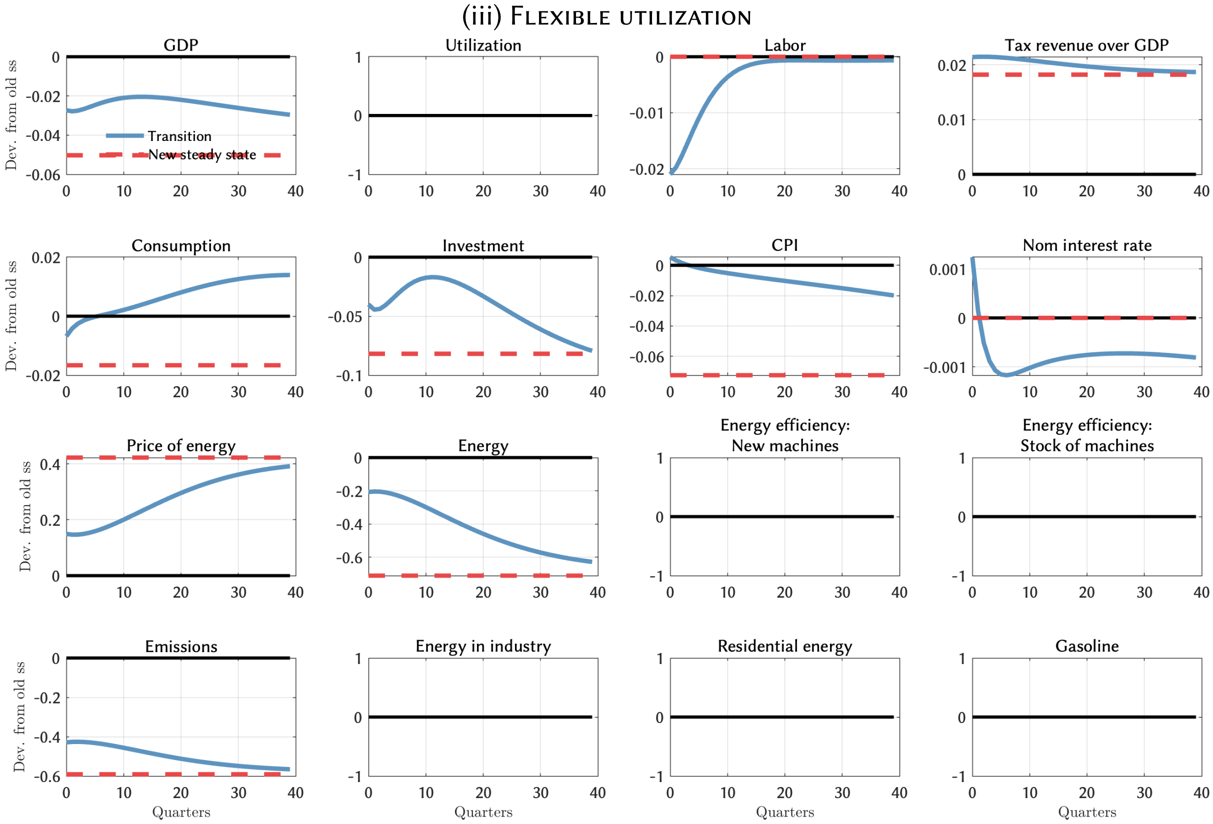


(ii) No utilization

Figure A1: Response to Carbon Tax: Model variations (1)

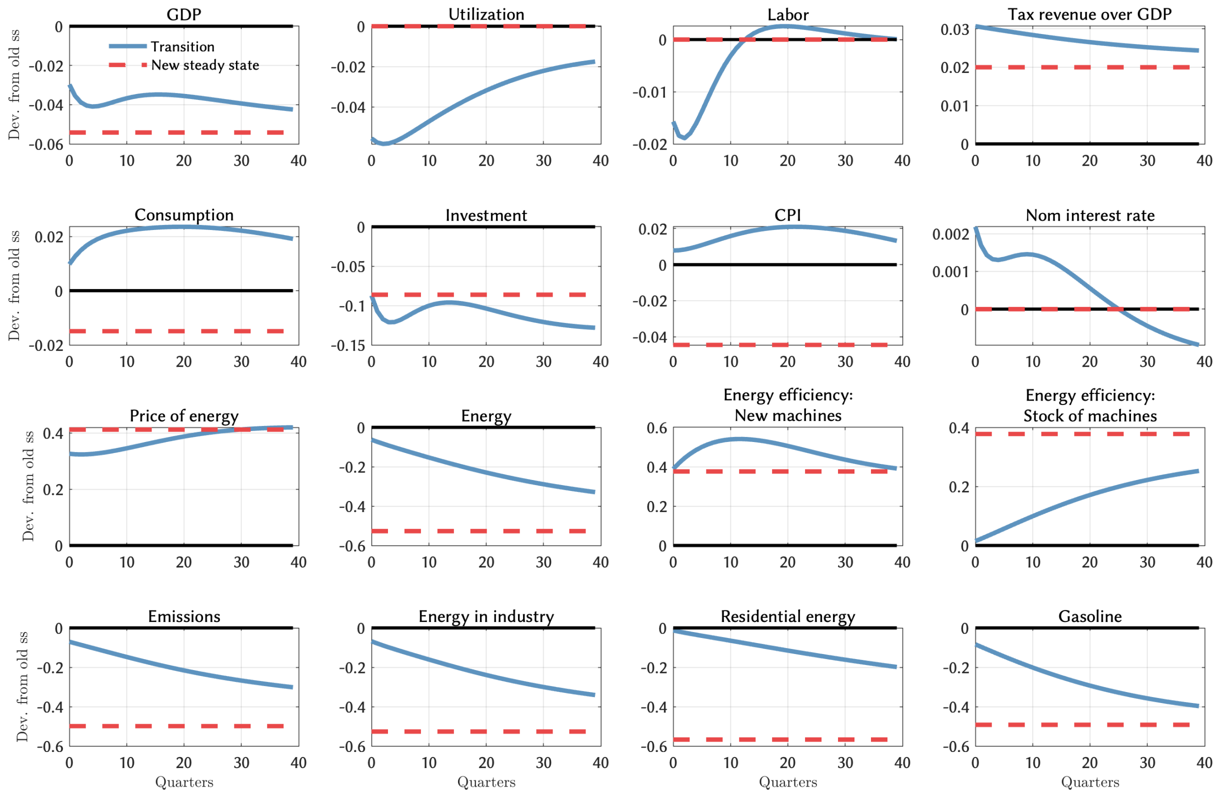


(iii) Flexible utilization

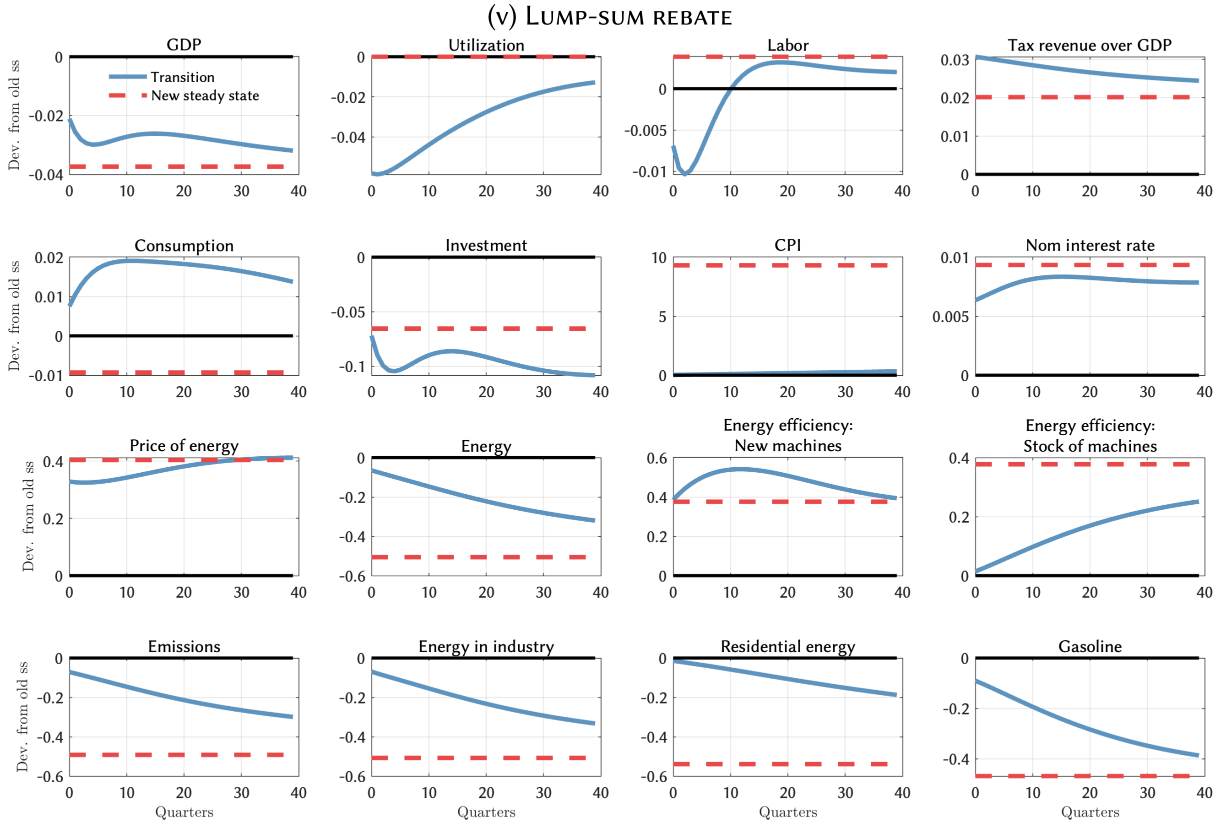


(iv) Cobb Douglas production function

Figure A1: Response to Carbon Tax: Model variations (2)

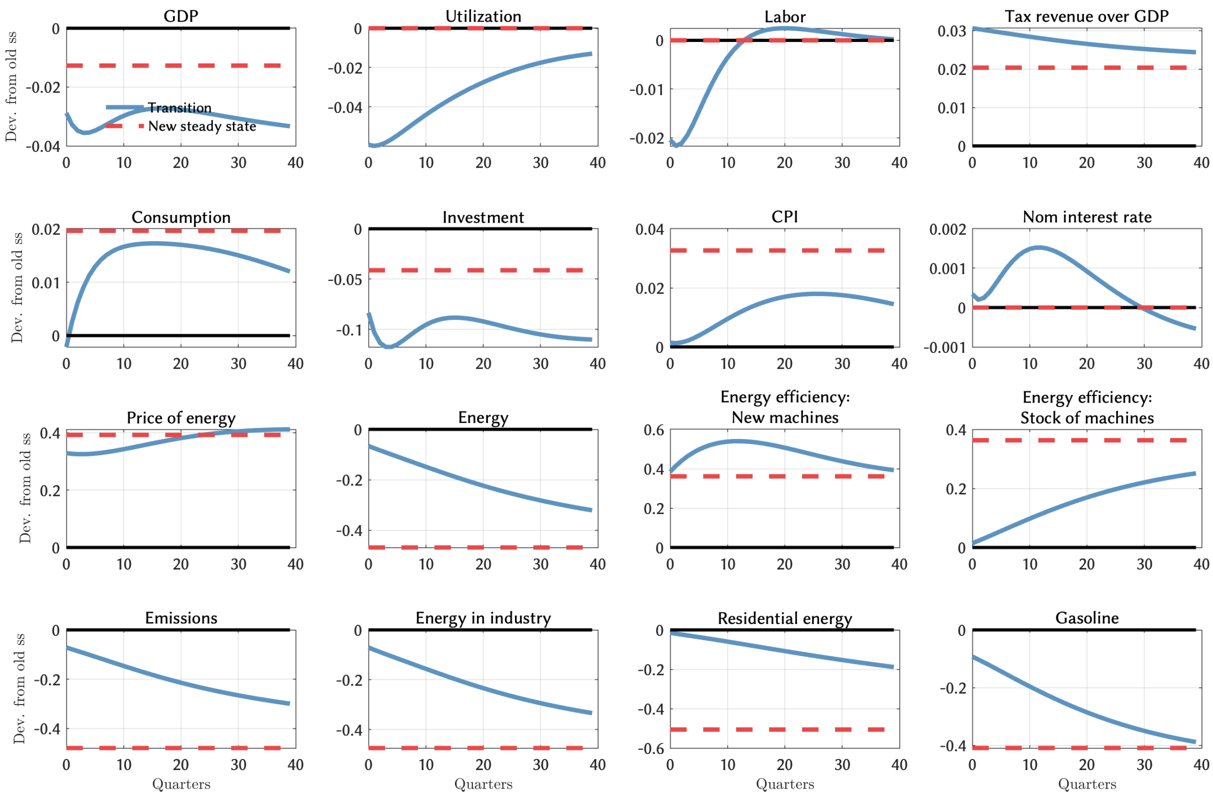


(v) Lump-sum rebate



(vi) Active monetary policy

Figure A1: Response to Carbon Tax: Model variations (3)



(vii) Damage from CO_2

Figure A1: Response to Carbon Tax: Model variations (4)

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